

ELECTROWEAK EVENT GENERATORS FOR LEP2 AND THE LINEAR COLLIDER*

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I discuss the state of the art and outline direction for research in event generation for electroweak physics at LEP2 and e^+e^- Linear Colliders.

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1. Introduction

With the start of experimentation at LEP1, a new era of precision measurements in high energy physics had begun. The potential of such machines calls for a new level of sophistication in the theoretical tools. The ability to measure parameters of the electroweak standard model at the permille level requires precise theoretical predictions of observable quantities.

Today's sophisticated multi purpose detectors have a complicated geometry that can not be described by simple acceptance and efficiency functions, but require detailed simulation instead. For this purpose, Monte Carlo event generators and sophisticated integration algorithms are indispensable.

The theoretical issues involved in the calculation of differential cross sections have been covered by other speakers at this conference [1] and need not be repeated here. I will therefore concentrate on the more technical issues involved in turning these theoretical predictions into numbers that can be compared with experimental results.

I will start in Section 2 with reviewing the motivation for Monte Carlo methods and I will describe the methods in use today in Section 3, including Quasi Monte Carlo in Subsection 3.3. I will continue with radiative corrections in Section 4 and discuss specific problems with automated calculations

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of radiative corrections in Subsection 4.1. After reviewing the status of event generators for LEP2 in Section 5, I describe the new challenges at a Linear Collider in Section 6, including a discussion of beamstrahlung in Section 6.2. After a brief look into the future in Section 7 and a few bits and pieces in Section 8, I will conclude.

2. The need for Monte Carlo

The increasing energy of e^+e^- storage rings and Linear Colliders opens up channels with many particles in the final state, that are calculable in the electroweak standard model. Continuing the precision tests of the standard model at higher energies requires a precise calculation of the cross sections for these channels. In addition, these channels provide backgrounds for new particle searches, which calls for theoretical control of these cross sections as well.

Numerical integration estimates the integral of a function f by a weighted sum of the functions' values sampled on a set $\{x\}$ of points

$$\int dx f(x) = \langle f \rangle = \sum_{x \in \{x\}} w(x) f(x). \quad (1)$$

Monte Carlo integration is the special case of (1), in which $\{x\}$ is a sample of pseudo random numbers and $w(x) = 1/|\{x\}|$, where $|\{x\}|$ denotes the number of points in $\{x\}$. This case is of special importance, because the law of large numbers guarantees the convergence of (1) for any f , only the scale of the error depends on f . In addition, the statistical nature of the method allows reliable error estimates by repeating the evaluation with varying random sets and checking the Gaussian distribution of the results.

Monte Carlo is the only known method that allows the integration of differential cross sections for arbitrary final states with arbitrary phase space cuts. Monte Carlo event generation is required for realistic simulations of the acceptance and efficiency of modern detectors with their complicated geometry. Thus, Monte Carlo is *the* universal tool for turning *actions* into measurable *numbers*.

2.1. Discovery vs. precision physics

Discovery physics and precision physics call for different approaches to event generation. Precision tests of the standard model require complete calculations, that include radiative corrections and "background" diagrams. Since the number of diagrams for many particle final states is large (up to 144 for four fermions and thousands for six fermions), such calculations

are technically demanding. Fortunately, the parameter space is restricted and allows a comprehensive analysis.

On the other hand, the search for physics beyond the standard model in general does not need radiative corrections other than the initial state radiation of photons. This makes the calculations technically easier, but the need to cover a lot of models with a vast parameter space creates other problems. Since tree level calculations folded with initial state structure functions are usually sufficient, computer aided approaches can help to cover the models and their parameter spaces.

3. Twelve roads from actions to answers

There are twelve different methods for getting answers (cross sections and event rates) from actions (the definition of the physical model in perturbative calculations). These can be factorized in three roads from actions to amplitudes and four ways from amplitudes to answers.

