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PRECISE CALCULATIONS OF THE BHABHA PROCESS * **

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We present the current status of the YFS Monte Carlo approach to high precision calculations of both small and large angle Bhabha scattering at high energies. We discuss comparisons which we have made with the available literature where possible, at both LEP1/SLC energies and at LEP2 energies. Throughout the discussion, we pay attention to the luminosity determination at LEP1/SLC for the small angle process and the analysis of wide angle Bhabha Z physics parameters and possible new LEP2 physics backgrounds for the wide angle process. In this way, the role of high precision Bhabha scattering calculations in exploration of the Standard Model and its possible extensions is made more manifest.

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1. Introduction

As the final LEP1 data analysis and the initial stages of LEP2 materialize while the SLD prepares for the beginning of what may be its final phase, the subject of precision calculations of small- (low) and large- (wide)

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angle Bhabha scattering continues to be interesting, particularly from the standpoint of realizing such calculations via a Monte Carlo event generator that would allow a comparison between theory and experiment at the level of events in the presence of *arbitrary* detector cuts. In what follows, we shall present the current status of the YFS exponentiated Monte Carlo approach to such calculations.

Specifically, the process we discuss is $e^+ + e^- \to e^+ + e^- + n(\gamma)$ at both small and large angles, where by *small* angles we intend the regime $\theta_{e^\pm} \lesssim 150$ mrad and by *large* angles we intend the regime $\theta_{e^\pm} \gtrsim 150$ mrad if $\theta_{e^\pm}$ are the respective $e^\pm$ c.m.s. scattering angles. We recall that the former regime (SABH) is relevant to the precision LEP1/SLC luminosity measurements [1] while the latter (LABH) is relevant to the precision measurements of $\Gamma_{e\bar{e}}$ and related quantities at LEP1/SLC and to new particle searches at LEP1 and LEP2 [2]. In the luminosity regime, as explained in Ref. [1], the current experimental error on $\sigma_{\mathcal{L}}$ is now 0.05%; we will show in what follows that, while the current theoretical error is 0.11%, the regime of a $\sim$ 0.05% theoretical error is not far away. In the wide-angle regime, at LEP1/SLC the current error [3] on $\sigma_{e\bar{e}}$ is $\sim$ 0.34% and there is no published Monte Carlo event generator that quotes an error below 1.0% at the time of writing; there are semi-analytical programs [4–8], which quote $\sim$ 0.5% for symmetrically cut cross sections. In what follows, we will present results showing that three of us (S.J., W.P. and B.F.L.W.) have recently achieved 0.3% and 1.5%, precision, respectively, on arbitrarily cut wide-angle Bhabha cross sections at the Z peak and at LEP2 energies.

The basic theoretical approach that we use is the YFS Monte Carlo approach developed by two of us in Refs. [9–12], in which the Yennie–Frautschi–Suura (YFS) [13] exponentiated cross sections of the respective processes are realized by Monte Carlo event generator methods. Here, we discuss the application of this approach to the process $e^+(p_1) + e^-(q_1) \longrightarrow e^+(p_2) + e^-(q_2) + \gamma_1(k_1) + \ldots + \gamma_n(k_n)$. We refer the reader to Refs. [9, 10, 13–17] for the precise definition of the respective YFS exponentiated cross section and associated infrared functions, and to Refs. [10, 14, 16, 18, 19] for the explicit realization of this cross section by Monte Carlo methods, in the event generators BHLUMI and BHWIDE for low- and wide-scattering angles, respectively. In the next sections we present the current status of these realizations.

Our discussion is organized as follows. In the next section, we discuss our recent results on the exact calculation of the $\mathcal{O}(\alpha)$ virtual correction to the single hard bremsstrahlung process in SABH. In Section 3, we review our recent analysis of the missing $\mathcal{O}(\alpha^3 L^3)$ leading log (LL) contribution to the bremsstrahlung corrections in BHLUMI 4.04 [19]. In Section 4, we present our recent results on the soft pairs effects in the luminosity cross section.

Section 5 contains our state-of-the-art analysis of the $\delta_{\gamma Z}$, the contribution of the γ–Z interference effect to the luminosity cross section. This leads us naturally to a discussion of the present theoretical error on the LEP/SLC luminosity cross section in Section 6. In Section 7, we turn to the wide-angle Bhabha scattering and present the current state of the art from the standpoint of Monte Carlo event generators. Section 8 contains our conclusions and outlook.

2. Exact results on the $\mathcal{O}(\alpha)$ virtual correction to $e\bar{e} \to e\bar{e} + \gamma$

In this section, we present the results of four of us (S.J., M.M., B.F.L.W. and S.A.Y.) on the exact virtual one-loop correction to the hard bremsstrahlung process in low-angle Bhabha scattering [20, 21]. These results are needed to complete the exact treatment of the $\mathcal{O}(\alpha^2)$ bremsstrahlung correction to the luminosity cross section $\sigma_{\mathcal{L}}$ prediction in BHLUMI 4.xx. We stress that, prior to our work, no published, exact, completely differential result for this correction was available.

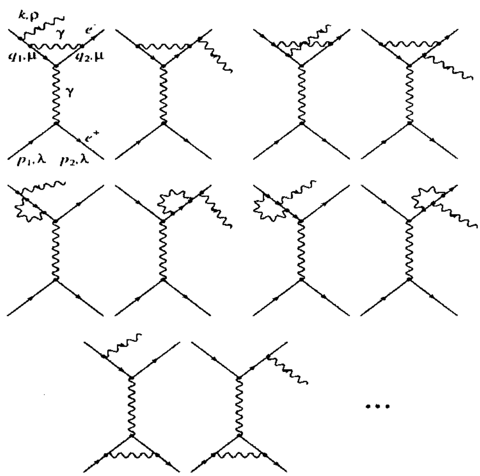

Fig. 1. $\mathcal{O}(\alpha^2)$ single bremsstrahlung correction in $e^+e^- \to e^+e^-$ at low-angles. Only electron line emission graphs are shown.

