AUTOMATIC COMPUTATION OF RADIATIVE CORRECTIONS*

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Automated systems are reviewed focusing on their general structure and requirement specific to the calculation of radiative corrections. Detailed description of the system and its performance is presented taking GRACE as a concrete example.

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1. Introduction

The development of high energy accelerator and experimental technology urges theorists to make large scale computation for a precise comparison between the experimental data and the theory. In high energy collisions we observe multi-particle channels and good experiments provide us with data of high accuracy. This means that the cross sections for complex processes must be calculated including higher-order corrections. Such a huge computation sometimes exceeds limit of desk work by theorists. Since perturbative calculation of quantum field theory is a well established algorithm, it is natural to expect automated systems on computer are able to solve the problem.

An example of such large scale computation is the four-fermion production in e^+e^- collision summarized in the LEP-II report [1]. It has been

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demonstrated that automated systems are quite efficient for the purpose among many codes contributed to this calculation. In this case one should have generated all the 76 channels of four-fermion final states and the maximum number of Feynman diagrams reaches 144 for two electron pairs [1]. Automated systems, for example `grc4f` [2] spawned from `GRACE` [5], make such a computation possible even keeping the fermion masses non-zero.

In its first stage the automated systems are developed for the computation of tree processes. Then they should be extended to the radiative corrections in one-loop and beyond, because one meets also large scale computation. Such a sample is the one-loop corrections to $\gamma\gamma \to W^+W^-$. This requires the computation of ~500 Feynman diagrams [3, 4].

In this report we review the general structure of the automated system by taking `GRACE`[5] as an concrete example. This system is designed to be used as an event generators for processes of tree level. One–loop calculation is possible, though restricted to 2-body processes at the moment.

Besides `GRACE` system, several automated systems have been developed in the world. We only quote some major systems below:

(1) FeynArts/FeynCalc [6] is constructed on Mathematica and can calculate one-loop amplitudes. Results for radiative corrections to $e^+e^- \to HZ$, $\gamma\gamma \to t\bar{t}$, $\gamma\gamma \to W^+W^-$ have been published. Extension to higher loops is under study.

(2) CompHEP system [7] provides an interactive interface to the user. It can generate events and has been applied to many physical processes. Though restricted to the tree-level at present the extension to the one-loop is in progress.

(3) MadGraph [8] is also for the tree calculation where the amplitude is evaluated by HELAS [9] library.

(4) ALPHA [10] system is for the tree calculation. It is unique in the sense that it does not use the perturbation by Feynman diagrams. The method is based on the fact that only discrete number of momenta defined by external particles appear in the intermediate states for tree level. The field operators are expanded by these discrete modes and the scattering amplitude is directly calculable by the generating functional. This method has shown to be efficient to save the computation time for the amplitude calculation.

(5) Wang's system [11] is written in RLISP+REDUCE. It also aims the one-loop automatic computation.

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[1] The number of diagrams depends on the choice of gauge.

2. Structure of automatic system

In this report we confine the discussion to the automation for the radiative corrections in the electro-weak (EW) theory but not those in QCD. The reason is that in higher order calculations the most adequate way of computation and necessary technology usually differ between these two theories. So many types of particles and vertices in the EW theory force us to develop an automated system. QCD is, however, much simpler even in the one-loop level.

Here we give some basic assumptions for the following discussions. (1) As we have seen it is possible to construct an automatic system without recourse to Feynman diagrams. However we skip the discussion for this direction since at present the radiatiove correction is only calculated by the conservative method based on the perturbation using Feynman diagram. (2) Feynman rules to generate diagrams are usually implemented in a code by hand. Recently a system [12] has been proposed which is designed for automatic generation of Feynman rules from a given Lagrangian. Though this will be certainly powerful for more complicated theories such as SUSY and extended models, the discussion is skipped in this report. (3) Also we skip all the discussion on kinematics, integration method and the way of event generation, against their importance. It should be stressed that these items are never trivial but only deep consideration and experience would be able to construct a realistic system.

Now we list the required ingredients of the automated system for one-loop.

