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# RELATIVISTIC WAVE EQUATION FOR HYPOTHETIC COMPOSITE QUARKS*

W. Królikowski

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00–681 Warszawa, Poland

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A two-body wave equation is derived, corresponding to the hypothesis (discussed already in the past) that $u$ and $d$ current quarks are relativistic bound states of a spin-1/2 preon existing in two weak flavors and three colors, and a spin-0 preon with no weak flavor nor color, held together by a new strong *but* Abelian, vectorlike gauge force. Some nonconventional (though somewhat nostalgic) consequences of this strong Abelian binding within composite quarks are pointed out. Among them are: new tiny magnetic-type moments of quarks (and nucleons) and new isomeric nucleon states possibly excitable at some high energies. The latter may arise through a rearrangement mechanism for quark preons inside nucleons. In the interaction $q\,\bar{q} \to q\,\bar{q}$ of preon-composite quarks, beside the color forces, there act additional exchange forces corresponding to diagrams analogical to the so called dual diagrams for the interaction $\pi\pi \to \pi\pi$ of quark-composite pions.

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## 1. Introduction

The idea of composite quarks and/or leptons returns from time to time to the physicists' attention, in spite of its rather poor predictive power, at any rate, in comparison with the Standard Model and even its supersymmetric and grand-unification extensions. The reason, not only of psychological nature, is that such an idea corresponds to the main avenue of historical developments in particle physics, from its composite molecular and atomic levels to its composite nuclear and hadronic levels. Therefore, some recent high-energy signals of possible excess of jet rate (at large transverse momenta) over predictions of the conventional perturbative QCD [1], although tenuous (and probably premature), are welcome by a considerable part of

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the physical community. However, it was already argued that this apparent deviation, if real, might still have another (than compositness), more standard origin: an additional interaction of elementary quarks, mediated by a new, very heavy, neutral vector boson [2], practically not interacting with leptons (leptophobic). A possibility of new, not leptophobic, vector bosons exists *a priori* as well.

In Ref. [3] we constructed and discussed at some preliminary level a relativistic two-body wave equation corresponding to the particular hypothesis [4, 5, 6] that $u$ and $d$ current quarks are, in fact, very tight relativistic bound states of two (more elementary) very heavy constituents: one with spin 1/2 and one with spin 0. More precisely, in Ref [3] we assumed that these constituents are: ($i$) a spin-1/2 preon $Q$ with charge (2/3,−1/3) in units $e$ and baryon number 1/3, existing in two weak flavors $U$ , $D$ and three colors, and ($ii$) a spin-0 preon $S$ with charge 0 and no weak flavor nor color. They are held together by a new, strong Abelian gauge force (called "ultra-electromagnetic" force), mediated by some massless neutral vector bosons ("ultraphotons" $\Gamma$). On the level of quantum mechanics, this interaction is approximately (in fact, rather qualitatively) described by a new, strong Coulomb-type potential $-\alpha^{(\mathrm{u})}/r$ with $\alpha^{(\mathrm{u})} \equiv e^{(\mathrm{u})2}$ (an analogue of $-\alpha/r$ with $\alpha \equiv e^2$). This is the static, one-ultraphoton-exchange attraction between both preons, $Q$ and $S$, supposed to carry opposite "ultracharges" $e^{(\mathrm{u})}$ and $-e^{(\mathrm{u})}$, respectively (*a priori*, $e^{(\mathrm{u})} > 0$ or $< 0$). Thus, $u$ and $d$ quarks are the bound states[1]

$$u = (U\,S) \quad , \quad d = (D\,S) \quad . \tag{1}$$

Then, their charge (in units $e$) and weak hypercharge follow correctly as $Q = I_3^{(\mathrm{L})} + Y/2 = (2/3\,,\,-1/3)$ and $Y = 2I_3^{(\mathrm{R})} + B = 1/3$ or $(4/3\,,\,-2/3)$ with $I_3^{(\mathrm{L})} = (\pm 1/2)$ or 0 and $I_3^{(\mathrm{R})} = 0$ or $(\pm 1/2)$, depending on their left (L) or right (R) chirality, respectively.

We assumed that the new vectorlike gauge group U(1), generated by the ultracharge, describes a new local symmetry in the physical world, and that it commutes with the Standard Model group SU(3)×SU(2)×U(1) [7]. This acts, of course, in the standard way on the color-triplet "ultraquarks" $U$ , $D$, forming one left weak isodoublet and two right weak isosinglets with baryon number 1/3, as well as on the color-singlet "ultrascalar" $S$ being a weak isosinglet with weak hypercharge 0. In contrast to $\gamma$, the ultraphoton $\Gamma$, being the gauge boson of the new U(1) group, is a total scalar of

[1] Another option (not discussed in this paper; *cf.* Ref. [4,5,6]) is that the ultracharged constituents of $u$ and $d$ quarks are: ($i$) a spin-1/2 preon $L$ with charge (1 , 0) in units $e$ and lepton number —1, existing in two weak flavors $L^+$ $L^0$, and ($ii$) a spin-0 preon $S$ with charge —1/3 in units $e$, baryon number 1/3 and lepton number 1, appearing in three colors. They may be called "ultraleptons" and "ultraleptoquarks", respectively. In this option $u = (L^+\,S)$ and $d = (L^0\,S)$.

the Standard Model group. It is also a strong isoscalar. This forbids the ultraelectromagnetic decays $\Sigma^0 \rightarrow \Lambda\Gamma$ and $\pi^0 \rightarrow 2\Gamma$, in contrast to the electromagnetically allowed familiar processes $\Sigma^0 \rightarrow \Lambda\gamma$ and $\pi^0 \rightarrow 2\gamma$ [2]. At the end of Section 4 we shall briefly comment on the possibility of breaking our U(1) local symmetry, leading to a massive ultraphoton.

Let us stress that the preon-composed quarks $q = (Q\,S)$ and antiquarks $\bar{q} = (\bar{Q}\,\bar{S})$ can interact strongly $q\,\bar{q} \rightarrow q\,\bar{q}$ not only chromodynamically *i.e.*, through the exchange of gluons coupled to their colored constituents $Q$ and $\bar{Q}$, but also through the exchange of $Q\,\bar{Q}$ and $S\,\bar{S}$ pairs emerging from the lines of their $Q$ and $S$ constituents [in analogy to the so called *dual diagrams* $\pi\,\pi \rightarrow \pi\,\pi$ for quark-composed pions $\pi = (q\,\bar{q})$]. In the case of very heavy preons and also their bound states $(Q\,\bar{Q})$ and $(S\,\bar{S})$ [in contrast to the light $(Q\,S)$; *cf.* Section 3], this exchange leads to an effective contact interaction for composite quarks $q = (Q\,S)$. Obviously, such an interaction should be significant only at energies high enough (if we were lucky, this might happen already in the experiment of Ref. [1]).

In general, according to our hypothesis, the up and down quarks of three generations ought to be composed of ultraquarks $U$, $D$ of three corresponding generations and the ultrascalar $S$.

It should be emphasized that this particular preon model, containing the color ultraquarks $U$, $D$ (appearing within composite color quarks) and ordinary ultracharge-neutral leptons $l^0 \equiv \nu_l$, $l^-$ (assumed to be elementary), is chiral-anomaly free in each of three fermion generations if considered in the Standard Model sector. As a whole, however, our model is not chiral-anomaly free because of the nonvanishing triangle anomalies $W^+W^-\Gamma$ and $ZZ\Gamma$, unless the contributions from three generations of ultraquarks $Q_i = (U_i\,,\, D_i)$ $(i = 1,2,3)$ cancel. This happens, if the ultracharges of $Q_i$ are $Q_i^{(u)} e^{(u)}$ with $\sum_i Q_i^{(u)} = 0$ $(Q_i^{(u)} \neq 0)$.