Specifically, the diagrams of interest to us are illustrated in Fig. 1, along with the relevant kinematics. The algebraic manipulation program FORM [22] is used in evaluating our one-loop virtual corrections. The resulting on-shell renormalized amplitude is then further reduced using the methods of Xu *et al.* [23] (Kleiss and Stirling [24] have introduced a similar simplification of the original CALKUL spinor methods [25]). *At no point are inherently 4-dimensional identities used in the presence of our 't Hooft–Veltman [26] n-dimensional regularization for our one-loop integrals.* The detailed formulas for our result for the amplitude in Fig. 1 can be found in Ref. [21]. Here, we will illustrate the effect of the corresponding $\mathcal{O}(\alpha^2)$ correction to the prediction of BHLUMI 4.xx for $\sigma_{\mathcal{L}}$ in the realistic ALEPH SICAL luminometer acceptance.

Turning to this illustration, we show in Fig. 2 the comparison, for the NW ALEPH SICAL luminometer acceptance, of the pure second-order contribution to the hard-photon residual $\bar{\beta}_1$ (see Refs. [10, 13] for the precise definition of $\bar{\beta}_n$) represented by the difference $\bar{\beta}_1^{(2)} - \bar{\beta}_1^{(1)}$ obtained from experimental version of BHLUMI 4.xx, with the exact results in Ref. [21].

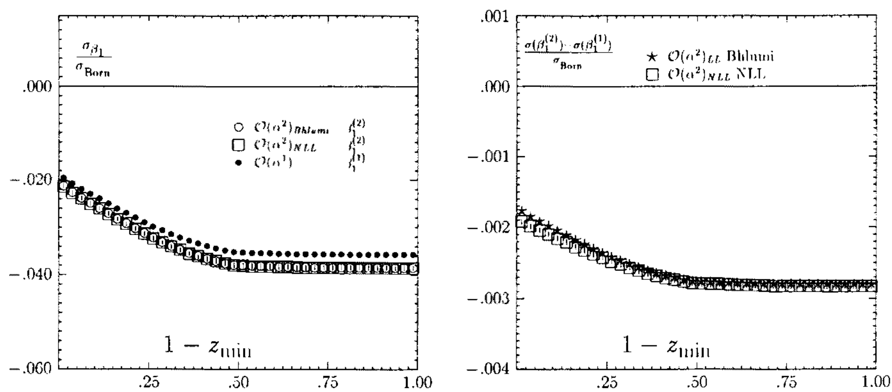

Fig. 2. Monte Carlo result for entire $\bar{\beta}_1^{(r)}, r = 1, 2$ and for the pure second-order contribution $\bar{\beta}_1^{(2)} - \bar{\beta}_1^{(1)}$ for the SICAL Wide-Narrow event selection. Mass terms are not included. All results are divided by the Narrow–Narrow Born cross section. Energy cut variable $z_{\min}$ is defined in Fig. 2 of Ref. [18].

In the first plot in the figure, we also plot the two results for $\bar{\beta}_1^{(2)}$ from BHLUMI 4.xx and from our exact prediction in Ref. [21], respectively denoted by $\mathcal{O}(\alpha^2)_{\text{BHLUMI}}$, $\mathcal{O}(\alpha^2)_{\text{NLL}}$, as well as the $\mathcal{O}(\alpha^1)$ result $\bar{\beta}_1^{(1)}$ in BHLUMI 4.xx separately. What we see is that the size of the missing part of the $\bar{\beta}_1^{(2)}$ in the BHLUMI 4.xx is below 2×10^{-4} of the respective Born cross section in the massless limit, consistent with our earlier naive expectations [18]. We are currently implementing an analogous comparison for the massive case, which would then allow us to arrive at the final precision on the bremsstrahlung correction for BHLUMI 4.xx [27]. We should note that the authors of Refs. [28, 29] have also recently obtained results for the next-to-leading-log contribution to the SABH process. We will compare our results with their final formulas elsewhere [27].

3. $\mathcal{O}(\alpha^3 L^3)$ bremsstrahlung: What does BHLUMI miss?

In this section, we present the results of two of us (S.J. and B.F.L.W.) for the calculation done in Ref. [30] to quantify the part of the $\mathcal{O}(\alpha^3 L^3)$ bremsstrahlung that is not generated by the YFS exponentiation in BHLUMI 4.xx. We recall that the bremsstrahlung matrix element in BHLUMI 4.xx is exact to $\mathcal{O}(\alpha^2 L^2)$; its $\mathcal{O}(\alpha^3 L^3)$ content is entirely due to YFS exponentiation.

Our starting point is a new version [30] of the LL MC event generator LUMLOG, which is available as an option in BHLUMI 4.xx [19] and which contains the entire LL series from both initial and final state bremsstrahlung in the SABH process. By truncating the BHLUMI 4.xx bremsstrahlung corrections to $\mathcal{O}(\alpha^3 L^3)$ and comparing them with the similar $\mathcal{O}(\alpha^3 L^3)$ content of LUMLOG, we have arrived at the results shown in Fig. 3. We refer

