BUNCHING PARAMETERS AND MULTIPLICITY FLUCTUATIONS IN HADRON–HADRON COLLISIONS

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We apply the bunching-parameter analysis to the hadron–hadron collisions within the FRITIOF model. The monofractal structure of intermittency is observed, in contrast to the multifractal structure in the e^+e^- annihilation. The unusual enhancement of the second-order bunching parameter is a direct manifestation of the enhanced void probability.

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1. Introduction

The analogy between photon-counting in quantum optics and multiple production in high-energy physics has been known for quite a long time [1]. Recently the stochastic method of continuous measurement in quantum optics has been proposed to analyze the fractal structure of multiplicity distributions in high energy collisions[2, 3]. The intermittent structure of the multiplicity distribution is studied in terms of the bunching parameters η_q defined as

$$\eta_q = \frac{q}{q-1}\,\frac{P(q)\;P(q-2)}{[P(q-1)]^2}\;, \qquad q \geq 2\;, \tag{1}$$

where $P(n)$ is the multiplicity distribution. Conventionally the fractal behavior in multiparticle production has a straightforward manifestation in the study of the normalized factorial moments F_q defined as[4]

$$F_q = \frac{\sum\limits_{n=q}^{\infty} n(n-1)\cdots(n-q+1)\;P(n)}{\left[\sum\limits_{n=1}^{\infty} n\;P(n)\right]^q}\;, \qquad q \geq 2\,. \tag{2}$$

These two sets of parameters, η_q and F_q, carry the same information of the multiplicity distribution $P(n)$ and can be translated into each other. Given the set $\{F_q\}$ with all orders q, the multiplicity distribution $P(n)$ can be obtained with one more parameter, *e.g.*, the average multiplicity $\langle n \rangle$. Similarly, given the set $\{\eta_q\}$ with all orders q, $P(n)$ can also be determined up to one free parameter, *e.g.*, the ratio $P(1)/P(0)$. With one more parameter, the multiplicity distribution $P(n)$ can be reconstructed from either the complete set $\{F_q\}$ or $\{\eta_q\}$. For the incomplete sets, these two types of parameters have different applications. The normalized factorial moments F_q provide the information for the global shape of the multiplicity distribution with more weighting put on the large multiplicity tail. The bunching parameters η_q probe into the local structure around the multiplicity $n = q - 1$. These two approaches can be taken as complementary to each other.

In Ref. [3], the comparison of the normalized-factorial-moment and bunching-parameter analysis is studied in the JETSET 7.4 PS model [5]. The behavior of bunching parameters η_q for hadrons produced in e^+e^- annihilation at 91.2 GeV is analyzed, and the azimuthal angle ϕ is used as a phase-space variable. As the size of the phase space decreases, all orders of bunching parameters increase with a power-like behavior. The multifractal structure of intermittency is concluded as an inherent feature of fluctuations in the azimuthal angle.

In this paper, we apply the bunching-parameter analysis to the hadron–hadron collisions. The fluctuations of the final pions produced in the $\bar{p}p$ collisions are studied with the FRITIOF 7.02 model taken as the event generator [6]. Instead of the azimuthal angle ϕ, the pseudorapidity η is used as the phase-space variable. The monofractal structure of intermittency is observed, in contrast to the multifractal structure in the e^+e^- annihilation. The unusual enhancement of the second-order bunching parameter η_2 is a direct manifestation of the enhanced void probability, which is the characteristic of rapidity-gap events.

In Section 2, we review some of the fractal structures revealed by the bunching-parameters and normalized-factorial-moment analysis. In Section 3, we present the numerical results from the Monte Carlo simulations. In Section 4, we give the conclusion.

