MAINZ TWO-LOOP METHODS *

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In the recent past procedures for the calculation of two-loop Feynman diagrams were developed at the University of Mainz. They solve self-energy and vertex diagrams involving arbitrary massive particles. The procedures are bound together to a program package called XLOOPS which is designed to treat Feynman diagrams up to the two-loop level in a completely automatic way.

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1. Two-loop integrals

The aim of our two-loop routines is not only to solve special mass cases or kinematical regions. They also supply the general case where all internal masses are different and the external momenta can be completely arbitrary. Full tensor structure and not only the scalar case is supported. As a consequence these general routines cannot return analytic results for all topologies. Therefore we adopt the following strategy:

1. separate the divergent parts (in any case analytically calculable)
2. find a two-fold integral representation (in $D = 4$)
3. integrate numerically the two-fold integral

In the following we comment these three items.

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1.1. UV divergent integrals

For the separation of divergences it is necessary to find a convenient subtraction term. If a two-loop integral is multiplied by a term like the one in brackets

$$\int d^D l d^D k \, \frac{l_{\mu_1} \cdots l_{\mu_n}}{\mathcal{P}_1(l)\mathcal{P}_2(l)\mathcal{P}_3(l+k)\mathcal{P}_4(k)\mathcal{P}_5(k)}$$
$$\to \int d^D l d^D k \, \frac{l_{\mu_1} \cdots l_{\mu_n}}{\mathcal{P}_1(l)\mathcal{P}_2(l)\mathcal{P}_3(l+k)\mathcal{P}_4(k)\mathcal{P}_5(k)} \left(1 - \frac{\mathcal{P}_1(l)\mathcal{P}_2(l)\mathcal{P}_3(l+k)}{l^4(l+k)^2}\right)^m$$

it can be shown that the degree of divergence is decreased by m [1]. To be specific, in the case of a logarithmic divergent integral which corresponds to the case $m = 1$ one gets a difference of two terms which turns out to be convergent. To recover the original integral it is necessary to add the subtraction term again.

$$\underbrace{\int d^D l d^D k \frac{l_\mu l_\nu}{\mathcal{P}_1(l)\mathcal{P}_2(l)\mathcal{P}_3(l+k)\mathcal{P}_4(k)\mathcal{P}_5(k)} - \int d^D l d^D k \frac{l_\mu l_\nu}{l^4(l+k)^2\mathcal{P}_4(k)\mathcal{P}_5(k)}}_{\text{convergent}}$$
$$+ \underbrace{\int d^D l d^D k \, \frac{l_\mu l_\nu}{l^4(l+k)^2\mathcal{P}_4(k)\mathcal{P}_5(k)}}_{\text{divergent}} .$$

The first line which denotes the convergent part is treated in $D = 4$ and will be reduced to a two-fold integral representation which is solved numerically. The second line contains the divergent part which can be solved analytically in $D \neq 4$. This procedure can be extended to any UV divergent diagram.

1.2. Integration strategy

We integrate directly in momentum space. Therefore we split the momentum components in parallel and orthogonal space variables. The definition of this splitting is simple: The parallel space describes the sub-space spanned by the external momenta, whereas the orthogonal space is the orthogonal complement of the parallel space.

In the two-point case only one external momentum is present, therefore the parallel space is one-dimensional. The external momentum q and the loop momenta l and k can be written as

$$q^\mu = (q_0, 0, 0, 0) , \qquad l^\mu = (l_0, \vec{l}_\perp) , \qquad k^\mu = (k_0, \vec{k}_\perp) ,$$

so that the parallel space is represented by the 0-component of the four-vectors. The integration measure simplifies in the following sense:

$$\int d^4l \int d^4k = 8\pi^2 \int_{-\infty}^{\infty} dl_0 dk_0 \int_0^{\infty} l_\perp^2 dl_\perp k_\perp^2 dk_\perp \int_0^{\pi} \sin\vartheta d\vartheta ,$$

ϑ is the angle between $\vec{l}_\perp$ and $\vec{k}_\perp$.

In the case of three-point functions the parallel space is two-dimensional. If one chooses the 0- and 1-components as parallel space variables one gets

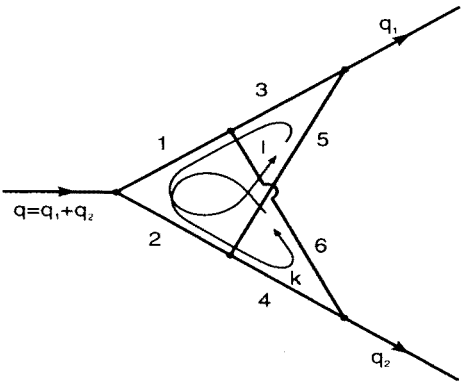
$$q_1^\mu = (e_1, q_z, 0, 0) , \quad q_2^\mu = (e_2, -q_z, 0, 0) , \quad q^\mu = q_1^\mu + q_2^\mu = (e_1 + e_2, 0, 0, 0) ,$$

$$l^\mu = (l_0, l_1, \vec{l}_\perp) , \qquad k^\mu = (k_0, k_1, \vec{k}_\perp) ,$$

$$\int d^4l \int d^4k = 4\pi \int_{-\infty}^{\infty} dl_0 dk_0 dl_1 dk_1 \int_0^{\infty} l_\perp dl_\perp k_\perp dk_\perp \int_0^{\pi} d\vartheta .$$