To set an example suppose that there are two broken horizontal-SU(3)-group triplets of preons, one $(Q_1\,,\, Q_2\,,\, Q_3)$ of ultraquarks and one $(\bar{S}_1\,,\, \bar{S}_2\,,\, \bar{S}_3)$ of antiultrascalars, both with ultracharges $(1\,,\,-1/2\,,\,-1/2)$ in units $e^{(u)}$ [or $(2/3\,,\,-1/3\,,\,-1/3)$ in units $(3/2)e^{(u)}$]. It means that $Q^{(u)} e^{(u)} \equiv \operatorname{diag}(Q_1^{(u)}\,,\, Q_2^{(u)}\,,\, Q_3^{(u)}) e^{(u)} = [(1/2)\lambda_3 + (1/2\sqrt{3})\lambda_8](3/2)e^{(u)}$ in terms of horizontal Gell–Mann matrices. Then, the composite quarks of three generations may be given, after breaking the horizontal SU(3) group, as the

[2] A similar argument cannot forbid the decays $\eta \rightarrow 2\Gamma$ and $\eta' \rightarrow 2\Gamma$, since both $\eta$ and $\eta'$ contain some flavor-SU(3)-singlet parts beside their flavor-SU(3)-octet parts. Here, the smallness of quark magnetic-type moments, responsible for a tiny coupling of ultracharge-neutral composite quarks to the ultraphoton $\Gamma$, must be invoked [*cf.* Eq. (40)].

bound states $q_1 = (Q_1 S_1)$, $q_2 = (Q_2 S_2)$, $q_3 = (Q_3 S_3)$ with masses $m_{q_1} < m_{q_2} \ll m_{q_3}$ (and very high preon masses $m_{Q_1} \simeq m_{S_1}$, $m_{Q_2} \simeq m_{S_2}$, $m_{Q_3} \simeq m_{S_3}$; *cf.* Section 3 as to the smallness of $m_{q_1}$ caused by the requirements of $m_{Q_1} \simeq m_{S_1}$ and $\alpha^{(u)} \simeq 2$). Perhaps, the ultracharge-neutral color-triplet bound states $(Q_2 S_3)$ and $(Q_3 S_2)$ may also exist (if, really, only $\Gamma$ contributes here to Coulombic potentials), but with very high masses of the order of preon masses (when $m_{Q_2}$ and $m_{Q_3}$ are significantly different from $m_{S_3}$ and $m_{S_2}$, respectively). Beside ordinary hadrons, there should also exist extra ultracharge-neutral colorless bound states $(Q_1 Q_2 Q_3)$, $(Q_i \bar{Q}_i)$ and $(S_1 S_2 S_3)$, $(S_i \bar{S}_i)$ with baryon number 1, 0 and 0, 0, respectively. Their spin would be 1/2 or 3/2, 0 or 1 and 0, 0, their charge in units $e$ — (1, 0) or (2, 1, 0, $-1$), (1, 0, $-1$) or 0 and 0,0. In the first category there should appear also the bound states $(Q_1 Q_2 Q_2)$, $(Q_1 Q_3 Q_3)$, $(Q_2 \bar{Q}_3)$ and, perhaps, in the second category — $(S_1 S_2 S_2)$, $(S_1 S_3 S_3)$, $(S_2 \bar{S}_3)$ (if only $\Gamma$ contributes here to Coulombic potentials).All these extra ultracharge-neutral colorless bound states would be expected to get very high masses of the order of preon masses and to be highly unstable in ultraelectromagnetic and color interactions.

In this context, notice that the Cabibbo–Kobayashi–Maskawa mixing of composite quarks $d$, $s$, $b$ requires (at the phenomenological level) the appropriate mixing of preon pairs $D_1 S_1$, $D_2 S_2$, $D_3 S_3$. This may suggest the existence of broken horizontal-SU(3)-group couplings $g^{(h)} \bar{Q}_i \gamma^\mu (1/2) \lambda_a^{ij} Q_j V_\mu^a$ and $-g^{(h)} \bar{S}_i^\dagger i \overleftrightarrow{\partial}^\mu (1/2) \lambda_a^{ji} S_j V_\mu^a$. Here, $\lambda_a$ $(a = 1, \ldots, 8)$ are (horizontal) Gell-Mann matrices and $V_\mu^a$ denote for $a \neq 3$, 8 massive ultracharged vector fields [with ultracharges $\pm 1$ in units $(3/2)e^{(u)}$], while the ultracharge-neutral vector fields $V_\mu^3$ and $V_\mu^8$ form two independent linear combinations, one definitely massless and one likely to be also massless (*cf.* Footnote[4] in Section 5), the former describing the ultraphoton $\Gamma$. It is so, if $g^{(h)} = (\sqrt{3}/2)e^{(u)}$ and the massless ultraphoton $\Gamma$ is given by $A_\mu^{(u)} = (\sqrt{3} V_\mu^3 + V_\mu^8)/2$. Then, the second combination $(-V_\mu^3 + \sqrt{3} V_\mu^8)/2$ describes a new horizontal ultracharge-neutral vector boson. This is coupled to a new diagonal horizontal charge, $[-(1/2\sqrt{3})\lambda_3 + (1/2)\lambda_8](3/2)e^{(u)} = \mathrm{diag}\,(0\,,1\,,-1)(\sqrt{3}/2)e^{(u)}$, like $\Gamma$ is coupled to the ultracharge $Q^{(u)} e^{(u)}$ (so it is *not active* for the first preon generation). Of course, in contrast to ultraquarks and ultrascalars, leptons of three generations (being ultracharge-neutral and elementary) do not interact with the horizontal vector bosons presented by $V_\mu^a$. It should be kept in mind that the transition amplitude for $q_j \to q_i$ $(j \neq i)$ is diminished by the factor $1/m_V^2$, where $m_V$ is the mass of the heavy vector boson $V^a$ $(a \neq 3, 8)$ exchanged between the ultraquark and ultrascalar, while the amplitude for $q_j \to q_i \Gamma$ $(j > i)$ is damped additionally by the average preon distance $\langle r \rangle$ of the order $O(1/2m_{\mathrm{preon}})$, because ultracharged very heavy preons are bound very tightly within composite quarks (and their virtual excited states acting effectively). Similarly, the amplitude for $q_j \to q_i \gamma$ $(j > i)$ is also damped additionally by $O(1/2m_{\rm preon})$ (in this case, it contains $\sqrt{\alpha} < \sqrt{\alpha^{(u)}}$, and, moreover, only ultraquarks, as charged, contribute to the $\gamma$ emission).

In consequence of the Abelian character of ultraelectromagnetic force, the ultracharged preons $U$, $D$ and $S$ would not be confined for ever within the ultracharge-neutral quarks [5], though their binding should be rather strong. The stronger this binding, the better description for hadronic phenomena (at energies not too high) would be provided by the effective QCD operating with composite quarks coupled effectively to ordinary gluons $g$ (assumed here to be elementary like all other gauge bosons: $\gamma$, $W^{\pm}$, $Z$, $\Gamma$ and, possibly, horizontal vector bosons other than $\Gamma$).