2. Multifractal and monofractal

For the distribution within a very small phase space, the average multiplicity is small and only the first few orders of the parameter are important, no matter the parameter-set is η_q, F_q, or $P(q)$. In such case, it had been shown that the following approximate relations are valid, [2]

$$F_q \sim \prod_{m=2}^{q} (\eta_m)^{q+1-m} \quad , \tag{3}$$

or equivalently

$$\eta_q \sim \frac{F_q \, F_{q-2}}{(F_{q-1})^2} \quad . \tag{4}$$

We notice that the above relations are exact for the binomial distributions, both for positive and negative binomial distributions. In general, the multiplicity distributions in high-energy collisions can be described quite well by the negative binomial distributions defined as [7]

$$P(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \left(\frac{\langle n \rangle}{k}\right)^n \left(1 + \frac{\langle n \rangle}{k}\right)^{-n-k} \quad , \tag{5}$$

where the two free parameters are $\langle n \rangle$ and k. For this distribution, we have

$$F_q = \prod_{i=1}^{q-1} \left(1 + \frac{i}{k}\right) \quad , \tag{6}$$

$$\eta_q = 1 + \frac{1}{k+q-2} \quad . \tag{7}$$

It is straightforward to check that Eqs. (3) and (4) become exact for the above expressions.

The fractal structure of intermittency is understood in the language of normalized-factorial-moments F_q as the power-law divergence of the moments when the phase-space δ decreases, *i.e.*,

$$F_q(\delta) \propto \delta^{-\phi_q} \quad , \tag{8}$$

where ϕ_q are the intermittency indices and can be related to the anomalous dimensions of the system. In the bunching-parameter analysis, with Eq. (4), the same fractal structure is revealed as

$$\eta_q(\delta) \propto \delta^{-\phi_q - \phi_{q-2} + 2\phi_{q-1}} \quad . \tag{9}$$

There are two kinds of fractal structures and we would like to determine which one belongs the high-energy multiproduction. The first kind is the multifractal structure characterized by the intermittency indices

$$\phi_q = \frac{d_2}{2}\, q(q-1) \tag{10}$$

This feature occurs in the random cascade model of the high-energy fluctuation phenomenology [4]. The corresponding structure in bunching parameters η_q is characterized by the same power-law behavior for all the bunching parameters

$$\eta_q(\delta) \;\propto\; \delta^{-d_2} \quad \text{for } q \geq 2 \ . \tag{11}$$

The second kind is the monofractal structure characterized by the intermittency indices

$$\phi_q = d_2 \ (q-1) \ . \tag{12}$$

This feature is expected for the fluctuations in the second-order phase transition [8]. The corresponding behaviors of the bunching parameters are

$$\eta_2(\delta) \;\propto\; \delta^{-d_2} \quad \text{and} \quad \eta_q(\delta) \sim const. \quad \text{for } q > 2 \ . \tag{13}$$

For high energy collisions, the fluctuations become more violent as the phase space decreases. The normalized factorial moments are observed to increase with the decreasing phase space. To determine the fractal structure of the system, one has to interpolate the slope of the power-like behavior of the normalized factorial moments to obtain the intermittency indices. In the above two cases, multifractal in Eq. (10) and monofractal in Eq. (12), all orders of F_q increase with the decreasing phase space. Though the increasing rates are different, it is not easy to discern the difference. On the contrary, the bunching-parameter analysis provides a clear way to discern one from the other. If all orders of η_q show the power-law behaviors, it is multifractal. If only the second-order η_2 diverges, it is monofractal.

In the case of the widely used negative binomial distributions, Eq. (5), the fractal structure is controlled by the parameter k. A power-like behavior of F_q is expected when the parameter k decreases with decreasing phase space. As only the second order η_2 diverges accordingly, see Eq. (7), it is the monofractal structure. However, the observation of monofractal structure will not guarantee the validity of negative binomial distributions, while the observation of multifractal structure would certainly imply that the negative binomial distributions are not valid.

3. Fluctuations in the FRITIOF 7.02 model

The general features of hadronic final-state fluctuations can be studied by simulating high-energy collisions according to Monte Carlo models. In this section, we apply the bunching-parameter analysis to the final charged pions produced in $\bar{p}p$ collisions using the FRITIOF 7.02 model [6]. It is expected that the behavior of the higher-order bunching parameters can provide a discriminator to distinguish the monofractal structure from the multifractal one.