3.1. The three roads from actions to amplitudes

The traditional textbook approach to deriving amplitudes from actions are *manual calculations*, which can be aided by computer algebra tools. The time-honored method of calculating the squared amplitudes directly using trace techniques is no longer practical for today's multi particle final states and has generally been replaced by helicity amplitude methods. Manual calculation has the disadvantage of consuming a lot of valuable physicist's time, but can provide insights which are hidden by the other approaches discussed below.

The currently most successful and increasingly popular technique is to use a well tested *library of basic helicity amplitudes* for the building blocks of Feynman diagrams and to construct the complete amplitude directly in the program in the form of function calls. A possible disadvantage is that the differential cross section is nowhere available as a formula, but the value of such a formula is limited anyway, since they can hardly be printed on a single page anymore.

Automatic calculations are a further step in the same direction. The Feynman rules (or equivalent prescriptions) are no longer applied manually, but encoded algorithmically. This method will become more and more important in the future, but more work is needed for the automated construction of efficient event generators and for the implementation of radiative corrections (see also Section 4.1).

3.2. The four roads from amplitudes to answers

Analytic and semi-analytic calculations have the potential to provide the most accurate results. Unfortunately, fully analytic calculations are not feasible for more than three particles in the final state, that is for most of the interesting physics at LEP2 and the Linear Collider. Still, semi-analytic calculations, in which some integrations are performed analytically and the remaining integrations are done numerically are possible for certain simple cuts and provide useful benchmarks with an accuracy unmatched by the other methods.

The most flexible approach is *Monte Carlo integration*, which converges under very general conditions as

$$\frac{\delta \langle f \rangle}{\langle f \rangle} \propto \frac{1}{\sqrt{N}} \sqrt{\langle f^2 \rangle - \langle f \rangle^2}, \quad (2)$$

with N denoting the number of function evaluations. As long as the integration variables can be transformed such that the integrand f does not fluctuate too wildly, this method can be implemented easily and efficiently. The quadratic increase on computing resource consumption with the precision puts a practical limit on the attainable precision, however.

Event Generation is a special case of Monte Carlo integration in which an ensemble of kinematical configurations is generated that is distributed according to the differential cross section. Such ensembles allow the straightforward simulation of the non-perturbative physics of the fragmentation and hadronization of strongly interacting particles and of the detector. If the weight function is bounded, rejection methods can be used to trivially turn a Monte Carlo code into an event generator. Hand tuning is required, however, to make the code efficient.

The rate of convergence in (2) is guaranteed by the law of large numbers and can only be improved if we turn away from pseudo random numbers and switch to *deterministic integration* and *Quasi Monte Carlo*. Empirical evidence from known quasi random number sequences suggests that

$$\frac{\delta \langle f \rangle}{\langle f \rangle} \propto \frac{\ln^n N}{N} \quad (3)$$

(c.f. (8), below) is possible by using point sets that are more uniform than both random sets and hyper-cubic lattices in high dimensions. Quasi Monte Carlo integration has been applied successfully to four fermion production at LEP2 [2]. Nevertheless, more work is needed, because there are too few theorems for realistic function spaces. Phase spaces of varying dimensionality, as in branching algorithms, have not been studied at all yet.

3.3. Quasi random numbers

It is intuitively clear that the best convergence will be gotten by using the most uniform point sets. It is less obvious how such uniform point sets look like and how to construct them. Let us therefore take a closer look at such point sets. More detail and references can be found in [3].

Let us assume for simplicity that we can map the integration region to the n -dimensional unit hypercube: $I = [0, 1]^{\times n}$. Using the characteristic function

$$\chi(y|x) = \prod_{\mu=1}^n \Theta(y^\mu - x^\mu), \quad (4)$$

we can define the *local discrepancy* of a point set $\{x\}$ for each $y \in I$:

$$g(y|\{x\}) = \frac{1}{|\{x\}|} \sum_{x \in \{x\}} \chi(y|x) - \prod_{\mu=1}^n y^\mu. \quad (5)$$