- Diagram generation. Diagrams to be calculated are generated according to the input which specifies the external particles and other control parameters. The diagrams are recorded in a structured format defined in the system.
- Viewer/Drawer of diagrams. Normally, as a service package, viewer/drawer to display generated diagrams is included in the system. This is necessary for the check and for the preparation of publication, though in a complicated process it is no more practical to inspect all diagrams by eyes.
- Amplitude generation. For each diagram codes are generated to evaluate the amplitudes. Intermediate expression is usually written in a symbolic manipulation language. Final object must be a code for numerical computation.
- Library for amplitude handling and computation. The structure of the library depends on the system. For example a set of functions

corresponding to parts, of which helicity amplitude is composed, should be included when the system is based on the helicity formalism.

- Library for loop integrals. A library is required to evaluate loop integrals for a given set of external momenta and internal masses.
- Monte Carlo integration and event generation.
  The phase space integration is to be done under various cuts required by experiments. Generally the Monte Carlo integration is the most convenient one for this purpose. Another important point, particularly from the experimental side, is to construct event generators which is able to produce events with weight one.
- Facility for calibration.
  In contrast to the manual calculation the automatic calculation proceeds in a sequence of operations on a computer terminal. The most serious problem is to confirm that the result is really correct. Hence any system should have some functions which allow the self-test. A few standard checks are the following.
  1. Renormalization. This implies to keep $1/\varepsilon$ as a variable in the program and to check whether the result is independent of this variable.
  2. Infra-red divergence. Infra-red divergence can be controlled by $1/\varepsilon$ (dimensional regularization), or by $\ln\lambda$ (the fictitious mass of a photon). This variable is also retained in the program and it is checked whether the dependence is canceled by the contribution from the soft radiation.
  3. Gauge invariance. A severe test will be done if the cross section is computed in different gauges. Usually one-loop computation is easy in 'tHooft-Feynman gauge due to its simple numerator, $(-g^{\mu\nu})$. In the unitary gauge $(-g^{\mu\nu} + p^\mu p^\nu/M^2)$, axial gauge and so on, the loop integration would become more complicate and tedious. A good candidate of gauge for the diagnostics is the non-linear gauge [13].
  4. Other (process dependent) checks. Other checks must be possible, though dependent on the process at hand. They include test of some symmetry, Furry's theorem, partial cancelation of divergence for a class of diagrams and so forth.
  5. Comparison with analytic result. This is a trivial idea but if analytic results, if any, are available even for some special cases or with some approximation, it is quite useful for a comparison.
  6. Comparison between systems. Ultimate comparison must be done between independent computations. For this it is indispensable

to have groups (at least two) competing in the development of automated systems.

- Acceleration. Generally the code generated by automated systems is less efficient than that written by hand and needs to accelerate the computation.
  - Theoretical aspect. One direction is, like ALPHA, to find other methods to compute matrix element than the conventional Feynman diagram. It might be possible to combine a set of graphs in a simpler form by using basic nature of the gauge theory [14]. Choosing an efficient gauge and development of efficient loop integral formula would be a possible improvement.
  - Software aspect. If one can factor out a common pattern among diagrams and/or generated codes, one can save the computational time [15]. It would not be an easy problem to find the best optimized factorization. However, some partial factorization is enough from practical point of view. Parallel computing is easier way for acceleration, assuming that the system is designed so as to suit the parallel computation. In fact a trial for PVM of `GRACE` was reported to be successful [16].
  - Hardware aspect. The solution is simple: The fastest computer is the best. Parallel computing technology is also favorable.

3. Example of application

In the following we present the function of `GRACE` as an example among existing automated systems. In the one-loop level this system has been tested for the following processes [17]: 1) $e^+e^- \to HZ$, 2) $e^+e^- \to q\bar{q}\gamma$ (QCD correction), 3) $e^+e^- \to t\bar{t}$, 4) $\gamma\gamma \to t\bar{t}$, 5) $\gamma\gamma \to W^+W^-$.

The system is based on the helicity formalism and computes amplitude for a given set of external helicity states. In addition the system includes the option to compute the matrix element (product of tree and one-loop amplitudes with spin summation) by symbolic manipulation which can be used for the self-check. We adopt the on-shell scheme given by Kyoto group [19].

The operation of `GRACE` is done as follows. Since it is a *full automatic* system, what the user has to do is almost to type a few commands at a terminal.