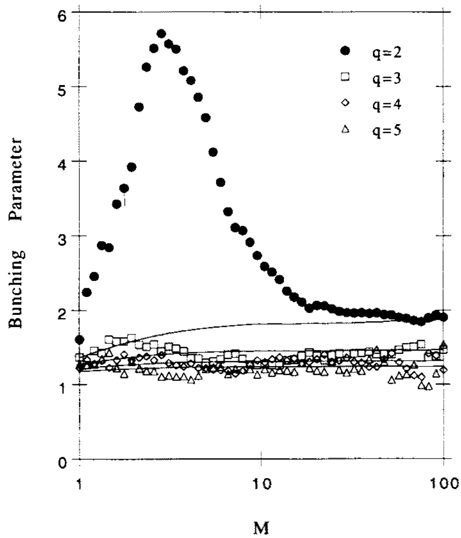

Fig. 1. Bunching parameter η_q as a function of the number of bins M_η in the pseudorapidity range $|\eta| \leq 5$. The solid lines are the predictions of the negative binomial distributions.

We generate 50,000 minimum biased events at CERN S$\bar{\text{p}}$pS energy $\sqrt{s} = 540$ GeV with the default parameters. The charged pions produced from the non-single-diffractive events are recorded within the pseudorapidity range $-5 \leq \eta \leq 5$. The multiplicity distributions $P(n)$ for various sizes of pseudorapidity intervals are obtained. The corresponding bunching parameters η_q and normalized factorial moments F_q are calculated with Eqs (1) and (2), respectively. In the following, we study and compare the M_η-dependences of η_q and F_q for symmetrical bins $|\eta| \leq \frac{5}{M_\eta}$. We note that in contrast to the usual data analysis, the average over M_η-bins is not performed. Fig. 1 shows the values of η_q as a function of M_η for the first few ranks q, where M_η is the number of partitions of the pseudorapidity range $-5 \leq \eta \leq 5$. Except η_2, all the higher-order η_q do not show M_η-dependence, which is a clear indication for the underlying monofractal structure. As the number of partitions increases, the second-order η_2 increases significantly to the value over 5 when the pseudorapidity range decreases from $\Delta\eta = 10$ down to $\Delta\eta = 3$. With further decreasing of the phase space the effect of antibunching begins to set in and η_2 decreases and then saturates at a value less than 2.

To further probe into this unusual enhancement of bunching and antibunching of the second order η_2, we estimate the behavior of η_q predicted by the negative binomial distributions. The results are also shown in the same figure. Note that the two free parameters of the negative binomial distributions are simply fixed by the observed average multiplicity and dispersion, not the best-fit of the data. For the higher-orders η_q, the predictions are

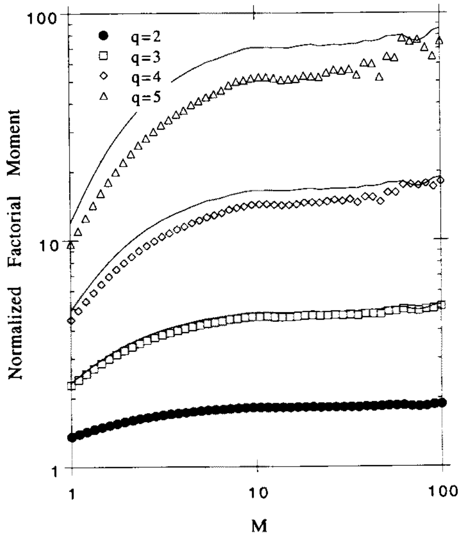

Fig. 2. Normalized factorial moments F_q as a function of the number of bins M_η in the pseudorapidity range $|\eta| \leq 5$. The solid lines are the predictions of the negative binomial distributions.

consistent with the data. In the case of η_2, the negative binomial distributions predict a slow and monotonous increasing. The enhancement of η_2 observed in the data is totally missed by the prediction of the negative binomial distributions.