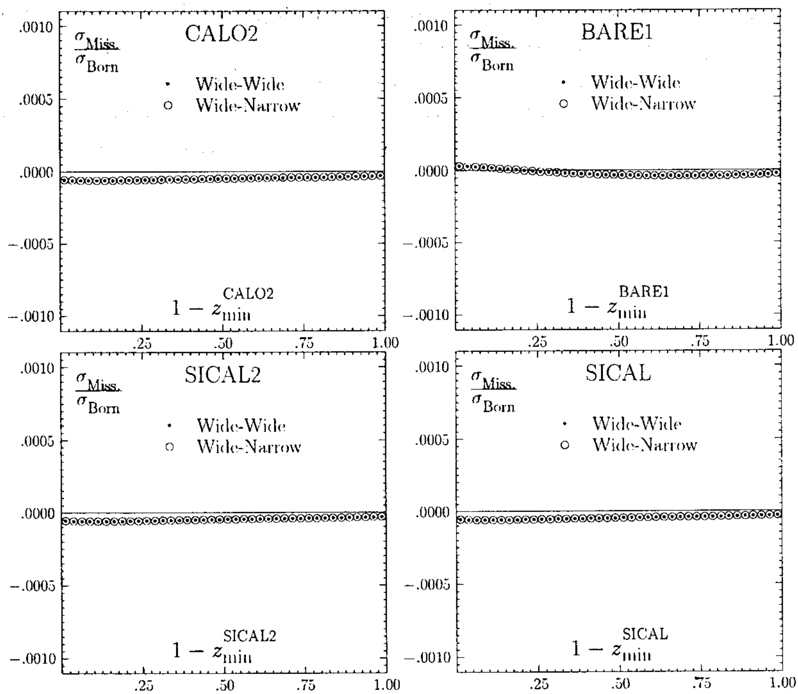

Fig. 3. Missing photonic leading-log $\mathcal{O}(\alpha^3)$ contribution $\sigma_{\rm Miss.}$ in the second-order BHLUMI 4.x with exponentiation calculated for various types of event selections using LUMLOG 4.x.

to Ref. [30] for the detailed formulas of this truncation of BHLUMI 4.xx, which involves a non-trivial integration over the transverse degrees of freedom, and of the new LUMLOG matrix element as well. Numerous cross checks, also described in Ref. [30], of the new LUMLOG matrix element with semi-analytical expectations have been made[1].

What we see is that the missing third-order LL contribution in the BHLUMI 4.xx bremsstrahlung correction is well below 0.01%, in four different types of event selection as defined in Refs. [32], for example. We shall discuss at present the implications of this result on the total precision of BHLUMI 4.xx.

4. Soft pairs contribution: YFS Monte Carlo approach

In this section, we present the results of three of us (S.J., M.S. and B.F.L.W.) [33] on the soft pairs effects in $\sigma_{\mathcal{L}}$. We have used the soft pairs extensions of the YFS infrared functions, which we derived in Ref. [34], to implement the soft pairs effects via the respective YFS exponentiation in a Monte Carlo event generator, in complete analogy with our YFS exponentiation of soft photons in our Monte Carlo event generator BHLUMI. In this way, multiple soft pairs effects in $\sigma_{\mathcal{L}}$, as illustrated in Fig. 4, are realized on an event-by-event basis. This is done in version 2.30 of BHLUMI [33], and the details can be found in Ref. [33]. We now illustrate some recent results obtained with this event generator.

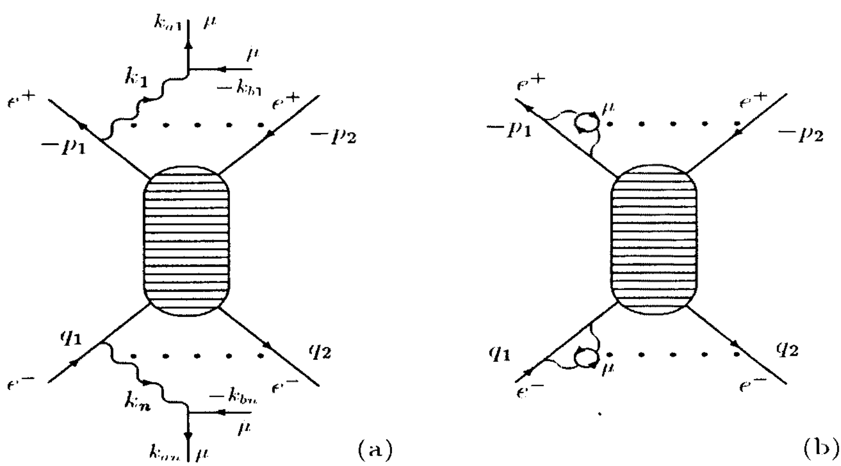

Fig. 4. (a) The class of real emission diagrams resummed in our soft pairs YFS exponentiation calculus; the kinematics is as indicated in the figure; (b) Typical virtual corrections, which are included in our soft pairs exponentiation calculus.

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[1] See also Ref. [31].

More precisely, we have made a number of cross checks and applications of BHLUMI 2.30. Here, we illustrate its use in the computation of the soft pairs effects in $\sigma_{\mathcal{L}}$, for the specific case of the ALEPH LCAL acceptance, for the interesting NW or M case as defined in Ref. [33] and references therein. We show in Fig. 5 the result of our BHLUMI 2.30 simulation for two different clustering realizations of the LCAL acceptance (event selection $\equiv$ ES) for the size of the soft pairs effects, in units of the respective Born cross section. Also shown for reference is the result of our semi-analytical calculation in Ref. [35]. We see that, at the typical experimental cut value $z = 0.5$ of the energy variable $z = 1 - s'/s$, which corresponds to the energy fraction of the outgoing clusters into the LCAL acceptance, the soft pairs effects are -1.3×10^{-4}. From this and related results in Ref. [33], we are able to corroborate the size of the soft pairs effects contribution to the precision tags of BHLUMI 2.02, 4.04 in Refs. [14, 19]. We may further stress that the detector simulation of the effects is now possible as needed.