1. The user creates a file to specify the process. An example of input file is shown in Fig. 1.
2. The user invokes the command `grc` to generate Feynman diagrams. Efficient algorithm is exploited here for graph generation as explained

in subsection 3.1. A data file to describe the structure of diagrams is created.

3. The user can view generated diagrams on the display by `gracefig` command. [18] This also creates postscript files when the user needs printed diagrams.
4. By `grcop` command REDUCE source code for the helicity amplitude is generated for each diagram. The description in this code is explained in detail in subsection 3.2. After the execution of REDUCE, the source code for numerical computation is obtained. In the generated FORTRAN code extended `CHANEL` library and loop integral library are called. Multiplication of tree amplitudes and those in one-loop is done numerically in the FORTRAN code.
5. Peripheral codes are also generated including the definition of masses, couplings and the renormalization constants, interface to Monte Carlo integration and so forth. Concerning the process at hand, the user can specify an appropriate definition of kinematics among the library of kinematics.
6. Some manual editing is required here for the specification of $\sqrt{s}$ and other parameters and inclusion of codes for special checks or codes for experimental cuts and so on.
7. After typing `make` all the object files are ready. By invoking command `integ`, the cross section will be calculated by calling adaptive Monte Carlo integration package `BASES` [20]. It also generates a parameter file to store the distribution of the integrand. The parameter file can be used by `SPRING` which generates events with weight one [20].