To compare with the normalized-factorial-moment analysis, we present in Fig. 2 the behavior of F_q as a function of M_η. The normalized factorial moments tend to increase with the decreasing of phase space. Again, the predictions from the negative binomial distributions are also shown in the same figure. The predictions overestimate the fluctuations in the higher moments; note that we do not perform the best fit of the data. However, the features observed in the data can be fully reproduced in the predictions of the negative binomial distributions.

In Fig. 3 we present the M_ϕ-dependence of η_q in the azimuthal angle ϕ, where M_ϕ is the number of partitions of the full azimuthal angle $-\pi \leq \phi \leq \pi$. It is observed that all the bunching parameters are independent of the size of the phase space, *i.e.*, no fractal structure in the transverse direction. Similar results are also observed in the M_ϕ-dependence of F_q. We note that these observations only imply that there is no M_ϕ-dependence when η_q and F_q are evaluated within the full pseudorapidity range $|\eta| \leq 5$, *i.e.*, M_η=1. For the restricted pseudorapidity intervals, $M_\eta \neq 1$, the M_ϕ-dependence is observed in both η_q and F_q.

To present the fractal structure in both longitudinal and transverse directions simultaneously, we show in Fig. 4 the (M_η, M_ϕ)-dependence of the second order η_2, where M_η and M_ϕ are the numbers of partitions in longitu-

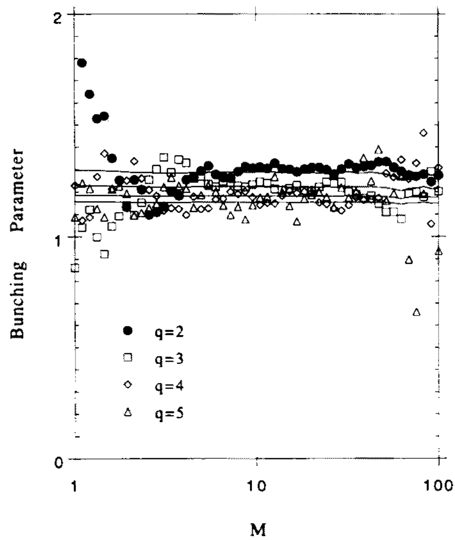

Fig. 3. Bunching parameter η_q as a function of the number of bins M_ϕ in the azimuthal angle ϕ. The solid lines are the predictions of the negative binomial distributions. Notice that the scale is different from that of Fig. 1.

dinal and transverse directions, respectively. The total number of partitions in the phase space is then $(M_\eta \times M_\phi)$. The dependences on both M_η and M_ϕ are clear observed. In the special cases of $M_\eta = 1$ and $M_\eta >> 1$, the values of η_2 become independent of M_ϕ. Also in the case of $M_\phi >> 1$, the values of η_2 become independent of M_η. The effect of η_2-enhancement in the longitudinal direction decreases as the partitions in transverse direction increases. All the higher-orders η_q do not show significant dependence on the partitions in both directions.

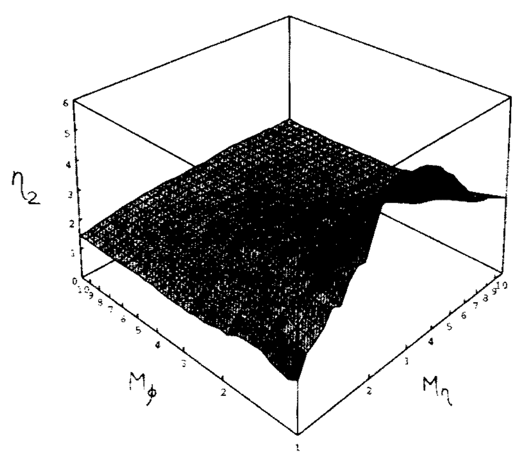

Fig. 4. Bunching parameter η_2 as a function of the number of bins (M_η, M_ϕ) in both longitudinal and transverse directions.