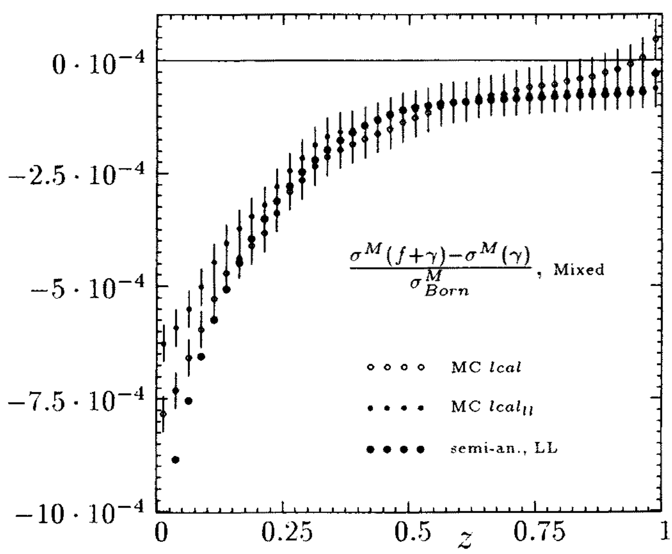

Fig. 5. The difference $[\sigma^M_{f+\gamma} - \sigma^M_\gamma]/\sigma^M_B$ for the mixed LCAL acceptance described in Ref. [33]. Here, $\sigma^M_{f+\gamma}, \sigma^M_\gamma$ are respectively the cross sections with soft pairs and photons and with only multiple photons included, and σ^M_B is the corresponding Born cross section. The small solid dots (open circles) correspond to calculations done with BHLUMI 2.30 for the *lcal* ($lcal_{II}$) versions of the LCAL trigger and the large solid dots correspond to calculations done with our semi-analytical LL formulas, as described in the text.

5. $\delta_{\gamma Z}$ interference effect: YFS exponentiated $\mathcal{O}(\alpha)$ implementation

In this section we present the results of three of us (S.J., W.P. and B.F.L.W.) [36] on the precision calculation of the important Z contribution to $\sigma_{\mathcal{L}}$, which is manifested via the interference between the t-channel photon and the s-channel Z exchange graphs. We denote this contribution to $\sigma_{\mathcal{L}}$ as $\delta_{\gamma Z}$ in units of the Born cross section. Prior to our work, the most precise computation of this effect in $\sigma_{\mathcal{L}}$ was that in Refs. [37, 38], which was based on the exact $\mathcal{O}(\alpha)$ calculation of the event generator BABAMC of Ref. [39]. Our calculation gives the YFS exponentiated exact $\mathcal{O}(\alpha)$ result for this effect as it is realized by Monte Carlo event generator methods.

Our starting points are the exact $\mathcal{O}(\alpha)$ results for the virtual and real bremsstrahlung corrections to $\delta_{\gamma Z}$. For the former, we use two different results already in the literature, that in BABAMC by the authors of Ref. [39] and that in the semi-analytical program ALIBABA by the authors of Refs. [4, 5]; the latter is the more up to date of the two and we use it as our default value; the former is useful for technical precision cross checks with the results of Refs. [37, 38], for example. Concerning the real bremsstrahlung correction to $\delta_{\gamma Z}$, we used the spinor methods of Ref. [23] to calculate it exactly; the resulting formulas are given in Ref. [36]. These exact $\mathcal{O}(\alpha)$ results have then been used to compute the $\mathcal{O}(\alpha)$ corrections to the hard photon residuals $\bar{\beta}_{0,1}$ for BHLUMI 4.xx due to $\delta_{\gamma Z}$, yielding the exact $\mathcal{O}(\alpha)$ YFS exponentiated calculation of the γZ interference effect in $\sigma_{\mathcal{L}}$ on an event-by-event basis [36]. We now turn to some illustrations of this calculation of $\delta_{\gamma Z}$.

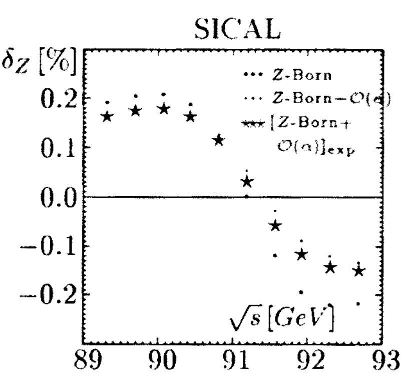

Fig. 6. Z exchange contributions to low-angle Bhabha scattering for the SICAL acceptance with the energy cut $U_{\rm min} = 0.43$ taken from BHLUMI, for $M_Z = 91.187\,{\rm GeV}$, $\Gamma_Z = 2.490\,{\rm GeV}$ and $\sin^2\theta_W = 0.2319$. The results are presented as a fraction (in %) of the Born cross section for the pure t-channel γ exchange.

In Fig. 6 we show the results of BHLUMI 4.xx for $\delta_{\gamma Z}$ for the ALEPH SICAL ES; the analogous results for the ALEPH LCAL ES can be found in Ref. [36]. Corresponding results hold for the other LEP Collaborations' luminometers in the respective angular ranges. In the figure, three sets of predictions are plotted, corresponding to the Born, Born+$\mathcal{O}(\alpha)$, and the *exact* $\mathcal{O}(\alpha)$ YFS exponentiated [Born+$\mathcal{O}(\alpha)$]$_{\rm exp}$ results. The difference $\delta''_{\gamma Z}$ between the [Born+$\mathcal{O}(\alpha)$]$_{\rm exp}$ and Born+$\mathcal{O}(\alpha)$ results is at most 0.03%. We have shown in Ref. [36], using the wealth of cross checks on the results for $\delta_{\gamma Z}$ available in BHLUMI 4.xx, that the realistic total error on the accuracy of our new exact $\mathcal{O}(\alpha)$ YFS exponentiated result for it is 0.015%. This is a significant improvement over the previous the best calculation, that of Refs. [37,38], which quotes 0.042% for this error.

