ON THE KINEMATIC RECONSTRUCTION OF $e^+e^- \to W^+W^- \to jj\tau\overline{\nu}_\tau$ EVENTS

R. Ryłko

Department of Physics, Brunel University, West London
Uxbridge UB8 3PH, UK

(Received June 21, 1996)

We show that the kinematic reconstruction of the $e^+e^- \to W^+W^- \to jj\tau^-\overline{\nu}_\tau$ events have a one-parameter ambiguity when reconstructed from the momentum of all measured W^- decay products. We propose a *hybrid* method of reconstruction of the $e^+e^- \to W^+W^- \to jj\tau\overline{\nu}_\tau$ events. This is based on the observation that the difference between the τ production angles and the production angles of the sum of its visible decay products is small, whilst the τ energy is poorly reconstructed. This method consists of taking the τ production angles from those measured for the sum of the visible τ decay products and reconstructing the τ energy from energy-momentum conservation constraints. A reconstruction using this method is found to be well-defined and possess a unique solution for the τ momentum range at LEP2 and NLC.

PACS numbers: 13.38. Be, 13.65. +i

One of the subjects of LEP2 is to study the three gauge couplings (TGC) of the γ/Z and W bosons. Due to limited statistics at LEP2, one of the crucial points will be to include as many W decay channels as possible for these studies. Ongoing studies use semileptonic $WW \to jjl\overline{\nu}_l$ (with l=e or μ) and hadronic $WW \to 4j$ events. The four jet channel, although the most abundant, is likely to be affected by the difficulties in jet tagging. Thus, folded angular distributions will have to be used, which result in a significant increase of the error for the fitted couplings [1].

It is widely believed that $WW \to jj\tau\overline{\nu}_\tau$ events may be used with the other semileptonic $WW \to jjl\overline{\nu}_l$ (with l=e or μ) events, to increase statistics for studies of the triple gauge couplings and/or the W boson mass measurements. Indeed, for the semileptonic $e^+e^- \to W^+W^- \to jj\tau^-\overline{\nu}_\tau$ (and charge-conjugate) events there are six unmeasured variables, namely the momentum of the anti-neutrino from W^- decay, $\boldsymbol{p}_{\overline{\nu}}^{\rm lab}$, and the momentum of neutrino from τ^- decay, $\boldsymbol{p}_{\nu}^{\rm lab}$. There are also six constraints, namely (in the narrow W width approximation):

$$\boldsymbol{p}_{\rm vis}^{\rm lab} + \boldsymbol{p}_{\nu}^{\rm lab} + \boldsymbol{p}_{\overline{\nu}}^{\rm lab} + \boldsymbol{p}_{W}^{\rm lab} = 0 \tag{1}$$

$$E_{\rm vis}^{\rm lab} + p_{\nu}^{\rm lab} + p_{\overline{\nu}}^{\rm lab} + E_{W}^{\rm lab} = \sqrt{s} \tag{2}$$

$$(E_{\rm vis}^{\rm lab} + p_{\nu}^{\rm lab} + p_{\overline{\nu}}^{\rm lab})^2 - (\boldsymbol{p}_{\rm vis}^{\rm lab} + \boldsymbol{p}_{\nu}^{\rm lab} + \boldsymbol{p}_{\overline{\nu}}^{\rm lab})^2 = m_W^2 \tag{3}$$

$$(E_{\rm vis}^{\rm lab} + p_{\nu}^{\rm lab})^2 - (\boldsymbol{p}_{\rm vis}^{\rm lab} + \boldsymbol{p}_{\nu}^{\rm lab})^2 = m_{\tau}^2 \tag{4}$$

where $E_{\rm vis}^{\rm lab}$ and $\boldsymbol{p}_{\rm vis}^{\rm lab}$ are the energy and momentum of the visible (*i.e.* charged and neutral τ^- decay products other than neutrino) τ^- decay products respectively, $E_W^{\rm lab}$ and $\boldsymbol{p}_W^{\rm lab}$ are the energy and momentum of the other boson (W^+) reconstructed from the two jets respectively, $\sqrt{s}$ is the e^+e^- center of mass energy and m_τ is the τ lepton mass. The above equations can be reduced to equations for the energies of the anti-neutrino and neutrino and three angles. Thus, it is difficult to see whether or not there is an ambiguity in the above equations, and what the character of this ambiguity might be. As the above equations contain quadratic terms of momenta, one may expect a two-fold ambiguity.

The problem simplifies when solved in the W^- rest frame. The boost to this frame is uniquely defined by the momentum of the W^+ boson determined from the measurement of the two jets. The above set of constraints now reduces to the following equations:

$$\boldsymbol{p}_{\rm vis} + \boldsymbol{p}_{\nu} + \boldsymbol{p}_{\overline{\nu}} = 0 \tag{5}$$

$$E_{\rm vis} + p_{\nu} + p_{\overline{\nu}} = m_W \tag{6}$$

$$(E_{\rm vis} + p_{\nu})^2 - (\boldsymbol{p}_{\rm vis} + \boldsymbol{p}_{\nu})^2 = m_{\tau}^2 \ . \tag{7}$$