Obviously, the local discrepancy measures how uniformly $\{x\}$ covers the hypercube. We can now define various versions of the *global discrepancy* of the point set $\{x\}$:

$$D_m(\{x\}) = \int dy (g(y|\{x\}))^m, \quad (6)$$

$$D_\infty(\{x\}) = \sup_y |g(y|\{x\})|. \quad (7)$$

A lower bound for D_∞ can be established

$$D_\infty(\{x\}) \geq C(n) \frac{\ln^{(n-1)/2} |\{x\}|}{|\{x\}|}, \quad (8)$$

where $C(n)$ depends only on the dimension of the hypercube and is in particular independent of $\{x\}$. The global discrepancy D_2 of a regular hypercubic lattice and a random point set can easily be calculated

$$D_2(\{\text{lattice}\}) = \frac{n}{4 \cdot 3^n} \frac{1}{|\{\text{lattice}\}|^{2/n}} + \dots, \quad (9)$$

$$D_2(\{\text{random}\}) = \left(\frac{1}{2^n} - \frac{1}{3^n} \right) \frac{1}{|\{\text{random}\}|}. \quad (10)$$

This shows that hypercubic lattices are *less* uniform than random point sets for more than two dimensions. This result is not surprising, however, because we know from experience that Monte Carlo works better than uniform integration formulae in higher dimensions.

Unfortunately, while it is intuitively obvious that discrepancy and integration error are related, it is much harder to derive mathematically rigorous results for realistic integrands, in particular for those with singularities and discontinuities. This is the price to pay for potentially faster convergence and more research is needed.

4. Radiative corrections

At high energies, electromagnetic radiative corrections are enhanced by large logarithms $\ln(s/m^2)$ and precision calculations of the hard cross section are useless if the radiative corrections are not under control.

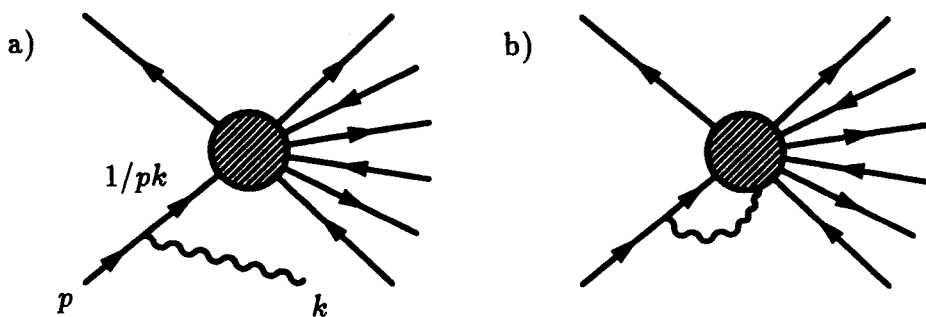


Fig. 1. Cancellations of real (a) and virtual (b) singularities.

Perturbative calculations at fixed order order in perturbation theory are not sufficient, because amplitudes for the emission of photons (Fig. 1a) have $1/\omega$ -soft and $1/\theta$ -collinear singularities. These singularities are cancelled

$$\frac{1}{\omega} \rightarrow \left(\frac{1}{\omega}\right)_+ = \lim_{\epsilon \rightarrow 0} \left[\frac{1}{\omega} \Theta(\omega - \epsilon) \Big|_{\text{real}} + \ln \epsilon \cdot \delta(\omega) \Big|_{\text{virtual}} \right] \quad (11)$$

in the cross section by opposite-sign singularities in the virtual corrections (Fig. 1b) to the indistinguishable process without emission. While sufficiently integrated cross sections remain positive, the differential cross section without emission must be negative for the cancellation to take effect. Only after resummation of the perturbation series, these singularities become integrable like $1/\omega^{1-\beta}$ and result in strictly positive differential cross sections. The combination of today's collider energy and of the energy- and angular resolution of today's detectors have made this resummation a practical necessity.