6. Total theoretical precision: current status

In this section, we put together the results discussed above, augmented by some further cross checks with the results of the SABSPV, BHAGEN95 and NLLBHA groups [32] to arrive at the current theoretical precision of the BHLUMI 4.xx prediction for the LEP luminosity cross section in the ALEPH SICAL-type acceptance. We stress that this is the total precision of $\sigma_{\mathcal{L}}$ for the SICAL-type ES as calculated by BHLUMI 4.xx [19].

Specifically, in Ref. [32], we made several cross checks of the formulas in BHLUMI 4.xx against various results now available from the SABSPV, BHAGEN95 and NLLBHA groups. An example of such a cross check on the pure bremsstrahlung contribution to $\sigma_{\mathcal{L}}$ is shown in Fig. 7; there, not only are the results for these collaborations shown, but also the result of our OLDBIS+LUMLOG solution described in Ref. [19], for example, using the BHLUMI 4.03 result as the reference cross section[2].

The box indicates the 1 per mille level in the comparisons, which are shown for several ESs, ranging from the more unrealistic BARE1 to the realistic SICAL2 ES, as described in Ref. [32] for example. On the basis of these comparisons, we have reduced the pure bremsstrahlung error on the calculation done by us in BHLUMI 4.xx to 0.10% compared with our earlier estimate [18] of 0.15%, for example. From the discussion above, evidently, this error will be reduced significantly again in the near future.

Continuing in this way, using the above results and making further cross checks with our own semi-analytical results and with the available results from the literature as described in Ref. [32], we finally arrive [40] at the total theoretical precision budget shown in Table I, which also contains our theoretical precision results from Ref. [18] and for the LEP2 regime as indicated.

[2] In this figure we also include the SABSPV2 result, which is obtained according to SABSPV recipe but taking raw (input) cross sections from OLDBIS and LUMLOG.

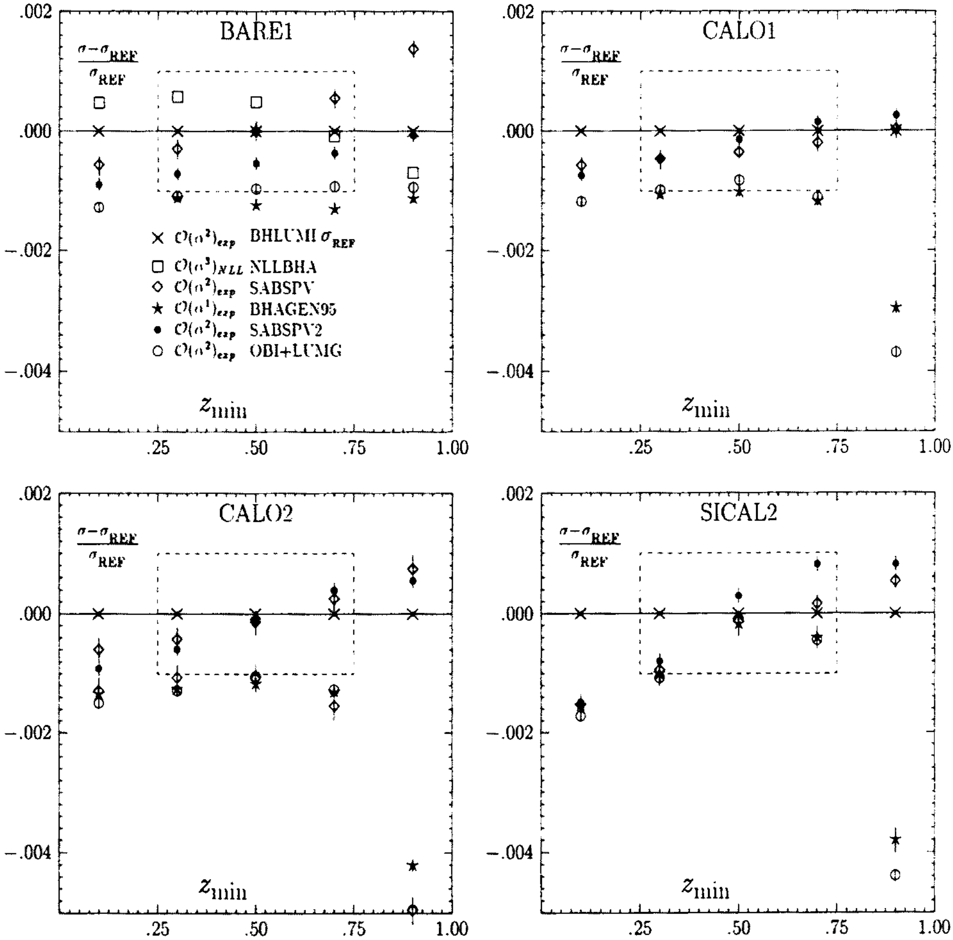

Fig. 7. Monte Carlo results for the symmetric Wide–Wide ESs BARE1, CALO1, CALO2 and SICAL2, for matrix elements beyond first-order. Z exchange, up-down interference and vacuum polarization are switched off. The centre-of-mass energy is $\sqrt{s} = 92.3$ GeV. In the plot, the $\mathcal{O}(\alpha^2)^{\rm YFS}_{\rm exp}$ cross section $\sigma_{\rm BHL}$ from BHLUMI 4.03 is used as a reference cross section.

Evidently, as Dallavalle [1] explained, the 0.11% result for the current precision at LEP1 is in need of improvement to the 0.05% regime soon, since this is the regime of the combined experimental error on $\sigma_{\mathcal{L}}$ at this point. We hope to achieve this level soon [41].

TABLE I

Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range 1°–3°; for LEP2 it covers energies up to 176 GeV, and angular ranges of 1°–3° and 3°–6° (see the text for further comments).