In fact, at energies high enough, the ultracharge-neutral quarks could be split within highly excited hadronic states into ultracharged preons $U$, $D$ and $S$. If the energy is sufficient, this splitting might cause the "ultraionization" of hadrons into some ultracharged (though always colorless) debris, possibly accompanied (if decelerated) by ultraphotons $\Gamma$ forming then "ultrabremsstrahlung". In particular, the colliding nucleons

$$p = [(U\,S)\ (U\,S)\ (D\,S)] \quad , \quad n = [(U\,S)\ (D\,S)\ (D\,S)] \quad , \tag{2}$$

where $U$, $D$ (and $S$) are ultraquarks (and an ultrascalar) of the first generation, might lead, for example, to the following ultraionization processes:

$$p + p \to p + [U\ (U\,S)\ (D\,S)] + S \tag{3}$$

and

$$p + \bar{p} \to p + [\bar{U}\ (\bar{U}\,\bar{S})\ (\bar{D}\,\bar{S})] + \bar{S}\ . \tag{4}$$

In a similar way, the colliding pions

$$\begin{aligned}
\pi^+ &= [(U\,S)\ (\bar{D}\,\bar{S})]\,, \\
\pi^0 &= \frac{1}{\sqrt{2}}\,[(U\,S)\ (\bar{U}\,\bar{S}) - (D\,S)\ (\bar{D}\,\bar{S})]\ , \\
\pi^- &= [(D\,S)\ (\bar{U}\,\bar{S})]\ .
\end{aligned} \tag{5}$$

might result, for example, in the following ultraionization reactions:

$$p + \pi^+ \to p + [U\ (\bar{D}\,\bar{S})] + S \tag{6}$$

and

$$p + \pi^- \to p + [(D\,S)\ \bar{U}] + \bar{S}\ . \tag{7}$$

Perhaps, much below their ultraionization energies but still at some high energies, colliding nucleons, due to a tunnelling mechanism for quark preons, might be excited to their rearranged isomeric states

$$p^* = [(U\,U\,D)\;S\,S\,S]\;,\;n^* = [(U\,D\,D)\;S\,S\,S]\;. \tag{8}$$

In these new, hypothetic nucleonic states a triple-ultracharged colorless core $(Q\,Q\,Q)$ would be surrounded by three colorless ultrascalars S bound through the "ultraelectric" attraction of this core. Of course, the wave function of the colorless bound state $(Q\,Q\,Q)$ of three color-triplet fermions $Q$ should include (in the ground state) a fully symmetric spin-1/2×isospin-1/2 part, analogical to that of the nucleon (naturally, the excitation of this part to a fully symmetric spin-3/2×isospin-3/2 part, analoguous to that of the $\Delta$ nucleon isobar, would occur frequently).

Thus, it might happen at some high energies that, for instance, $p\,p \to p\,p^*$ or $p^*\,p^*$ and $p\,\bar{p} \to p\,\bar{p}^*$ or $p^*\,\bar{p}$ or $p^*\,\bar{p}^*$ as well as $p\pi^0 \to p^*$ and $p\,\pi^- \to n^*$. Of course, $p^*$ and $n^*$ could be excited also in lepton-nucleon and photon-nucleon scattering. Since they would be highly unstable in ultraelectromagnetic and color interactions, in some cases they might play the role of broad heavy resonances. *A priori*, the excitation of individual quarks $q = (QS)$ might be also taken into account [*c.f.*, however, the comment after Eq. (29)].

Similarly, due to a tunnelling mechanism, colliding pions might excite at some high energies their rearranged isomeric states

$$\begin{aligned}
\pi^{+*} &= (U\,\bar{D}) + (S\,\bar{S})\;,\\
\pi^{0*} &= \frac{1}{\sqrt{2}}\left[(U\,\bar{U}) - (D\,\bar{D})\right] + (S\,\bar{S})\;,\\
\pi^{-*} &= (D\,\bar{U}) + (S\,\bar{S})
\end{aligned} \tag{9}$$

which here would be split into pairs of simpler spin-0 particles, still ultracharge-neutral and colorless, but likely to be unstable with strong decay rates. In fact, by ultraelectromagnetic and/or color interactions $(S\,\bar{S}) \to 2\Gamma$ (and/or hadrons) and $(U\,\bar{D}) \to \pi^+\,\Gamma$ (and/or hadrons), if the masses are sufficient.

The Abelian nature of ultraelectromagnetic force has also another, in principle observable consequence that the ultracharge-neutral, preon-composite quarks (and so, the quark-composite nucleons too) should display nonzero internal "ultramagnetic" moments [4], much like the charge-neutral, quark-composite neutron displays a nonzero internal magnetic moment. Then, the ultracharge–neutral nucleons should reveal an additional interaction (with each other and with ultraphotons) caused by their ultramagnetic moments.

While the ultramagnetic moments of spin-1/2 preons $U$, $D$ have the same Dirac-type value $\mu^{(u)}_{\rm preon} = e^{(u)}/2m_{\rm preon}$ (if $U$, $D$ are assumed to have the same $e^{(\mathrm{u})}$ and $m_{\mathrm{preon}}$), the ultramagnetic moments of ultracharge-neutral $u$ and $d$ quarks are both equal to $\mu^{(\mathrm{u})}_{\mathrm{quark}} \equiv e^{(\mathrm{u})}_{\mathrm{eff}}/2M_{\mathrm{quark}}$. Here, $M_{\mathrm{quark}} \simeq M_{\mathrm{nucleon}}/3$ stands for the constituent quark mass and $e^{(\mathrm{u})}_{\mathrm{eff}}$ has to be calculated from the wave equation for composite quarks (*cf.* Section 4). Such a calculation shows that the ultramagnetic moments of proton and neutron are both equal to

$$\mu^{(\mathrm{u})}_{\mathrm{nucleon}} = \mu^{(\mathrm{u})}_{\mathrm{quark}} \to -\mu^{(\mathrm{u})}_{\mathrm{preon}} \tag{10}$$

when $\alpha^{(\mathrm{u})} \equiv e^{(\mathrm{u})\,2} \to 2$. Thus, when $\alpha^{(\mathrm{u})} \to 2$,

$$\left(-e^{(\mathrm{u})}_{\mathrm{eff}}\right) : e^{(\mathrm{u})} \to M_{\mathrm{quark}} : m_{\mathrm{preon}} \ll 1\ . \tag{11}$$

Hence, the proton magnetic moment $\mu_p \simeq e/2M_{\mathrm{quark}} \simeq 3e/2M_{\mathrm{nucleon}}$ is expected to be much larger in its magnitude than the proton ultramagnetic moment that is equal to $\mu^{(\mathrm{u})}_{\mathrm{nucleon}} = e^{(\mathrm{u})}_{\mathrm{eff}}/2M_{\mathrm{quark}} \simeq 3e^{(\mathrm{u})}_{\mathrm{eff}}/2M_{\mathrm{nucleon}}$. It is so, because from Eq. (11)

$$|\mu^{(\mathrm{u})}_{\mathrm{nucleon}}| : \mu_p = |e^{(\mathrm{u})}_{\mathrm{eff}}| : e \to \sqrt{\frac{2}{\alpha}}\,(M_{\mathrm{quark}} : m_{\mathrm{preon}}) \ll 1 \tag{12}$$

when $\alpha^{(\mathrm{u})} \to 2$, although $\sqrt{\alpha^{(\mathrm{u})}/\alpha} \to 16.6 > 1$. For instance, if $m_{\mathrm{preon}} = O(1\ \mathrm{TeV})$ to $O(10\ \mathrm{TeV})$, what gives the reasonable quark size $1/m_{\mathrm{preon}} = O(10^{-16}\ \mathrm{cm})$ to $O(10^{-17}\ \mathrm{cm})$, one gets $|\mu^{(\mathrm{u})}_{\mathrm{nucleon}}| : \mu_p \to O(10^{-2})$ to $O(10^{-3})$ when $\alpha^{(\mathrm{u})} \to 2$.

In the next Section we will present an improved derivation of the relativistic two-body wave equation in order to describe more precisely composite quarks corresponding to our particular preon model.