The most popular approach today is the summation of the leading logarithmic initial state contributions through structure functions

$$\frac{d\sigma}{d\Omega}(s) = \int dx_+ dx_- \left| \frac{\partial \Omega'}{\partial \Omega}(x_+, x_-) \right| D(x_+, s) D(x_-, s) \frac{d\sigma^0}{d\Omega'}(s'). \quad (12)$$

The structure functions D can be obtained from finite order perturbation theory with resummed (“exponentiated”) singularities, from Yennie-Frautschi-Suura summation, from the DGLAP evolution equations or from parton shower algorithms.

A drawback of this approach is that the transversal momentum carried away by the photons is ambiguous in leading logarithmic approximation. For practical purposes, most authors use the $1/(p \cdot k)$ behaviour suggested by the most singular pieces. This problem is worrisome for experiments that measure the amount of initial state radiation by tagging photons in order to select an event sample with effective center-of-mass energy below the nominal machine energy. Theoretically most satisfactory are algorithms that match the hard matrix element outside the singular region to the resummed cross section in the singular region.

Another universal correction appears in the W^+W^- -production close to threshold at LEP2. The long range Coulomb interaction of slow W 's gives rise to a substantial change of the cross section. This correction is easy to implement and available in most computer codes.

The issue of QCD corrections was the subject of some debates during the 1995 LEP2 workshop. The “naive” QCD correction

$$\left(1 + \frac{\alpha_{\text{QCD}}}{\pi} \right) \cdot \Gamma(W/Z \rightarrow q\bar{q}') \quad (13)$$

is obtained by integrating over the gluon in the decay of a vector boson into quark pairs. The effect on the width and the branching ratios is substantial and must be included in some way in the calculations. At the same time, this naive approach is theoretically questionable, because the unavoidable experimental cuts will invalidate the fully inclusive calculation. Since there are again cancellations between real and virtual contributions at work, this is not a priori a numerically small effect. Furthermore, the resonant diagrams do not form a gauge invariant subset and a more complete analysis is called for.

Such studies are under way [4]. But until their results are included in the standard computer codes, the naive QCD corrections are available in most programs because the effect is substantial.

Weak radiative corrections are certainly more interesting from the physics perspective, but they will hardly be measurable at LEP2. Their calculation

is very tough, because effective Born approximations similar to those successful at LEP1, do *not* exist for differential cross sections in W^+W^- pair production. Since complete calculations do not yet exist except for the on-shell production of stable W 's, gauge dependent *ad-hoc* resummations of the propagators are needed.

For practical purposes at LEP2, the gauge dependence problem has been solved by the fermion loop scheme [5], but more complete investigations and implementations remain desirable, because stronger effects are expected at the Linear Collider.

4.1. Radiative corrections in automatic calculations

Radiative corrections in gauge theories pose a particular problem for automatic calculations (see [6] for a general review of automatic calculations). Formally [7], loops can be incorporated in existing tree level generators by replacing the classical action $S(A, \psi, \bar{\psi})$ with the generating functional $\Gamma(A, \psi, \bar{\psi})$ of one particle irreducible diagrams. Unfortunately, this approach works only for loops consisting of heavy particles.

As mentioned above, the amplitude for the emission of massless particles (Fig. 1a) contains infrared and collinear singularities, which are cancelled in the cross section by the virtual contributions (Fig. 1b) for degenerate final states. The problem is that both contributions have to be regularized and that the appropriate regularization of the loop in diagram 1b depends on which external leg it is attached to. Therefore it is not possible to describe the loop diagrams by a position independent generating functional $\Gamma(A, \psi, \bar{\psi})$. More sophisticated algorithms which analyze the topology of the graphs are needed. A new approach to this problem for QCD jet cross sections has been presented at this conference [8].

Until this problem is solved, automatic calculations are useful for the emission of hard photons which are separated from all charged particles. Integrated cross sections can be calculated in the leading logarithmic approximation only by folding hard cross sections with structure functions (12).

5. Status of event generators for LEP2

The event generators available for WW -physics at LEP2 have been described in [9]. An updated summary of the properties of the available codes is presented in Table I.