In the W^- rest frame all three momenta lie in a plane. The boosted visible τ decay products' momentum $\boldsymbol{p}_{\rm vis}$ fixes the plane defined by eq. (5), but there is a freedom of rotation of the plane around $\boldsymbol{p}_{\rm vis}$. The solution of these equations is:

$$p_\nu = \frac{m_W^2 - 2m_W E_{\rm vis} + m_\tau^2}{2m_W} \tag{8}$$

and

$$p_{\overline{\nu}} = \frac{m_W^2 - m_\tau^2}{2m_W} \ , \tag{9}$$

and the cosines of angles between $\boldsymbol{p}_{\rm vis}$, $\boldsymbol{p}_{\nu}$ and $\boldsymbol{p}_{\overline{\nu}}$ may be expressed in terms of p_ν and $p_{\overline{\nu}}$. The other solutions may be obtained from the one above by rotations of the decay plane around $\boldsymbol{p}_{\rm vis}$. Thus, the W^- helicity angle θ^*, *i.e.* the angle between W^- laboratory momentum and the momentum of τ in the W^- rest frame, as well as the helicity angle ϕ^*, lie in a certain range resulting from the full rotation of $\boldsymbol{p}_\nu$ around $\boldsymbol{p}_{\text{vis}}$. This ambiguity in the reconstruction of the τ momentum ($\boldsymbol{p}_\tau = \boldsymbol{p}_{\text{vis}} + \boldsymbol{p}_\nu$) in the W^- momentum rest frame, when boosted back to the laboratory frame, leads to ambiguous energy and production angles for τ^-. Thus, the events may not be fully reconstructed kinematically, *i.e.* there is a one parameter ambiguity for a kinematic fit similar to the one for the $W^+W^- \to jjl\bar{\nu}_l$ (with l=e or μ) case.

One may consider the τ reconstruction method based on finding its flight vector from the secondary vertex. Although the momentum, and thus the decay length of the τ is large, however, due to small branching ratio for the τ decaying to more than one charged particle, the momentum estimators (see *e.g.* [2] and references cited therein) seem unlikely to be applied successfully. On the other hand, at LEP2 or higher energies, the τ lepton from $e^+e^- \to W^+W^- \to jj\tau^-\bar{\nu}_\tau$ will have high momentum compared with the transverse (to the direction of τ) momenta of its decay products, which are of the order of $m_\tau/2$. Thus, one may expect the laboratory τ production angles are almost the same as the production angles of the sum of its visible decay products. The energy of the τ is, however, substantially different than the energy of its visible decay products.

This suggests two methods. The first method is to study the distribution of the production angles only (assuming the angles of $\boldsymbol{p}_{\text{vis}}^{\text{lab}}$ to be the τ production angles), resulting in a fit with less angular information, and thus a greater degree of inaccuracy. The other *hybrid* method is to use the measured production angles of the visible τ decay products as the τ production angles and reconstruct the τ energy. Let $\hat{\boldsymbol{\tau}}$ be the unit vector in the direction of the sum of momenta of the visible τ^- decay products. Now, assuming that $\hat{\boldsymbol{\tau}}$ is the true τ^- direction gives the following constraints for the laboratory τ^- momentum, p_τ^{lab}, and the anti-neutrino laboratory momentum vector, $\boldsymbol{p}_{\bar{\nu}}^{\text{lab}}$:

$$p_\tau^{\text{lab}}\hat{\boldsymbol{\tau}} + \boldsymbol{p}_{\bar{\nu}}^{\text{lab}} = -\boldsymbol{p}_W^{\text{lab}} \tag{10}$$

and

$$E_\tau^{\text{lab}} + p_{\bar{\nu}}^{\text{lab}} = \sqrt{s} - E_W^{\text{lab}} \ , \tag{11}$$

where E_W^{lab} and $\boldsymbol{p}_W^{\text{lab}}$ are the laboratory energy and momentum of the other (W^+) boson reconstructed from the jj final state, respectively. The above four equations may be reduced to one quadratic equation. For $p_W^{\text{lab}} < (m_{W^-}^2 - m_\tau^2)/2m_\tau$, *i.e.* for $p_W^{\text{lab}} < (1.8 \pm 0.1)$ TeV for nominal W boson mass (where a change in the nominal W mass within one width produces this variation of 0.1 TeV), there are two solutions of the above equations, namely:

$$p^{\rm lab}_{\tau\pm} = \frac{-p_{||}\Sigma^2 \pm (\sqrt{s} - E^{\rm lab}_W)\sqrt{\Sigma^4 - 4m^2_\tau[(\sqrt{s} - E^{\rm lab}_W)^2 - p^2_{||}]}}{2[(\sqrt{s} - E^{\rm lab}_W)^2 - p^2_{||}]}\,, \tag{12}$$

where $p_{||} = p^{\rm lab}_W \cos\theta_{\tau - W^+}$ and $\theta_{\tau - W^+}$ is the angle between $\hat{\tau}$ and the other W boson (W^+) momentum, and

$$\Sigma^2 = (\sqrt{s} - E^{\rm lab}_W)^2 - p^2_W + m^2_\tau \ . \tag{13}$$

However, for the W^- boson energies

$$E^{\rm lab}_{W^-} < \frac{m^2_{W^-} + m^2_\tau}{2m_\tau}\ , \tag{14}$$

one of the above solutions is negative and the other is positive, thus there is only one physical solution to the above problem.

The above property shows that the proposed *hybrid* method may be used for kinematic fitting, where the τ production angles are fitted to the measured values of the production angles of the sum of the visible τ decay products, and the τ momentum value $p^{\rm lab}_\tau$ is fitted using the constraints given by eq.(10) and eq.(11). As a narrow W width approximation is not implied in eq.(10) and eq.(11), the above method may be used to exploit the $e^+e^- \to W^+W^- \to jj\tau^-\overline{\nu}_\tau$ (and charge-conjugated) channel for TGC studies as well as for W boson mass measurements.

The author thanks P. Clarke, J. Conboy, A. Hasan, R. Sekulin, P. Singh and A. Skillman for remarks.

REFERENCES

[1] R.L. Sekulin, *Phys. Lett.* **B338**, 369 (1994).
[2] K. Rybicki, R. Ryłko, *Phys. Lett.* **B353**, 547 (1995).