## 2. Composite-quark wave equation

Consider a system of one spin-1/2 particle and one spin-0 particle which, if isolated from each other, are described by the Dirac equation and the Klein–Gordon equation, respectively. As was shown some years ago (*cf.* the second Ref. [8]), such a system can be described (in the stationary case) by the following set of first-order wave equations:

$$\begin{aligned}
&\left\{E - V - \vec{\alpha}\cdot\left[\vec{p}_1 - e_1\vec{A}(\vec{r}_1)\right] - \beta m_1\right\}\phi(\vec{r}_1,\vec{r}_2) = m_2\phi^0(\vec{r}_1,\vec{r}_2)\ ,\\
&\left[\vec{p}_2 - e_2\vec{A}(\vec{r}_2)\right]\phi(\vec{r}_1,\vec{r}_2) = m_2\vec{\phi}(\vec{r}_1,\vec{r}_2)\ ,\\
&\left\{E - V - \vec{\alpha}\cdot\left[\vec{p}_1 - e_1\vec{A}(\vec{r}_1)\right] - \beta m_1\right\}\phi^0(\vec{r}_1,\vec{r}_2)\\
&-\left[\vec{p}_2 - e_2\vec{A}(\vec{r}_2)\right]\cdot\vec{\phi}(\vec{r}_1,\vec{r}_2) = m_2\phi(\vec{r}_1,\vec{r}_2)\ ,
\end{aligned} \tag{13}$$

where $\vec{\alpha}$ and $\beta$ are the familiar Dirac matrices. This set corresponds to the Klein–Gordon sector of the more extended, reducible set of wave equations for a system of one Dirac particle and one Duffin–Kemmer–Petiau particle (the remaining part is the Proca sector, where the bosonic partner of Dirac particle corresponds to the Proca particle).

In the five-component wave equation (13) there are taken into account an external Abelian gauge field $(A_\mu) = (A^0, -\vec{A})$ (say, ultraelectromagnetic or electromagnetic) as well as a total (internal and external) vector-like potential $V(\vec{r}_1, \vec{r}_2)$ including among others the "ultraelectric" attraction $-\alpha^{(\mathrm{u})}/|\vec{r}_1 - \vec{r}_2|$ ($\alpha^{(\mathrm{u})} \equiv e^{(\mathrm{u})\,2}$) and the term $e_1 A^0(\vec{r}_1) + e_2 A^0(\vec{r}_2)$. Of course, all five wave-function components $\phi$, $\phi^0$, $\vec{\phi}$ are here Dirac bispinors.

Eliminating from the set (13) four wave-function components $\phi^0$, $\vec{\phi}$ we get the second-order wave equation for $\phi$:

$$\left(\left\{E - V - \vec{\alpha}\cdot\left[\vec{p}_1 - e_1\vec{A}(\vec{r}_1)\right] - \beta m_1\right\}^2 - \left[\vec{p}_2 - e_2\vec{A}(\vec{r}_2)\right]^2 - m_2^2\right)\phi(\vec{r}_1, \vec{r}_2) = 0 \,. \tag{14}$$

This will be our basic two-body wave equation. In some situations, the case of $m_1 = m_2$ (and $e_1 = -e_2$) may be referred to as the (ideal) case of a supersymmetric particle-antiparticle pair (*cf.* the first Ref. [8]).

After an algebraic manipulation, Eq. (14) can be equivalently rewritten in the form

$$\begin{aligned}\Bigg( & E - V - 2\left\{\vec{\alpha}\cdot\left[\vec{p}_1 - e_1\vec{A}(\vec{r}_1)\right] + \beta m_1\right\} - \frac{e_1}{E-V}\vec{\sigma}\cdot\vec{B}(\vec{r}_1) \\ & + \frac{1}{\sqrt{E-V}}\left\{\left[\vec{p}_1 - e_1\vec{A}(\vec{r}_1)\right]^2 + m_1^2 \right. \\ & \left. - \left[\vec{p}_2 - e_2\vec{A}(\vec{r}_2)\right]^2 - m_2^2\right\}\frac{1}{\sqrt{E-V}}\Bigg)\psi(\vec{r}_1, \vec{r}_2) = 0 \,,\end{aligned} \tag{15}$$

where $\vec{B}(\vec{r}_i) = \mathrm{rot}_i\vec{A}(\vec{r}_i)$ $(i = 1, 2)$ and

$$\psi(\vec{r}_1, \vec{r}_2) \equiv \sqrt{E-V}\,\phi(\vec{r}_1, \vec{r}_2) \;\;, \quad \int d^3r_1\, d^3r_2 |\psi(\vec{r}_1, \vec{r}_2)|^2 = 1 \,, \tag{16}$$

the normalization condition being valid for bound states, both with respect to the internal and the external potential.

Further, introduce to Eq. (15) the total momentum and the momentum transfer of two partons

$$\vec{P} = \vec{p}_1 + \vec{p}_2 \;\;, \quad \vec{p} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_2) \;\;, \tag{17}$$

as well as their canonically conjugate coordinates:

$$\vec{R} = \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2) \ , \ \ \vec{r} = \vec{r}_1 - \vec{r}_2 \ . \tag{18}$$

Obviously, only in the case of equal preon masses $m_1 = m_2 \equiv m$, the coordinate $\vec{R}$ describes the centre-of-mass position and the momentum $\vec{p}$ refers to the relative momentum (although $\vec{P}$ and $\vec{r}$ are always the centre-of-mass momentum and relative coordinate, respectively). In the case of a composite quark confined within a hadron, it is convenient to include the confining interaction through the substitution $m_i \to m_i + S(\vec{r}_i)$ $(i = 1, 2)$, where $S(\vec{r}_i) = M_q/2$ or $\infty$ inside or outside the hadron, respectively, while $M_q$ denotes the constituent quark mass (in the case of a nucleon, $M_N \simeq 3M_q$ is the nucleon mass). Then, Eq. (15) transits into the following form valid for our composite quark inside the confining hadron:

$$\begin{aligned}\{E \ &- V - \vec{\alpha}\cdot\left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right] - \beta M_q \\ &-\vec{\alpha}\cdot\left[2\vec{p} - e_1\vec{A}(\vec{r}_1) + e_2\vec{A}(\vec{r}_2)\right] - \beta 2m_1 - \frac{e_1}{E-V}\vec{\sigma}\cdot\vec{B}(\vec{r}_1) \\ &+\frac{1}{\sqrt{E-V}}\left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right] \\ &\cdot\left[2\vec{p} - e_1\vec{A}(\vec{r}_1) + e_2\vec{A}(\vec{r}_2)\right]\frac{1}{\sqrt{E-V}} \\ &+\left.\frac{(m_1 + \frac{1}{2}M_q)^2 - (m_2 + \frac{1}{2}M_q)^2}{E-V}\right\}\psi(\vec{r}_1, \vec{r}_2) = 0 \ .\end{aligned} \tag{19}$$

Note that in the case of a homogeneous external magnetic-type field $\vec{B}$, where $\vec{A}(\vec{r}_i) = (1/2)(\vec{B}\times\vec{r}_i)$ $(i = 1, 2)$, we can write in Eq. (19)

$$e_1\vec{A}(\vec{r}_1) \pm e_2\vec{A}(\vec{r}_2) = \tfrac{1}{2}(e_1 \pm e_2)(\vec{B}\times\vec{R}) + \tfrac{1}{2}(e_1 \mp e_2)\tfrac{1}{2}(\vec{B}\times\vec{r}) \ . \tag{20}$$

Thus, in the case of $e_1 = -e_2$ there is no magnetic-type external contribution to $\vec{P}$ and also no magnetic-type internal contribution to $\vec{p}$.