	LEP1		LEP2
Type of correction/error	Past	Present	Present
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10%	0.20%
(b) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	0.015%	0.03%
(c) Vacuum polarization	0.05%	0.04%	0.10%
(d) Light pairs	0.01%	0.03%	0.05%
(e) Z-exchange	0.03%	0.015%	0.0%
Total	0.16%	0.11%	0.25%

7. Wide-angle Bhabha scattering

In this section we present the recent results of three of us (S.J., W.P. and B.F.L.W.) [16] on the wide-angle Bhabha scattering process (LABH) at LEP1/SLC and at LEP2 energies. Our motivation was as explained in the introduction and as is illustrated in the contribution of Kawamoto [2] to these Proceedings: to improve on the precision of the available MC event generators for LEP1/SLC and LEP2 LABH. Indeed, until recent work done in the context of the Physics at LEP2 Workshop [32], BABAMC [39], with $\sim 1.0\%$ precision on its QED aspect, was the most widely used LABH MC event generator.

Specifically, our starting point was the $\mathcal{O}(\alpha)$ YFS exponentiated Monte Carlo (MC) event generator BHLUMI 1.xx developed by two of us (S.J. and B.F.L.W.) in Ref. [10] for the SABH process. In order to arrive at an event generator valid for wide-angles, we have had to introduce the effects of the Z exchange graphs for the Born level and bremsstrahlung graphs and the attendant pure weak one-loop corrections into the calculations presented in Ref. [10]. We provide the user with a choice between the pure weak libraries of BABAMC [39, 42] and of ALIBABA [4]; the default choice is the latter. The complete exact hard bremsstrahlung amplitude we computed [16] explicitly using the methods of Ref. [23]; it was checked excellently against the CALKUL result of Ref. [25]. Using these exact $\mathcal{O}(\alpha)$ results, we then compute the corresponding $\mathcal{O}(\alpha)$ correction to $\bar{\beta}_0$, and the corresponding extension of $\bar{\beta}_1$ to wide-angles. The detailed formulas are given in Ref. [16]. It is in this way that we have arrived at our new $\mathcal{O}(\alpha)$ YFS exponentiated

No.	E_{CM}	TOPAZ0	BHWIDE	BHAGENE3	ALIBABA	BHAGEN95
			(a) BARE $acol_{max} = 10°$			
1.	88.45	0.4579 ± .0003	0.4560 ± .0004	0.4495 ± .0016	0.4575 ± .0003	0.4578 ± .0002
2.	89.45	0.6452 ± .0002	0.6429 ± .0006	0.6334 ± .0023	0.6440 ± .0003	0.6445 ± .0003
3.	90.20	0.9115 ± .0002	0.9087 ± .0008	0.8997 ± .0033	0.9090 ± .0004	0.9095 ± .0004
4.	91.19	1.1846 ± .0002	1.1797 ± .0010	1.1847 ± .0033	1.1840 ± .0004	1.1822 ± .0005
5.	91.30	1.1639 ± .0002	1.1592 ± .0009	1.1667 ± .0033	1.1636 ± .0005	1.1619 ± .0005
6.	91.95	0.8738 ± .0002	0.8711 ± .0007	0.8856 ± .0028	0.8769 ± .0003	0.8742 ± .0004
7.	93.00	0.4771 ± .0002	0.4761 ± .0005	0.4808 ± .0019	0.4814 ± .0001	0.4796 ± .0002
8.	93.70	0.3521 ± .0002	0.3512 ± .0004	0.3521 ± .0013	0.3556 ± .0001	0.3550 ± .0001
			(b) BARE $acol_{max} = 25^0$			
1.	88.45	0.4854 ± .0003	0.4808 ± .0005	0.4699 ± .0016	0.4833 ± .0003	0.4826 ± .0002
2.	89.45	0.6746 ± .0003	0.6699 ± .0006	0.6593 ± .0023	0.6727 ± .0003	0.6710 ± .0003
3.	90.20	0.9438 ± .0003	0.9387 ± .0008	0.9279 ± .0033	0.9425 ± .0003	0.9384 ± .0004
4.	91.19	1.2198 ± .0003	1.2130 ± .0010	1.2169 ± .0034	1.2187 ± .0004	1.2133 ± .0005
5.	91.30	1.1989 ± .0003	1.1924 ± .0010	1.1995 ± .0034	1.1982 ± .0004	1.1928 ± .0005
6.	91.95	0.9054 ± .0002	0.9011 ± .0007	0.9124 ± .0026	0.9089 ± .0003	0.9014 ± .0003
7.	93.00	0.5040 ± .0002	0.5013 ± .0005	0.4996 ± .0019	0.5054 ± .0002	0.5027 ± .0002
8.	93.70	0.3777 ± .0002	0.3749 ± .0004	0.3689 ± .0013	0.3782 ± .0001	0.3771 ± .0001

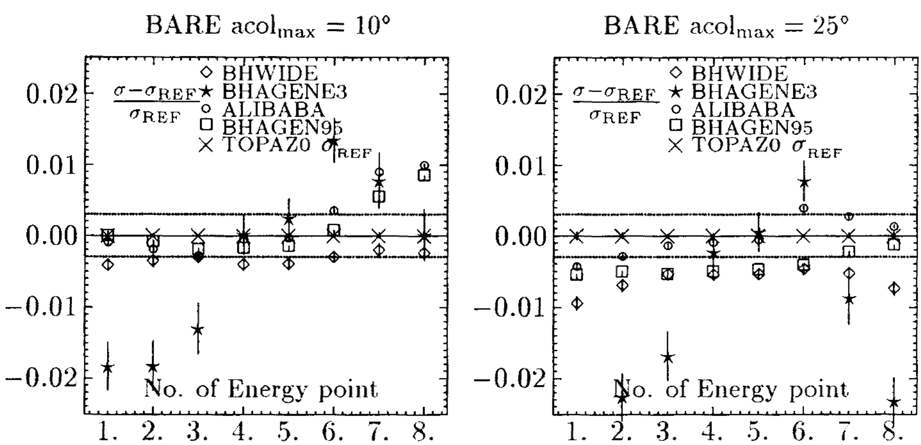

Fig. 8. Monte Carlo results for BARE trigger, for two values (10° and 25°) of acollinearity cut. Center of the mass energies (in GeV) close to Z peak. In the plots, the cross section σ_{REF} from TOPAZ0 is used as a reference cross section. The cross sections are in nb. The two horizontal dotted lines indicate the 0.3% band, for reference.