TABLE I

Properties of the available computer codes for W^+W^- -physics at LEP2. In the ‘Type’ column ‘MC’ stands for Monte Carlo integration, ‘EG’ for event generation, ‘SA’ for semi-analytic and ‘Int.’ for deterministic integration. Implemented subsets of diagrams are denoted by ‘CC03’ for doubly resonant W^+W^- , ‘CC11’ for singly resonant W^+W^- , ‘CC20’ for final states including electrons or positrons, ‘NCnn’ for various neutral current diagrams and ‘NCC’ for various neutral and charged current diagrams. The implementation of initial state radiation is denoted by ‘SF’ for structure functions, ‘FF’ for flux functions, ‘PS’ for parton showers, ‘YFS’ for Yennie-Frautschi-Suura, ‘ME’ for matrix element and ‘ $\mathcal{O}(\alpha)$ ’ for one photon bremsstrahlung. The ‘NQCD’ column applies to naive inclusive QCD corrections. For fermion masses, ‘ \pm ’ denotes massless matrix elements with massive kinematics. See [9] for references and more details.

Program	Type	Diag.	ISR	FSR	NQCD	Clb.	AC	m_f	Jets
ALPHA	MC	all	—	—	—	—	—	+	—
COMPHEP	EG	all	SF	—	—	—	—	+	—
ERATO	MC	all	SF	—	+	—	+	—	+
EXCALIBUR	MC	all	SF	—	+	+	+	—	—
GENTLE	SA	NCC	SF/FF	—	+	+	+	PS	—
GRC4F	EG	all	SF/PS	PS	+	+	+	+	+
HIGGSPV	EG	NCnn	SF	—	—	n/a	—	\pm	—
KORALW	EG	all	YFS	SF	+	+	—	\pm	+
LEPWW (+)	EG	CC03	$\mathcal{O}(\alpha)$	+	+	—	+	—	+
LPWW02	EG	CC03	SF	SF	+	+	—	\pm	+
PYTHIA	EG	CC03	SF+PS	PS	+	+	—	\pm	+
WOPPER	EG	CC11	PS	—	+	+	+	\pm	+
WPHACT	EG	all	SF	—	+	+	+	+	+
WTO	Int.	NCC	SF	—	+	+	—	—	—
WWF	EG	CC20	SF+ME	ME	+	+	+	+	+
WWGENPV	EG	CC20	SF _{p_T}	SF _{p_T}	+	+	—	\pm	+

In [9], the predictions for LEP2 have been compared in detail. In a “tuned” comparison with a prescribed calculational scheme, the implementations have been tested and the modern dedicated $e^+e^- \rightarrow 4f$ codes agreed at a level far better than required at LEP2. In a second “unleashed” comparison, the different theoretical approaches of the codes have been compared and the results have shown that the predictions are under control at the level required for LEP2.

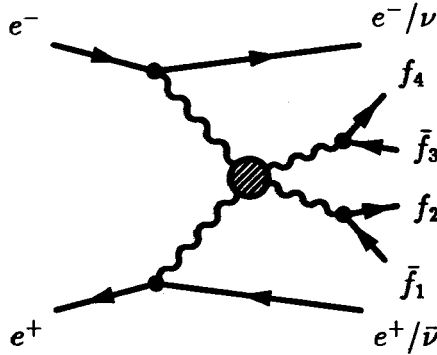


Fig. 2. $VV \rightarrow VV$ scattering.

6. Towards the linear collider

We have to wait another decade until a Linear Collider will be available for experimentation. Nevertheless, the design of the interaction region and of detectors needs firm predictions for the expected physics [10] *today*.

Regarding two gauge boson physics (which for precision measurements is really four fermion physics), most of the LEP2 Monte Carlos in Table I can simply be run at higher energies, provided that they can be interfaced to beamstrahlung codes, as discussed in Subsection 6.2 below.

It is unreasonable to expect deviations of the three gauge boson couplings from the Standard Model values (*anomalous couplings*) that will be measurable at LEP2 (though this assertion has to be checked anyway). This will change at the Linear Collider, because reasonable values of $\mathcal{O}(1/(16\pi^2))$ become accessible and event generators have to support anomalous couplings. Fortunately, the majority of the event generators supports them today and preliminary cross checks have been satisfactory.