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Model="canon.mdl";
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Process;
  ELWK={4,2};
  Initial={photon, photon};
  Final  ={W-plus,W-minus};
  Expand=Yes;
  OPI=No;
  Block=No;
  Extself=No;
  Tadpole=No;
Pend;
```

Fig. 1. An example of input file for $\gamma\gamma \to W^+W^-$.

3.1. Graph generation

GRACE can generate Feynman diagrams to any order in an efficient way based on the orderly algorithm [21].

First it generates topology and removes identical graphs. For instance we consider the two-loop graph in Fig. 2.

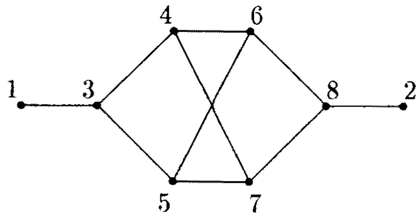

Fig. 2. Example of a graph.

The graph can be represented by the following matrix which expresses the connection of edges (external particles and vertices).

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

This allows a mapping of a graph to a number by regarding this matrix as a sequence of binary number.

$$f(G) = 00100000000000110011000\ldots .$$

Then we have to eliminate graphs with duplicate topology. We perform all permutations for vertices.

$$F(G) = \max_p f(pG) \qquad (p = \text{permutation of vertices})\,.$$

If $F(G) > f(G)$, we discard the graph.

This is simple but the operation is $O(N!)$ where N is the number of vertices. We introduce the orderly algorithm with vertex classification. Then vertices are divided into groups, so that the number is not $O(N!)$ but $O(N_1!N_2!\cdots)$, $N_1 + N_2 + \cdots = N$. Also a special trick eliminates the duplicated graphs at the intermediate stage of graph generation.

For each generated skeleton graph, we assign particles by the list of vertices (Feynman rules). Duplication is again checked by a similar method. The symmetry factor of the graph is also determined at this step.

3.2. Procedure to produce amplitude

In GRACE the helicity amplitude is calculated. The ultra-violet divergence is kept by a special variable assigned to $C_{uv} = 1/\varepsilon - \gamma_E + \log 4\pi$, $n = 4 - 2\varepsilon$. The infra-red divergence is regulated by the fictitious mass λ. As is described before we have a symbolic code for each Feynman diagram. A sequence of procedures is necessary to produce FORTRAN source code. We present below the operations applied in the symbolic code.

1. The numerator structure is generated symbolically.
2. Indices inside loop are contracted in n dimension.
3. If there is a fermion loop, trace is taken. The cyclicity of γ matrix is *not* used to avoid the anomaly from γ_5. The ordering of γ's is common to a set of graphs with same topology.
4. Shift momentum is determined by inspecting the denominator structure after combining propagators by Feynman parameters. Then loop momentum, ℓ, is shifted.
5. Terms in odd power of ℓ are dropped.
6. The following replacement of even ℓ is done:

$$\ell^\mu \ell^\nu \to \frac{\ell^2}{n} g^{\mu\nu} ,$$

$$\ell^\mu \ell^\nu \ell^\rho \ell^\sigma \to \frac{(\ell^2)^2}{n(n+2)} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\rho\nu}) .$$

7. Numerator becomes polynomials in Feynman parameters. The set of coefficient of polynomial are extracted. Following is an example of a vertex integral.

$$\int dxdy \Gamma(3) \int d^n \ell \frac{(g(x,y) + \varepsilon g_1(x,y))\ell^2 + f(x,y)}{(D_3 - i\varepsilon)^3} .$$

8. Here $f(x,y)$, $g(x,y)$, ... are products of coupling constants, invariants, gamma matrices and wave functions of external particles.
9. Each fermionic string is replaced by a symbolic expression with appropriate arguments. This expression is computed numerically by extended CHANEL library, where CHANEL [22] is used in GRACE for tree diagrams.
10. Summation is done for coefficients over all combination of internal helicity freedom for a given set of external helicities.
11. Loop library is called to compute loop integral with (numerical) polynomials in the numerator.

3.3. Numerical loop integration

For the loop integration GRACE system uses a numerical method in the Feynman parameter space [23]. One algorithm is to use the symmetrization to regulate singularity. Another one is a kind of hybrid method in which the singularity is integrated up to logarithms:

$$\int_a^b dx \frac{1}{x - c - i\varepsilon} = \log(x - c - i\varepsilon) \,\Big|_a^b \,.$$

We can integrate the loop integral with a numerator in Feynman parameter space directly without reductions.

For the large scale problem we found that Monte Carlo method is quite efficient to integrate both the phase space variables and the Feynman parameters at the same time. This technique can be extended to two–loop and higher loops. It should be noted that the integrand is exact (including that of loop) and the result is obtained within the error given by Monte Carlo method. When the final state is multi-body there is no need to have precise value of the loop integrals, because the phase space integral destroys the precision after all as long as the Monte Carlo method is employed.

3.4. Results for $\gamma\gamma \to W^+W^-$

As an example of the performance of GRACE we present the results for the radiative correction to $\gamma\gamma \to W^+W^-$. The checks of the result are made in the following way: First, C_{uv} independence and λ independence were confirmed. Partial check for t–u symmetry, C_{uv} independence for vertex, Furry's theorem, finiteness of Higgs vertex were done successfully.