MC event generator BHWIDE 1.xx, which simulates realistic multiple photon radiative effects for wide-angle Bhabha scattering in the LEP1/SLC and LEP2 energy regimes.

We have compared [16] our results with those of BABAMC, of the Monte Carlo integrator program SABSPV of Ref. [43], of the semi-analytical pro-

No.	E_{CM}	TOPAZ0	BHWIDE	BHAGENE3	UNIBAB	BHAGEN95
	(a) CALO $acol_{max} = 10°$					
1.	88.45	0.4533 ± .0004	0.4523 ± .0004	0.4467 ± .0008	0.4490 ± .0010	0.4524 ± .0001
2.	89.45	0.6387 ± .0004	0.6377 ± .0006	0.6302 ± .0011	0.6358 ± .0012	0.6370 ± .0002
3.	90.20	0.9023 ± .0003	0.9016 ± .0008	0.8920 ± .0015	0.9021 ± .0014	0.8990 ± .0003
4.	91.19	1.1725 ± .0001	1.1707 ± .0010	1.1767 ± .0021	1.1772 ± .0016	1.1689 ± .0004
5.	91.30	1.1520 ± .0001	1.1505 ± .0009	1.1571 ± .0020	1.1559 ± .0016	1.1491 ± .0004
6.	91.95	0.8649 ± .0001	0.8646 ± .0007	0.8795 ± .0015	0.8689 ± .0012	0.8660 ± .0003
7.	93.00	0.4723 ± .0001	0.4725 ± .0005	0.4796 ± .0008	0.4733 ± .0008	0.4761 ± .0001
8.	93.70	0.3486 ± .0001	0.3486 ± .0004	0.3507 ± .0006	0.3486 ± .0007	0.3526 ± .0001
	(b) CALO $acol_{max} = 25°$					
1.	88.45	0.4769 ± .0004	0.4742 ± .0004	0.4696 ± .0008	0.4733 ± .0010	0.4741 ± .0001
2.	89.45	0.6638 ± .0003	0.6615 ± .0006	0.6556 ± .0011	0.6619 ± .0012	0.6599 ± .0002
3.	90.20	0.9297 ± .0003	0.9278 ± .0008	0.9207 ± .0012	0.9302 ± .0014	0.9236 ± .0003
4.	91.19	1.2025 ± .0003	1.1994 ± .0010	1.2074 ± .0021	1.2073 ± .0016	1.1954 ± .0004
5.	91.30	1.1819 ± .0003	1.1790 ± .0010	1.1879 ± .0021	1.1860 ± .0016	1.1753 ± .0004
6.	91.95	0.8924 ± .0003	0.8909 ± .0007	0.9058 ± .0016	0.8965 ± .0012	0.8899 ± .0003
7.	93.00	0.4964 ± .0003	0.4954 ± .0005	0.5004 ± .0009	0.4976 ± .0008	0.4971 ± .0001
8.	93.70	0.3717 ± .0003	0.3704 ± .0004	0.3690 ± .0006	0.3720 ± .0007	0.3730 ± .0001

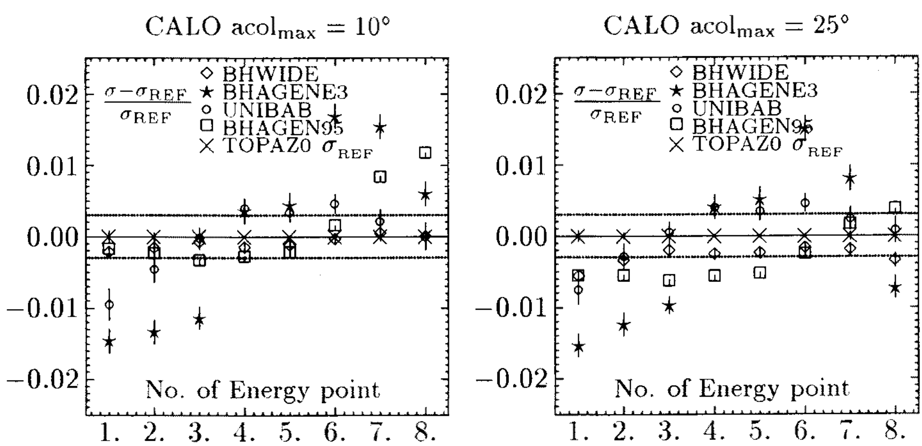

Fig. 9. Monte Carlo results for CALO trigger, for two values (10° and 25°) of the acollinearity cut. The center of the mass energies (in GeV) are close to the Z peak. In the plots, the cross section σ_{REF} from TOPAZ0 is used as a reference cross section. The cross sections are in nb. The two horizontal dotted lines indicate the 0.3% band, for reference.

gram TOPAZ0 of Refs. [7, 8], of ALIBABA, of the MC event generator BHAGENE3 of Refs. [44, 45], of the MC event generator BHAGEN95 of Refs. [46–49], and of the MC event generator UNIBAB of Ref. [50]. We now illustrate only the main features of these comparisons.