As mentioned above, weak radiative corrections will also become relevant. Here more work is still needed.

6.1. $e^+e^- \rightarrow 6f + n\gamma$

At a 500 GeV Linear Collider interesting $e^+e^- \rightarrow 6f$ channels open up. For the first time, precision measurements of $t\bar{t}$ production at a e^+e^- -collider will be possible. An event generator at the signal diagram level, including bound state effects, is available. The study of background diagrams will be necessary and a cross check of the generator will be desirable. At high energies, vector boson scattering (*c.f.* Fig. 2), becomes an interesting $e^+e^- \rightarrow 6f$ channel as a probe of the electroweak symmetry breaking sector. Work in this direction has started this year.

The general case of $e^+e^- \rightarrow 6f$ is a formidable computational challenge, because a huge number of diagrams has to be calculated. At the moment four approaches are being discussed:

1. start from on-shell $e^+e^- \rightarrow VVV$, $VV \rightarrow VV$ and $e^+e^- \rightarrow t\bar{t}$ Monte Carlos and add $V \rightarrow f\bar{f}'$ decays in a second step. This approach is probably not useful for obtaining a complete calculation in the end, but it can provide reasonable descriptions of the most important, resonant contributions soon.
2. extent the **EXCALIBUR** [9] algorithm for massless four fermion production to six fermions. It is still unclear how to deal with quark masses efficiently in this approach.
3. start with complete calculations of specific final states for which the number of diagrams is manageable. Work in this direction has started and is showing first promising results [11].
4. perform a fully computerized calculation. Work in this direction using the **GRACE** [9] system has started as well.

These projects will certainly keep the aficionados of standard model calculations entertained for some years.

6.2. Beamstrahlung

The experimental environment at the Linear Collider causes a new phenomenon that event generators have to deal with. The largest (two fermion) standard model cross sections are of the order

$$\frac{4\pi}{3} \frac{\alpha^2}{s} \approx \frac{100\text{fb}}{(\sqrt{s}/\text{TeV})^2}, \quad (14)$$

and four and six fermion cross sections are suppressed by further factors of α . It is therefore clear that interesting physics at the Linear Collider will require large luminosities of the order of

$$10^{34} \text{cm}^{-2} \text{sec}^{-1} \approx 100\text{fb}^{-1} v^{-1}, \quad (15)$$

where $v = 10^7 \text{sec} \approx \text{year}/\pi$ corresponds to one “effective” year of running. These luminosities can only be achieved with extremely dense bunches of particles.

Such bunches experience a strong electromagnetic interaction with non-trivial consequences. A desired effect of this interaction for oppositely charged particles is the “pinch effect”, which increases the luminosity by further collimating the bunches through the reciprocal attraction. But the same physics gives rise to undesirable side effects as well. The deflection of

the particles in the bunches causes them to loose several percent of their energy as synchrotron radiation (*beamstrahlung*). Therefore, the colliding particles will have a non-trivial energy spectrum. This spectrum has to be included in the event generators for realistic predictions. At the same time, the radiated photons take part in γe^\pm - and $\gamma\gamma$ -collisions and therefore their energy spectrum has to be known as well.

Quantitatively, the effect of the beamstrahlung is of the same order as the effect of initial state radiation. But unlike ordinary bremsstrahlung, beamstrahlung can not be treated by ordinary perturbation theory, because the underlying physics is the interaction of a particle with *all* the particles in the colliding bunch. Approximate analytical treatments of this collective effect exist, but full simulations [12] show that they are not adequate.

TABLE II

Three prototypical linear collider designs at 500 GeV: **SBAND** and **TESLA** are DESY's room temperature and superconducting options, **XBAND** is for KEK's and SLAC's plans.