Now, due to the weakness of all external interactions of our composite quark inside the confining hadron (in comparison with the internal interactions of preons within the quark), we can replace approximately $E - V$ in the denominators in Eq. (19) by $M_q - V^{\text{in}}$ where $V^{\text{in}}(\vec{r})$ denotes the internal potential dominating the total potential $V(\vec{r}_1, \vec{r}_2)$, while for $u$ and $d$ quarks the internal energy eigenvalues $E^{\text{in}}$ (giving in ground states the quark current masses) are much smaller than their constituent masses $M_q$. The eigenvalues $E^{\text{in}}$ correspond to the situation, when in Eq. (19) we put

$\vec{A}(\vec{r}_i) = 0 \ \ (i = 1,2)$, $V(\vec{r}_1, \vec{r}_2) = V^{\rm in}(\vec{r})$ , $S(\vec{r}_i) = 0 \ \ (i = 1,2)$ and $\vec{P} = 0$:

$$\left[E^{\rm in} - V^{\rm in} - 2(\vec{\alpha}\cdot\vec{p} + \beta m_1) + \frac{m_1^2 - m_2^2}{E^{\rm in} - V^{\rm in}}\right]\psi^{\rm in}(\vec{r}) = 0 \,. \tag{21}$$

This is the relativistic wave equation for internal motion of preons within our composite quark. If $m_1 = m_2 \equiv m$, Eq. (21) becomes the Dirac equation. If $m_1 \neq m_2$ (but $|m_1 - m_2| \ll m_1 + m_2$), the last term in Eq. (21) (with $E^{\rm in}$ replaced there by $E^{\rm in}_{m_1=m_2}$ or even neglected *versus* $V^{\rm in}(\vec{r})$ ) may be treated as a perturbation, and then in the lowest order

$$E^{\rm in} = E^{\rm in}_{m_1=m_2} + \delta E^{\rm in}$$

with

$$\delta E^{\rm in} = -(m_1 - m_2)\langle\psi^{\rm in}|(m_1 + m_2)(E^{\rm in} - V^{\rm in})^{-1}|\psi^{\rm in}\rangle_{m_1=m_2} \,.$$

We will assume that $m_1 = m_2 \equiv m$ or, at least, $m_1 \simeq m_2$. Obviously, this is an *ad hoc* assumption. Note that, in contrast to the bispinor ultraquarks $U$, $D$, a nonzero mass for the (scalar or pseudoscalar) ultrascalar $S$ is, in principle, allowed even before standard SU(2)×U(1) symmetry breaking. However, the bilinear Higgs mechanism may be the origin of ultrascalar mass. Then, the assumption $m_1 \simeq m_2$ may be not unnatural.

It is interesting to note that the hamiltonian resulting now from Eq. (19) implies the following velocity operators:

$$\dot{\vec{R}} \equiv \frac{1}{i}\left[\vec{R}\,,\,H\right] = \vec{\alpha} - \frac{1}{\sqrt{M_q - V^{\rm in}}}\left[2\vec{p} - e_1\vec{A}(\vec{r}_1) + e_2\vec{A}(\vec{r}_2)\right]\frac{1}{\sqrt{M_q - V^{\rm in}}} \tag{22}$$

and

$$\frac{1}{2}\dot{\vec{r}} \equiv \frac{1}{i}\left[\frac{1}{2}\vec{r}\,,\,H\right] = \vec{\alpha} - \frac{1}{\sqrt{M_q - V^{\rm in}}}\left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right]\frac{1}{\sqrt{M_q - V^{\rm in}}}\,, \tag{23}$$

due to specific coupling of the external and internal motion in the last term in Eq. (19) (notice that in Eq. (23), and also later on, $V^{\rm in}(\vec{r})$ can be freely commuted with $\vec{P}$). From Eqs. (22) and (23) we obtain the particle velocity operators

$$\dot{\vec{r}}_{1,2} = \dot{\vec{R}} \pm \dot{\vec{r}}/2 = \vec{\alpha} \pm \vec{\alpha} \mp 2(M_q - V^{\rm in})^{-1/2}\left[\vec{p}_{1,2} - e_{1,2}\vec{A}(\vec{r}_{1,2})\right](M_q - V^{\rm in})^{-1/2}\,,$$

where $(M_q - V^{\rm in})^{-1}$ is of the order $O(1/2m)$ (if $m_1 = m_2 \equiv m$) and $2m \gg M_q > m_q$. Thus, making use of Eq. (22), we can rewrite now the wave equation (19) in the form:

$$\begin{aligned}\{E & - V - \dot{\vec{R}} \cdot \left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right] - \beta M_q \\ & -\vec{\alpha} \cdot \left[2\vec{p} - e_1\vec{A}(\vec{r}_1) + e_2\vec{A}(\vec{r}_2)\right] - \beta 2m_1 - \frac{e_1}{M_q - V^{\rm in}}\vec{\sigma} \cdot \vec{B}(\vec{r}_1) \\ & + \left.\frac{(m_1 + M_q/2)^2 - (m_2 + M_q/2)^2}{M_q - V^{\rm in}}\right\} \psi(\vec{r}_1, \vec{r}_2) = 0 \end{aligned} \tag{24}$$

with $\dot{\vec{R}}$ as given in Eq. (22). We can see that the Dirac bispinor degrees of freedom, connected with the parton 1, are involved both in the external and internal motion as it is defined by the coordinate $\vec{R}$ and $\vec{r}$, respectively. That makes the very concept of these motions physically unclear, in spite of their independent coordinates $\vec{R}$ and $\vec{r}$, even in the case of nonrelativistic external approximation (if only the external motion can be distinguished from rest). However, we will undertake the task of clarifying this concept in Eq. (26).

Consistently with the relative weakness of all external interactions of our composite quark, let us assume that the quark centre-of-mass motion inside the confining hadron is nonrelativistic in the sense that $\left[\vec{P}-e_1\vec{A}(\vec{r}_1)-e_2\vec{A}(\vec{r}_2)\right]^2$ is small enough in comparison with $M_q^2$ (and $\vec{P}-e_1\vec{A}(\vec{r}_1)-e_2\vec{A}(\vec{r}_2)$ is roughly equal to $M_q\dot{\vec{R}}$). More precisely, we begin with putting

$$\begin{aligned} & \vec{\alpha} \cdot \left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right] + \beta M_q \\ & = \left\{\left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right]^2 - \vec{\sigma} \cdot \left[e_1\vec{B}(\vec{r}_1) + e_2\vec{B}(\vec{r}_2)\right] + M_q^2\right\}^{1/2} \\ & \simeq M_q + \frac{1}{2M_q}\left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right]^2 \\ & -\frac{1}{2M_q}\vec{\sigma} \cdot \left[e_1\vec{B}(\vec{r}_1) + e_2\vec{B}(\vec{r}_2)\right], \end{aligned} \tag{25}$$

where the first step ought to be understood in the formal sense of Dirac square root, while the second is the familiar nonrelativistic expansion. Then, we consider the remaining part of the term $-\dot{\vec{R}} \cdot \left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right]$ in Eq. (24) that is

$$
\begin{aligned}
&(M_q - V^{\rm in})^{-\frac{1}{2}}\left[2\vec{p} - e_1\vec{A}(\vec{r}_1) + e_2\vec{A}(\vec{r}_2)\right]\left(M_q - V^{\rm in}\right)^{-\frac{1}{2}} \\
&\quad \cdot \left[\vec{P} - e_1\vec{A}(\vec{r}_1) - e_2\vec{A}(\vec{r}_2)\right] \\
&\quad = \left(M_q - V^{\rm in}\right)^{-\frac{1}{2}}\left[2\vec{P}\cdot\vec{p} - 2e_1\vec{A}(\vec{r}_1)\cdot\vec{p}_1 + 2e_2\vec{A}(\vec{r}_2)\cdot\vec{p}_2\right. \\
&\quad \left. + \; e_1^2\vec{A}^2(\vec{r}_1) - e_2^2\vec{A}^2(\vec{r}_2)\right]\left(M_q - V^{\rm in}\right)^{-\frac{1}{2}}
\end{aligned}
$$