No.	BHWIDE	TOPAZ0	BHAGENE3	UNIBAB	SABSPV	BHAGEN95
	(a) CALO $\mathrm{acol}_{\mathrm{max}} = 10°$					
1.	35.257±.040	35.455±.024	34.690±.210	34.498±.157	35.740±.080	35.800±.019
2.	29.899±.034	30.024±.020	28.780±.170	29.189±.134	30.270±.070	30.296±.016
3.	25.593±.029	25.738±.015	24.690±.150	24.976±.115	25.960±.060	25.958±.014
	(b) CALO $\mathrm{acol}_{\mathrm{max}} = 25°$					
1.	39.741±.049	40.487±.025	39.170±.280	39.521±.158	40.240±.100	40.463±.021
2.	33.698±.042	34.336±.017	32.400±.190	33.512±.135	34.100±.080	34.287±.018
3.	28.929±.036	29.460±.013	27.840± .160	28.710±.116	29.280±.070	29.409±.015

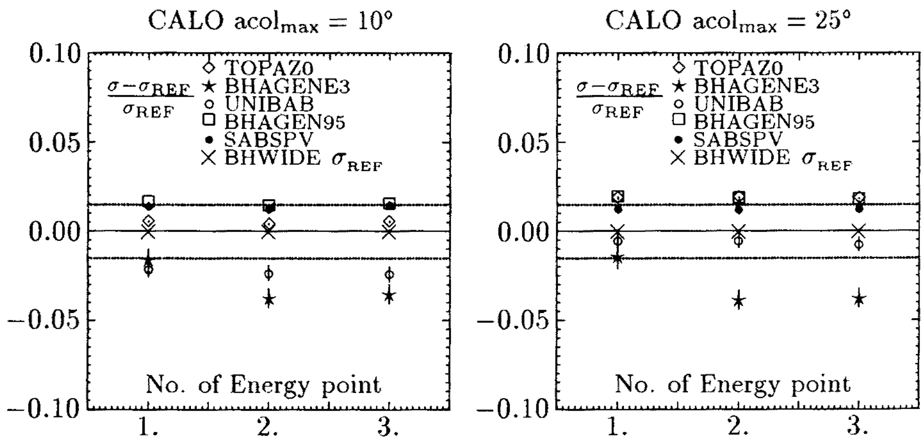

Fig. 10. Monte Carlo results for CALO trigger, for two values (10° and 25°) of the acollinearity cut. The center of the mass energies are close to the W-pair production threshold (E_{CM}: 1. 175 GeV, 2. 190 GeV, 3. 205 GeV). In the plots, the cross section σ_{REF} from BHWIDE is used as a reference. The cross sections are in pb. The two horizontal dotted lines indicate the 1.5% band, for reference.

Specifically, for our comparisons we use the same event selection (ES) cuts, BARE and CALO, as defined in Ref. [32]. For the BARE (CALO) ES acceptance cuts, we show in Fig. 8 (Fig. 9) the comparison of the BHWIDE results with those of ALIBABA, BHAGEN95, BHAGENE3 and TOPAZ0 (BHAGEN95, BHAGENE3, TOPAZ0 and UNIBAB), where for definiteness we plot the ratio $(\sigma_A - \sigma_{REF})/\sigma_{REF}$ for each calculation A, $A =$ ALIBABA, BHAGEN95[3], BHAGENE3, BHWIDE, UNIBAB, and TOPAZ0, using the TOPAZ0 cross section as σ_{REF}. This we do for the 8 CMS energy values: 88.45, 89.45, 90.20, 91.19, 91.30, 91.95, 93.00, 93.70 GeV, which are denoted in the figure as energy points $1, 2, \ldots, 8$, respectively. Based on the com-

[3] Here we use recently updated results of BHAGEN95, obtained from the authors.

parison of results illustrated in Figs. 8 and 9 and on related comparisons as described in Ref. [32], we conclude that, for the CALO ES, the total precision of BHWIDE is 3 per mille within ± 100 MeV of the Z peak; off peak, within $+2.5/-2.75$ GeV thereof, we set this precision at 5 per mille in the LEP1 energy regime. For reference the 3 per mille band is indicated by the two horizontal dotted lines in Figs. 8 and 9. This precision tag should be compared with that for BABAMC [39], whose total precision on pure QED was set at 1% in the Z peak region in Refs. [51, 52].

Turning next to LEP2 energies, we show in Fig. 10 the comparison of the results of the six programs BHAGEN95, BHAGENE3, BHWIDE, SABSPV, TOPAZ0, and UNIBAB in the same format as in Figs. 8 and 9, where BHWIDE is used for the reference cross section σ_{REF}, for the three CMS energy points 175, 190, and 205 GeV. We use the CALO ES only here. As we explain in Ref. [16] on the basis of the comparison of results illustrated by Fig. 10, and on the basis of related comparisons, we estimate the total precision of BHWIDE at the LEP2 energies as 1.5%, conservatively. For reference, the 1.5% band is indicated by the two horizontal dotted lines in Fig. 10.

8. Conclusions

In this work, we have presented the current status and the outlook for the YFS Monte Carlo approach to both SABH and LABH processes. For the SABH process, we showed that the total precision tag of 0.11% in version 4.03 (4.04) of BHLUMI has been achieved and that all formulas needed for achieving the total precision tag of $\sim 0.05\%$ are now being implemented and cross-checked. For the large-angle Bhabha scattering at LEP1/SLC and LEP2 energies, we presented a new exact $\mathcal{O}(\alpha)$ YFS exponentiated Monte Carlo event generator BHWIDE 1.00, in which the respective multiple photon effects are realized on an event-by-event basis and in which the infrared singularities are canceled to all orders in α. It features a 0.5% precision in general at LEP1 (0.3% on the Z peak itself) and 1.5% at LEP2 energies, both state-of-the-art results for MC event generators. Thus, in both the SABH and the LABH, we now have the state-of-the-art calculations of the respective radiative corrections realized on an event-by-event basis so that arbitrary detector cuts are accessible. We find this situation exciting indeed, and we look forward to the many further applications of our results at the various e^+e^- colliding beam facilities, such as LEP1, LEP2, SLC, SLAC B-Factory, BELLE, BES, *etc.*

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