	SBAND	TESLA	XBAND
E/GeV	250	250	250
$N_{\text{particles}}/10^{10}$	1.1	3.63	0.65
$\epsilon_x/10^{-6}\text{mrad}$	5	14	5
$\epsilon_y/10^{-6}\text{mrad}$	0.25	0.25	0.08
β_x^*/mm	10.98	24.95	8.00
β_y^*/mm	0.45	0.70	0.13
σ_x/nm	335	845	286
σ_y/nm	15.1	18.9	4.52
$\sigma_z/\mu\text{m}$	300	700	100
f_{rep}	50	5	180
n_{bunch}	333	1135	90

The full simulations consume too much computer time and memory for directly interfacing them to particle physics Monte Carlos. Also, the input parameters collected in Table II not familiar to most particle physicists. The pragmatical solution of this problem is to describe the result of the simulations by a simple *ansatz*, that captures the essential features.

The approximate solutions motivate a factorized *ansatz*, which should positive and have *integrable* singularities at the endpoints $x_{e^\pm} \rightarrow 1$ and $x_\gamma \rightarrow 0$

$$D_{p_1p_2}(x_1,x_2,s) = d_{p_1}(x_1)d_{p_2}(x_2) \, , \tag{16}$$

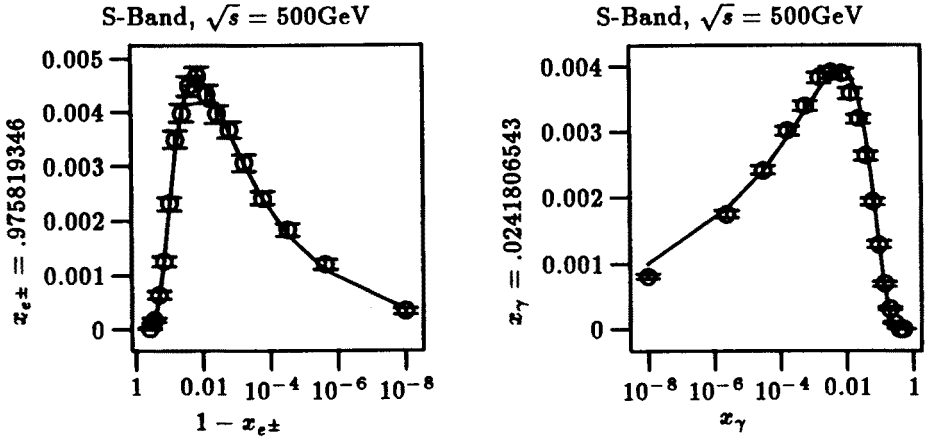


Fig. 3. Quality of the fits for the TESLA design at 500 GeV and 1 TeV.

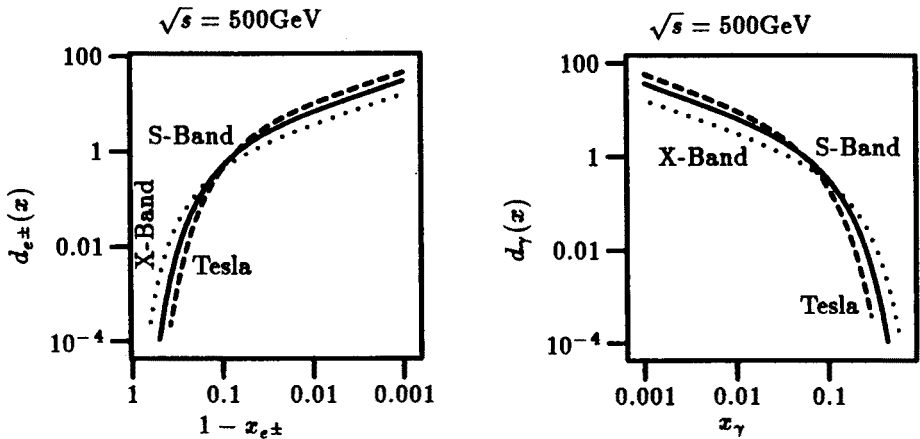


Fig. 4. Results of simulations and fits for the three prototypical linear collider designs at 500 GeV.

$$d_{e\pm}(x) = a_0\delta(1-x) + a_1x^{a_2}(1-x)^{a_3}, \quad (17)$$

$$d_\gamma(x) = a_4x^{a_5}(1-x)^{a_6}. \quad (18)$$

Fig. 3 shows that this ansatz works surprisingly well. It has therefore been made available as distribution functions and non-uniform random number generators in the *circe* library [13].