in the Coulomb gauge, where $\mathrm{div}_i\vec{A}(\vec{r}_i) = 0 \quad (i = 1,2)$. Thus, in the case of homogeneous external magnetic-type field $\vec{B}$, when $\vec{A}(\vec{r}_i) = (1/2)(\vec{B}\times\vec{r}_i) \quad (i = 1,2)$, this is equal to

$$
\begin{aligned}
&\left(M_q - V^{\rm in}\right)^{-\frac{1}{2}}\left\{2\vec{P}\cdot\vec{p} - \frac{1}{2}\left[(e_1 - e_2)\left(\vec{R}\times\vec{P} + \vec{r}\times\vec{p}\right)\right.\right. \\
&\quad \left.\left. + \; (e_1 + e_2)\left(\vec{R}\times 2\vec{p} + \frac{1}{2}\vec{r}\times\vec{P}\right)\right]\cdot\vec{B} + \mathrm{O}(B^2)\right\}\left(M_q - V^{\rm in}\right)^{-\frac{1}{2}} \quad .
\end{aligned}
$$

Assume now that $V(\vec{r}_1,\vec{r}_2) \equiv V^{\rm ex}(R) + V^{\rm in}(r)$ with $R = |\vec{R}|$ and $r = |\vec{r}|$. In such a case, our two-body problem considered in the external nonrelativistic approximation can be solved through the separable ansatz $E = E^{\rm ex} + E^{\rm in}$ and $\psi(\vec{r}_1,\vec{r}_2) = \psi^{\rm ex}(\vec{R})\psi^{\rm in}(\vec{r})$ with the Dirac bispinor index ascribed to $\psi^{\rm in}(\vec{r})$ [this is the first-order perturbative solution with respect to external-internal kinematic coupling where $(M_q - V^{\rm in})^{-1}$ and $r$ are of the order $\mathrm{O}(1/2\mathrm{m})$; as to the negligible second-order corrections *cf.* the discussion of the internal-energy spectrum (29)]. In fact, external expectation values of the vectors $\vec{R}$ and $\vec{P}$ linearly coupled with $\vec{\alpha}$, $\vec{r}$ and $\vec{p}$ vanish due to parity conservation, what leads to an internal wave equation for $\psi^{\rm in}(\vec{r})$, independent of integrals involving $\psi^{\rm ex *}(\vec{R})$ and $\psi^{\rm ex}(\vec{R})$. Therefore, the last expression above may be effectively abridged to

$$
-\tfrac{1}{2}(e_1 - e_2)\left(M_q - V^{\rm in}\right)^{-1}\left(\vec{R}\times\vec{P} + \vec{r}\times\vec{p}\right)\cdot\vec{B}\,.
$$

Thus, in the external nonrelativistic approximation, taking into account this abridged term as well as Eq. (25), we can reduce effectively our wave equation (24) to the following form valid in the case of homogeneous external magnetic-type field:

$$\Big\{E - M_q - V^{\rm ex}(R) - V^{\rm in}(r) - \frac{1}{2\,M_q}\left[\vec{P}^2 - (e_1+e_2)\vec{B}\cdot\left(\vec{R}\times\vec{P}\right)\right]$$
$$+ \frac{e_1+e_2}{2\,M_q}\vec{\sigma}\cdot\vec{B} - \vec{\alpha}\cdot\left[2\vec{p} - \frac{1}{2}(e_1+e_2)\frac{1}{2}\left(\vec{B}\times\vec{r}\right)\right] - \beta\,2m_1$$
$$- \frac{1}{M_q - V^{\rm in}(r)}\left[e_1\vec{\sigma} + \frac{1}{2}(e_1-e_2)\left(\vec{R}\times\vec{P}+\vec{r}\times\vec{p}\right)\right]\cdot\vec{B}$$
$$+ \frac{(m_1+M_q/2)^2 - (m_2+M_q/2)^2}{M_q - V^{\rm in}(r)}\Big\}\psi(\vec{R},\vec{r}) = 0 \ . \qquad (26)$$

Here, the kinetic-energy terms are also effectively abridged.

Evidently, the Hamiltonian resulting from Eq. (26) implies that in the external nonrelativistic approximation

$$\dot{\vec{R}} = M_q^{-1}\left[\vec{P} - \tfrac{1}{2}(e_1+e_2)\left(\vec{B}\times\vec{R}\right)\right] \ \text{ and } \ \tfrac{1}{2}\dot{\vec{r}} = \vec{\alpha}\ ,$$

where we neglected the terms

$$\tfrac{1}{2}(e_1-e_2)\left(M_q - V^{\rm in}\right)^{-1}\left(\vec{B}\times\vec{R}\right) \ \text{ and } \ \tfrac{1}{2}(e_1-e_2)\left(M_q - V^{\rm in}\right)^{-1}\left(\vec{B}\times\tfrac{1}{2}\vec{r}\right)\ ,$$

respectively, which are of the order O(1/2m). Hence, $\dot{\vec{r}}_{1,2} = M_q^{-1}$ $[\vec{P} - (1/2)(e_1+e_2)(\vec{B}\times\vec{R})] \pm \vec{\alpha}$. We can see that in the nonrelativistic approximation for the external (*i.e.*, centre-of-mass) motion the Dirac bispinor degrees of freedom — connected in fact with the preon 1 — are ascribed to the internal (*i.e.*, relative) motion within our composite quark.

In the case of our particular preon model for composite quarks we have $V^{\rm in}(r) = -\alpha^{(\rm u)}/r$ (ultraelectrostatic attraction between $Q$ and $S$ preons), and for $u$ and $d$ quarks we put (as our first guess) $m_1 = m_2 \equiv m$ (equal preon masses). If the composite quarks move in the external ultramagnetic field $\vec{B}^{(\rm u)}$, we have in addition $e_1 = -e_2 \equiv e^{(\rm u)}$ at $A_\mu^{(\rm u)}$ (opposite preon ultracharges). Then, the wave equation (26) takes the form:

$$[E - M_q - V^{\rm ex}(R) - V^{\rm in}(r) - \frac{1}{2\,M_q}\vec{P}^2$$
$$-\vec{\alpha}\cdot 2\vec{p} - \beta\,2m - \frac{e^{(\rm u)}}{M_q - V^{\rm in}(r)}\left(\vec{\sigma} + \vec{R}\times\vec{P} + \vec{r}\times\vec{p}\right)\cdot\vec{B}\,]\,\psi(\vec{R},\vec{r}) = 0\ . \qquad (27)$$

This is our wave equation for a $u$ or $d$ composite quark, moving slowly enough inside a confining hadron (*e.g.*, a proton or neutron). This quark moves in the (mean) external central potential $V^{\rm ex}(R)$ produced within the hadron as a whole, and also in the external homogeneous ultramagnetic field $\vec{B}^{(\rm u)}$. The latter is possibly created by a polarized nuclear surroundings, when nucleon ultramagnetic moments are polarized therein [4].