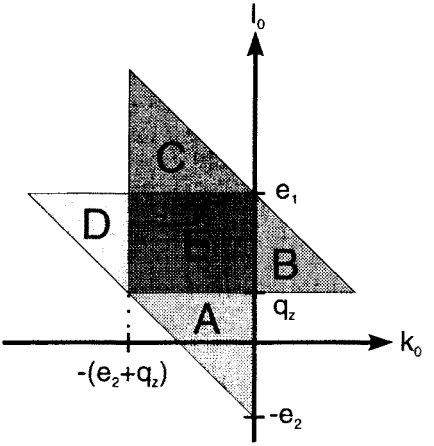
In any case the l_0 and k_0 integrations are left for numerical evaluation. All other integrations are performed analytically [2, 3].

To go into more detail we concentrate on the non-planar three-point function which is known to be the most challenging three-point topology. The integrand consists of six propagators:



$$\begin{aligned}
\mathcal{P}_1 &= (l + k - q_1)^2 - m_1^2 + i\varrho , \\
\mathcal{P}_2 &= (l + k + q_2)^2 - m_2^2 + i\varrho , \\
\mathcal{P}_3 &= (l - q_1)^2 - m_3^2 + i\varrho , \\
\mathcal{P}_4 &= (k + q_2)^2 - m_4^2 + i\varrho , \\
\mathcal{P}_5 &= l^2 - m_5^2 + i\varrho , \\
\mathcal{P}_6 &= k^2 - m_6^2 + i\varrho .
\end{aligned}$$

After having performed first the angular integration the residue theorem can be applied twice – for l_1 and k_1. One gets five different integration areas A–E in the l_0-k_0-plane:



With the help of Euler's change of variables [4] two more integrations ($l_\perp$ and $k_\perp$) can be performed analytically. The integrand involves then the dilogarithm and Clausen's function [5].

1.3. Numerical evaluation

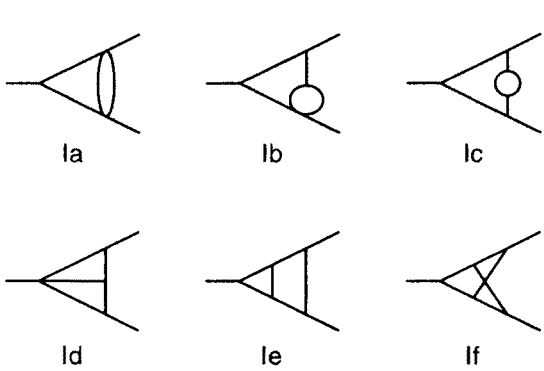
The two-fold integral representations can be directly taken for numerical evaluation. As far as there were independent data available we found perfect agreement [5–7].

1.4. Status

Up to now we solved all two-point topologies for scalar and tensor evaluations. In the three-point case all topologies are solved for the scalar case. At present we work on the extension to three-point tensor and four-point scalar integrals.

2. Decay of a heavy Higgs

As an application the decay of a Higgs into a Z or $W^\pm$ pair was calculated [8]. In the limit where the masses of the vector bosons are small compared to the energy the so-called equivalence theorem holds [9]. This means that the vector bosons $W^\pm, Z$ are replaced by the corresponding Goldstone modes $\phi^\pm, \phi^0$. As a consequence only scalar integrals occur. The only remaining mass scale is the Higgs mass. Six proper three-point topologies had to be calculated:



The result confirmed an earlier calculation by Ghinculov [10].

3. XLOOPS

The program package XLOOPS automates the integration methods described above. The package consists of the following parts:

- Input via Xwindows interface:
  In a window the user selects the topology which shall be calculated. A Feynman diagram pops up in which the particle names have to be inserted.
- Processing with MAPLE:
  The selected diagram is evaluated. The necessary steps for reducing the numerator (SU(N) algebra, Dirac matrices) are performed by routines written for MAPLE [11]. The result is expressed in terms of one- and two-loop integrals.
- Evaluation of one-loop integrals:
  One-loop one-, two- and three-point integrals are calculated analytically or numerically to any tensor degree using MAPLE. This part was already released separately [12].
- Evaluation of two-loop integrals:
  All two-loop two-point topologies including tensor integrals are supported by the MAPLE routines. For those topologies where no analytic result is known, XLOOPS creates either an analytic two-fold integral representation or integrates numerically with the help of VEGAS [13] using C++. The other topologies can be calculated analytically or numerically like the one-loop integrals.

At present a demo version covering one-loop diagrams is available on the net. Version 1.0 which contains all features described above is in preparation [14]. In any case the current version is available on

`http://dipmza.physik.uni-mainz.de/~franzkowski/xloops.html`

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