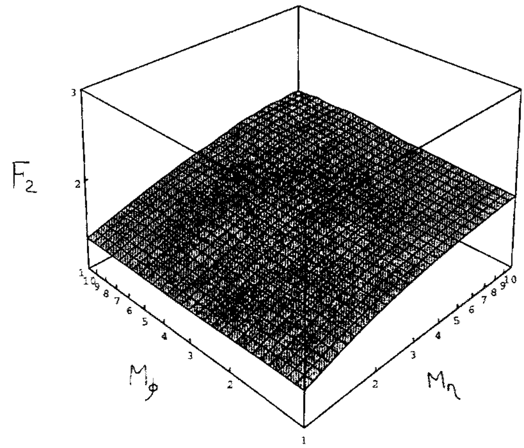

Fig. 5. Normalized factorial moment F_2 as a function of the number of bins (M_η, M_ϕ) in both longitudinal and transverse directions. Notice that the scale is different from that of Fig. 4.

In comparison, we present in Fig. 5 the behavior of F_2 as a function of (M_η, M_ϕ). The dependences on both M_η and M_ϕ are also observed, but not as strong as in the case of η_2. All the higher-orders F_q show a similar dependence on the partitions in both directions. We note that the observed power-like behavior has a much wider range of partitions, in this case $(M_\eta \times M_\phi)$.

4. Conclusion

The phenomena of intermittent dynamical fluctuations can be described in terms of bunching parameters, complementary to the normalized factorial moments. One of the important properties of the bunching parameters is that the analysis is not affected by the experimental statistical bias which arises in the normalized factorial moments when the bin size becomes very small. Moreover, there is a trivial tendency in the behavior of the normalized factorial moments that the value of the moment monotonously increases with its order. On the contrary, the bunching parameters can have any kind of behavior when taken as a function of its order.

The multiplicity fluctuations of final hadrons produced in the hadron–hadron collisions have been studied by means of the bunching parameters. The fractal behavior has been identified with the monofractal structure, in contrast to the multifractal structure observed in the e^+e^- annihilation. In the study of one dimensional partitions, M_η or M_ϕ, the fractal structure results from the multiplicity fluctuations in the longitudinal direction, *i.e.*, the pseudorapidity η. There is no fractal structure in the transverse direction, *i.e.*, the azimuthal angle ϕ. In the study of two dimensional partitions, both M_η and M_ϕ, the observed fractal structure is the combined results from both directions.

In studying the bin-size dependence of parameters η_q and F_q, in contrast to the conventional bin-averaged parameters, we focus on the symmetrical bins centered at $(\eta, \phi) = (0,0)$, *i.e.*, $|\eta| \leq \frac{5}{M_\eta}$ and $|\phi| \leq \frac{\pi}{M_\phi}$. In the two dimensional partitions, the values of η_2 reveal a strong dependence on the ways of partitions, see Fig. 4. With fixed number of partitions $(M_\eta \times M_\phi)$, η_2 has different values for different choices of M_η and M_ϕ. Such variations can be related to the different production mechanism within different kinematic regions. The observed features will be smeared when one studies the bin-averaged parameters.

Besides the fractal structure, the bunching-parameter analysis also reveals the unusual enhancement of the second order η_2, which has not been revealed in the normalized-factorial-moment analysis at all. This unusual behavior of η_2 is outside the validity of Eqs (3) and (4), where $\eta_2 \sim F_2$ is expected. This enhancement of η_2 can be traced to the corresponding behavior of the void probability $P(0)$. As the bunching parameters probe into the local structure of multiplicity distribution, the enhancement of the void probability has a direct manifestation on η_2, which is proportional to $P(0)$. As to the normalized factorial moments, on the contrary, the variation of $P(0)$ only causes a small change in the normalization factor, which is not so easy to discern. With recent interest in the rapidity-gap events, the bunching-parameter analysis is expected to provide a much more direct manifestation than the conventional normalized-factorial-moment analysis.

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