## 3. Composite-quark internal structure

In the case of our particular model, the relativistic wave equation (21) for internal motion of preons within a $u$ or $d$ composite quark becomes

$$\left[E^{\rm in} + \frac{\alpha^{(\rm u)}}{r} - 2\left(\vec{\alpha}\cdot\vec{p} + \beta m\right)\right]\psi^{\rm in}(\vec{r}) = 0 \,, \tag{28}$$

leading readily to the Sommerfeld-type energy spectrum:

$$E^{\rm in} = 2m\left[1 + \left(\frac{\alpha^{(\rm u)}/2}{n_r + \gamma}\right)^2\right]^{-1/2} \,, \quad \gamma \equiv \sqrt{(j+1/2)^2 - (\alpha^{(\rm u)}/2)^2} \,, \tag{29}$$

where $n_r = 0, 1, 2, \ldots$ and $j = 1/2, 3/2, 5/2, \ldots$. We can see that for the ground state ($n_r = 0$, $j = 1/2$) we get the internal energy eigenvalue $E^{\rm in}_0 = 2m\gamma_0$ with $\gamma_0 \equiv \sqrt{1 - (\alpha^{(\rm u)}/2)^2}$, thus $\gamma_0 \to 0$ and $E^{\rm in}_0 \to 0$ when $\alpha^{(\rm u)} \to 2$. Here, $\alpha^{(\rm u)} = 2$ is the critical value of $\alpha^{(\rm u)}$ giving the Klein-paradox behaviour of the wave function $\psi^{\rm in}_0(\vec{r})$ at $r \to 0$. However, a choice of value $\alpha^{(\rm u)}$ growing to 2 may be not unreasonable as, after all, the current masses $m_q$ of $u$ and $d$ quarks, identified here naturally with $E^{\rm in}_0$, are small, in contrast to their constituent mass $M_q \simeq M_N/3$. Note from Eq. (29) that all excited states correspond to internal energy eigenvalues $E^{\rm in}$ of the order $O(2m)$. This is an additional factor that makes small all second-order perturbative corrections (to the ground state) from external-internal kinematic coupling.

In the convenient representation of Dirac matrices, where

$$\vec{\alpha} = \begin{pmatrix} 0 & i\vec{\sigma}_P \\ -i\vec{\sigma}_P & 0 \end{pmatrix} \,, \; \beta = \begin{pmatrix} \mathbf{1}_P & 0 \\ 0 & -\mathbf{1}_P \end{pmatrix} \,, \; \vec{\sigma} = \begin{pmatrix} \vec{\sigma}_P & 0 \\ 0 & \vec{\sigma}_P \end{pmatrix} \tag{30}$$

with $\vec{\sigma}_P$ and $\mathbf{1}_P$ denoting four Pauli matrices, the ground–state wave function corresponding to $m_j = +1/2$ has the form

$$\psi^{\rm in}_0(\vec{r}) = \left[\frac{\left(2m\sqrt{1-\gamma_0^2}\right)^{2\gamma_0+1}(1+\gamma_0)}{2\Gamma(2\gamma_0+1)}\right]^{1/2} r^{\gamma_0-1}\exp(-m\sqrt{1-\gamma_0^2}\, r)$$

$$\times \begin{pmatrix} Y_{00}(\hat{r}) \\ 0 \\ \sqrt{\frac{1}{3}(1-\gamma_0)/(1+\gamma_0)}\, Y_{10}(\hat{r}) \\ -\sqrt{\frac{2}{3}(1-\gamma_0)/(1+\gamma_0)}\, Y_{11}(\hat{r}) \end{pmatrix} . \tag{31}$$

Here, $\langle \psi_0^{\rm in} | \psi_0^{\rm in} \rangle = 1$. From Eq. (31) it follows that

$$\begin{aligned} \langle \psi_0^{\rm in} |\, \beta \,| \psi_0^{\rm in} \rangle &= \gamma_0 \stackrel{\alpha^{(\rm u)} \to 2}{\longrightarrow} 0 , \\ \langle \psi_0^{\rm in} |\, \sigma_z \,| \psi_0^{\rm in} \rangle &= \frac{1}{3}(2\gamma_0 + 1) \stackrel{\alpha^{(\rm u)} \to 2}{\longrightarrow} \frac{1}{3} , \\ \langle \psi_0^{\rm in} | (\vec{r} \times \vec{p})_z | \psi_0^{\rm in} \rangle &= \frac{1}{3}(1 - \gamma_0) \stackrel{\alpha^{(\rm u)} \to 2}{\longrightarrow} \frac{1}{3} , \\ \langle \psi_0^{\rm in} | (\vec{r} \times \vec{\alpha})_z | \psi_0^{\rm in} \rangle &= \frac{1}{3m}(2\gamma_0 + 1) \stackrel{\alpha^{(\rm u)} \to 2}{\longrightarrow} \frac{1}{3m} \end{aligned} \tag{32}$$

and

$$\begin{aligned} \langle \psi_0^{\rm in} |\, r \,| \psi_0^{\rm in} \rangle &= \frac{1}{2m} \frac{2\gamma_0 + 1}{\sqrt{1-\gamma_0^2}} \stackrel{\alpha^{(\rm u)} \to 2}{\longrightarrow} \frac{1}{2m} , \\ \langle \psi_0^{\rm in} |\, \frac{1}{r} \,| \psi_0^{\rm in} \rangle &= -\langle \psi_0^{\rm in} | \frac{\partial}{\partial r} | \psi_0^{\rm in} \rangle = m \frac{\sqrt{1-\gamma_0^2}}{\gamma_0} , \\ \langle \psi_0^{\rm in} |\, \vec{r} \,| \psi_0^{\rm in} \rangle &= 0 = \langle \psi_0^{\rm in} |\, \vec{p} \,| \psi_0^{\rm in} \rangle , \\ \langle \psi_0^{\rm in} | \frac{1}{M_q + \alpha^{(\rm u)}/r} | \psi_0^{\rm in} \rangle &= \frac{1}{2m} \frac{1}{\Gamma(2\gamma_0 + 1)} \int_0^\infty dx \frac{x^{2\gamma_0 + 1} \exp(-x)}{(M_q/2m)x + 2(1-\gamma_0^2)} \\ &\stackrel{M_q \ll 2m}{\simeq} \frac{1}{4m} \frac{2\gamma_0 + 1}{1 - \gamma_0^2} \stackrel{\alpha^{(\rm u)} \to 2}{\longrightarrow} \frac{1}{4m} \end{aligned} \tag{33}$$

with $x \equiv 2m\sqrt{1-\gamma_0^2}\, r$.

Since $E_0^{\rm in} = 2m\gamma_0$, where $\gamma_0 = \sqrt{1 - (\alpha^{(\rm u)}/2)^2}$, the current masses of $u$ and $d$ quarks might be given as

$$m_u = 2m\sqrt{1 - \alpha^{(\rm u)\,2}/4} \ , \ \ m_d = 2m\sqrt{1 - \alpha^{(\rm u)\,2}/4} \, . \tag{34}$$

However, in reality there is a nonzero mass difference $m_d - m_u > 0$, that in our model must be caused by the electromagnetic differences between $U$ and $D$ preons (involved in $u$ and $d$ quarks).In the case of equal preon masses $m_{_U} = m_{_S} = m_{_D} (\equiv m)$, one may try to describe phenomenologically quark masses by the ansatz

$$m_u = 2m\sqrt{1 - (\alpha^{(\rm u)} + \delta\alpha_u)^2/4} \ , \ \ m_d = 2m\sqrt{1 - (\alpha^{(\rm u)} + \delta\alpha_d)^2/4} \, , \tag{35}$$

where the effective coupling constants $\alpha^{(\mathrm{u})} + \delta\alpha_u$ and $\alpha^{(\mathrm{u})} + \delta\alpha_d$ are to be determined. Then, using the popular, experimentally suggested figures $m_u = 4$ MeV and $m_d = 7$ MeV and putting reasonably $m = (1$ to $10)$ TeV, one gets

$$\alpha^{(\mathrm{u})} + \delta\alpha_u = 2 - 4\left(\frac{\mathrm{MeV}}{m}\right)^2 = 2 - 4 \times (10^{-12} \text{ to } 10^{-14}) \qquad (36)$$

and

$$\alpha^{(\mathrm{u})} + \delta\alpha_d = 2 - \frac{49}{4}\left(\frac{\mathrm{MeV}}{m}\right)^2 = 2 - 12.3 \times (10^{-12} \text{ to } 10^{-14}) \,. \qquad (37)$$

Hence,

$$\delta\alpha_u - \delta\alpha_d = \frac{33}{4}\left(\frac{\mathrm{MeV}}{m}\right)^2 = 8.3 \times (10^{-12} \text{ to } 10^{-14}) > 0 \,. \qquad (38)$$

Note here a gentle balance between very large $m$ and very small square roots appearing in Eq. (35).