TABLE I

Values of correction, $\delta_{\Delta E=0.1E}$. Comparison with Ref. [3]

W (GeV)	θ(deg)	Ref. [3](%)	GRACE(%)
500	5	0.02	0.016
500	20	−2.68	−2.710
500	90	−10.79	−10.790
1000	5	−2.06	−2.063
1000	20	−10.90	−11.900
1000	90	−31.68	−31.644
2000	5	−7.14	−7.132
2000	20	−30.31	−30.312
2000	90	−59.59	−59.573

For physical parameters we use $M_Z = 91.187$ GeV, $M_W = 80.333$ GeV, $M_H = 250$ GeV, $m_t = 170$ GeV, $k_{\rm cut} = 0.1 E_{\rm beam}$ to compare with Ref. [3] in Table I, and we use $M_Z = 91.187$ GeV, $M_W = 80.333$ GeV, $M_H = 300$ GeV, $m_t = 174$ GeV, $k_{\rm cut} = 0.1$ GeV to compare with Ref. [4] in Tables II and III.

TABLE II

Values of correction, $\delta^{\rm tot} - \delta^{\rm hard}$. Comparison with Ref. [4]

W (GeV)	θ(deg)	Ref. [4](%)	GRACE(%)
500	10	−7.27	−7.27
500	20	−9.38	−9.39
500	60	−16.12	−16.15
500	90	−17.42	−17.45
1000	10	−16.9	−16.89
1000	20	−23.6	−23.55
1000	60	−40.4	−40.42
1000	90	−43.2	−43.25
2000	10	−33.4	−33.37
2000	20	−47.7	−47.72
2000	60	−73.1	−73.13
2000	90	−77.0	−76.95

TABLE III

Table of radiative correction for various helicity amplitudes. Columns denoted by J and G are results in Ref. [4] and that by GRACE, respectively. Values of $\delta = \delta_{\rm soft} + \delta_{\rm bose} + \delta_{\rm fermi}$ is shown in %. Here $\delta = \sigma/\sigma^{\rm Born} - 1$ and $\sigma = \int_{10^\circ}^{170^\circ} \frac{d\sigma}{d\theta} d\theta$.

		$W = 500$ GeV	$W = 1000$ GeV	$W = 2000$ GeV
unpol	G	−10.9	−23.8	−44.6
	J	−11.0	−23.9	−44.5
$++TT$	G	−10.7	−24.5	−45.5
	J	−10.9	−24.5	−45.6
$++TL$	G	0	0	0
	J	0	0	0
$++LL$	G	−10.3	−22.1	−49.1
	J	−10.3	−22.0	−48.6
$+-TT$	G	−11.0	−23.0	−43.3
	J	−11.0	−23.1	−43.4
$+-TL$	G	−13.5	−29.4	−48.3
	J	−13.4	−29.3	−48.2
$+-LL$	G	−9.9	−24.0	−43.9
	J	−9.8	−24.4	−44.1

Ref. [4] also calculated the total cross section of the process $\gamma\gamma \to W^+W^-\gamma$ to make completion of the $O(\alpha)$ corrections. Since it is an easy task for GRACE to calculate a tree process, we also show the radiative correction due to the hard photon emission in Table IV. These tables demonstrate that GRACE produces the consistent results with other calculations.

TABLE IV

Table of hard photon contribution for various helicity amplitudes. Columns denoted by J and G are results in Ref. [4] and that by GRACE, respectively. Values of $\delta = \delta_{\rm hard}$ is shown in %.

		$W = 500$ GeV	$W = 1000$ GeV	$W = 2000$ GeV
unpol	G	7.87	13.4	20.0
	J	7.94	13.4	20.1
$++TT$	G	7.91	13.4	19.1
	J	7.94	13.5	20.1
$++TL$	G	0	0	0
	J	0	0	0
$++LL$	G	13.5	65.2	759
	J	13.7	64.2	748
$+-TT$	G	7.63	13.1	19.5
	J	7.69	13.1	19.6
$+-TL$	G	10.3	40.8	252
	J	10.2	40.6	248
$+-LL$	G	8.22	14.3	21.0
	J	8.22	14.2	21.0

4. Toward two-loop and beyond

We have discussed the automatic calculation of radiative correction in one-loop. Eventually this will have to be extended to higher loops.

For the graph generation there is no problem. GRACE graph generator is general enough to any order in the standard model. Next task is to process tensor structure in the numerator in general manner. This is realized up to now in a few systems, *e.g.*, TwoCalc [24] and XLOOPS [25].

Also two-loop integral library should be ready for general combination of external momenta and internal masses. It seems hopeless to obtain compact analytic formula except for a few lucky cases. Many methods are proposed for this target. They are *(i)* analytical methods [26], *(ii)* asymptotic expansion, up to box [27], *(iii)* recurrence relations [28], *(iv)* integration by parts [29], *(v)* momentum space integration [30], *(vi)* momentum expansion with Padé approximation [31], *(vii)* analytical/numerical method [32], *(viii)* Feynman parameter numerical integration [33]. Though some progress and applications are found in literature, the automatic system is not yet completed.

5. Summary

It is now recognized that automated system for the perturbative calculation of QFT is indispensable for high-energy physics.

Many systems have been used for the calculation of tree processes and some of them can handle up to 2-body → 5-body and more complicated reactions[2]. For the tree processes it can be concluded that the automation *has substantialy reduced man-power requirements.*

A few system are available for the one-loop. At least any 2 to 2 processes can be automated. It seems to *reach the level of man-power.*

A few advent toward higher loop automation is in progress. *More work is needed to exceed man-power.*

In the development of the automated system it is important to reduce intervention by man-hand, since a human makes error *at random.* Automated system is a complex of symbolic manipulation and numeric computation. One starts with Lagrangian and Feynman rules which are symbolic objects and ends numerical stuffs, cross sections and generated events. Their interplay is a key for the design of system.

Important issues include the calibration of the system which can be done best by comparison between independent systems. This will motivate the standardization of the inputs and outputs of various systems. Also we should expect a new technology for acceleration of computing time for practical application.

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