The resulting distributions are displayed in Fig. 4, which highlights the substantial differences in the designs.

7. Monte Carlo futures

The craft of Monte Carlo event generation for precision physics at high energy e^+e^- -colliders is practised successfully by many physicists today, as witnessed by the many high quality computer codes available for LEP2. Still there are areas where technical progress is desirable.

The most promising direction are *generator generators*, where amplitudes and event generation or integration algorithms are constructed algorithmically by a computer program from a Lagrangian for each desired final state. The systems available today have come a long way [6], but are still far from perfect.

While the combinatorics of the generation of the amplitude is implemented in several systems, most of these implementations do not scale well to final states with more than four particles. The number of diagrams will scale like $n!$ with the number n of particles. For small n , the resource consumption of this factorial growth can be matched by faster hardware in the future, but this is not obvious for the numerical stability of the code. Gauge theories have a good high energy behaviour because of strong cancellations among individual diagrams. The numerical problems resulting from these cancellations have to be controlled. This is only possible with more sophisticated algorithms that regroup and partially factorize the amplitudes. Such algorithms are non trivial and will need further research. Here progress is particularly important for radiative corrections.

A second area of research are adaptive methods for event generation and integration. Except for special cases, where the singularities in the amplitudes are known in advance, today's systems still need human intervention for finding optimal phase space variables that minimize the sampling error. Here more sophisticated algorithms are needed as well.

Technical advances in this direction will allow to efficiently produce reliable Monte Carlo codes for precision physics at high energy Linear Colliders. Hopefully, this will also propel the state of the art of event generation for discovery physics to the same level.

Another promising direction for research is the investigation of Quasi Monte Carlo methods for event generation and integration. Here more experience with realistic applications is needed.

There are however some areas where progress will be slow. In particular the interface of the perturbative precision calculations with the non perturbative simulation of strongly interacting final states is poorly understood. A lot of the sophistication in the calculation of interfering contributions is lost when the perturbative amplitude is matched to the classical simulation of the fragmentation and hadronization process. At LEP2, the problems of color reconnection and Bose-Einstein correlations are the limiting factor for

the W -mass measurements. When LEP2 data become plentiful, they might help to improve phenomenological models and give a better control of the systematical error.

8. Bits and pieces

Before concluding, I want to take this opportunity to advertize a welcome addition to the family of pseudo random number generators available for Monte Carlo integration and event generation.

It is well known that bad random number generators will spoil any simulation, while (some) good random number generators can consume macroscopic fractions of the total computer time.

Recently, Donald Knuth made errata [14] for his textbook available, which contain a gem of a portable generator. It is an extremely fast implementation of a lagged Fibonacci generator $X_j = (X_{j-100} - X_{j-37}) \bmod 2^{30}$, which is portable even for systems which, like **FORTRAN**, offer no unsigned 32-bit arithmetic. The generator passes all statistical tests, even (with a slight speed penalty) the birthday spacings test. But the most interesting property is the innovative initialization algorithm for which one can *prove* that it will generate $2^{30} - 2$ *statistically independent* sequences from a simple 30-bit integer seed. For most other generators, the statistical independence for different seeds is much harder to control.

The practical consequence is that Knuth's new algorithm allows parallel execution and reliable Monte Carlo estimation of errors, without having to worry about the statistical independence of the generated samples.

9. Conclusions

The Monte Carlo codes for precision physics at LEP2 physics are in excellent shape and the craft of their construction is well understood. Tools for the design of the Linear Collider are already available, but there is work left to do to make them as comprehensive as the tools available for LEP2.

Technical progress is still needed for streamlining the computational approaches that will eventually allow us to investigate the nature of electroweak symmetry breaking at the Linear Collider.

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