It is not surprising, of course, that our preon binding by means of Coulombic ultraelectrostatic potential and/or our guess of equal preon masses $m_1 = m_2$ are too simple to reproduce quantitatively the current quark masses $m_u$ and $m_d$. Nevertheless, from the above discussion of $m_u$ and $m_d$ we can probably draw the conclusion that the ultraelectromagnetic coupling constant $\alpha^{(\mathrm{u})}$ is almost as large as 2.

To try an improved ansatz, put for preon masses $m_U \neq m_S (\equiv m) \neq m_D$ and $(m_U - m_S) : (m_D - m_S) = e_U^2 : e_D^2 = 4$ , and then use for quark masses the perturbed formulae (34):

$$m_u = 2m\sqrt{1 - \alpha^{(\mathrm{u})\,2}/4} + \delta m_u \;\;, \quad m_d = 2m\sqrt{1 - \alpha^{(\mathrm{u})\,2})/4} + \delta m_d \,.$$

Since the first-order perturbative calculation with $m_U - m_S$ and $m_D - m_S$ treated as small quantities (*cf.* Eq. (21) and two next formulae) gives

$$\begin{aligned}\delta m_u &= -\left[(m_U - m_S)\,/\Gamma(2\gamma_0 + 1)\right] \int\limits_0^\infty dx x^{2\gamma_0 + 1} \exp(-x)/\left[\gamma_0 x + 2(1 - \gamma_0^2)\right] \\ &\simeq -\left[(2\gamma_0 + 1)\,/\left(2 - 2\gamma_0^2\right)\right](m_U - m_S) = -0.5\,(m_U - m_S)\end{aligned}$$

and similarly $\delta m_d = -0.5(m_D - m_S)$, we may roughly estimate $m_U - m_D \sim 2(m_u - m_d) = 6$ MeV, when $m_u = 7$ MeV and $m_d = 4$ MeV. Thus, $m_U - m_S \sim$

8 MeV, $m_D - m_S \sim 2$ Mev and $m_S \equiv m$. Then, with $m_u = 4$ MeV, $m_d = 7$ MeV and $m =$(1 to 10) TeV one gets

$$\alpha^{(\mathrm{u})} \sim 2 - 16\left(\frac{\mathrm{MeV}}{m}\right)^2 = 2 - 16 \times \left(10^{-12} \text{ to } 10^{-14}\right) ,$$

and

$$\gamma_0 \equiv \sqrt{1 - \alpha^{(\mathrm{u})\,2}/4} \sim 4\frac{\mathrm{MeV}}{m} = 4 \times \left(10^{-6} \text{ to } 10^{-7}\right) .$$

## 4. Calculating ultramagnetic moments

From Eq. (27) we can read off the (internal) ultramagnetic moments for $u$ and $d$ composite quarks as both equal to[3]

$$\vec{\mu}_q^{(\mathrm{u})} = -\frac{e^{(\mathrm{u})}}{M_q + \alpha^{(\mathrm{u})}/r}\left(\vec{\sigma} + \vec{r} \times \vec{p}\right) . \tag{39}$$

Hence, making use of the quark internal wave function (31), we obtain

$$\mu_q^{(\mathrm{u})} \equiv \frac{\langle \psi_0^{\mathrm{in}} |\, \mu_{qz}^{(\mathrm{u})}\, | \psi_0^{\mathrm{in}} \rangle}{\langle \psi_0^{\mathrm{in}} |\, \sigma_z\, | \psi_0^{\mathrm{in}} \rangle} = -\frac{e^{(\mathrm{u})}}{4m}\frac{\gamma_0 + 2}{1 - \gamma_0^2} \stackrel{\alpha^{(\mathrm{u})} \to 2}{\longrightarrow} \mp\frac{\sqrt{2}}{2m} \tag{40}$$

if $e^{(\mathrm{u})} > 0$ or $< 0$, respectively. Here, $2m \gg M_q \simeq M_N/3$. Note from Eq. (40) that $\mu_q^{(\mathrm{u})} = -\mu_{\mathrm{preon}}^{(\mathrm{u})}(\gamma_0/2 + 1)/(1 - \gamma_0^2) \to -\mu_{\mathrm{preon}}^{(\mathrm{u})}$ when $\alpha^{(\mathrm{u})} \to 2$ (here, $\mu_{\mathrm{preon}}^{(\mathrm{u})} = e^{(\mathrm{u})}/2m$).

In an analogical way, making use of Eq. (26), we can find the (internal) magnetic moments for $u$ and $d$ composite quarks moving in the external homogeneous magnetic field $\vec{B}$. In this case, $e_1 \equiv (2/3\,,\,-1/3)e$ and $e_2 \equiv 0$ at $A_\mu$. Then, with $e_q \equiv e_1 + e_2 = (2/3\,,\,-1/3)e$, we read off that[3]

$$\vec{\mu_q} = e_q\left(\frac{1}{2M_q}\vec{\sigma} + \frac{1}{4}\vec{r} \times \vec{\alpha}\right) - \frac{1}{M_q + \alpha^{(\mathrm{u})}/r}\left[e_1\vec{\sigma} + \frac{1}{2}(e_1 - e_2)\left(\vec{r} \times \vec{p}\right)\right] . \tag{41}$$

Hence, we calculate

[3] Notice that the formulae (39) and (41) are valid also in the case of the alternative option $q = (LS)$ mentioned in Footnote [1] (strictly speaking, for $u$ composite quark there appears then $\alpha^{(\mathrm{u})} + \alpha/3 \simeq \alpha^{(\mathrm{u})}$ in place of $\alpha^{(\mathrm{u})}$, since $e_1 \equiv (1\,,\,0)$ and $e_2 \equiv -1/3$).

$$\mu_q \equiv \frac{\langle\psi_0^{\rm in}|\mu_{qz}|\psi_0^{\rm in}\rangle}{\langle\psi_0^{\rm in}|\sigma_z|\psi_0^{\rm in}\rangle} = \frac{e_q}{2M_q} + \frac{e_q}{4m} - \frac{1}{8m}\frac{3e_1(1+\gamma_0) - e_2(1-\gamma_0)}{1-\gamma_0^2}$$
$$\simeq \frac{e_q}{2M_q} = \left(\frac{2}{3}, -\frac{1}{3}\right)\frac{\sqrt{\alpha}}{2M_q}, \tag{42}$$

since $2m \gg M_q \simeq M_N/3$ and $\sqrt{\alpha} = e$.

The composite-quark ultramagnetic moments (40) and magnetic moments (42) imply the following ultramagnetic and magnetic moments for the proton $p = (u\,u\,d)$ or neutron $n = (u\,d\,d)$:

$$\left.\begin{array}{l} \mu_p^{(\mathrm{u})} = \frac{2}{3}\left(2\mu_u^{(\mathrm{u})} - \mu_d^{(\mathrm{u})}\right) + \frac{1}{3}\mu_d^{(\mathrm{u})} \\ \mu_n^{(\mathrm{u})} = \frac{2}{3}\left(2\mu_d^{(\mathrm{u})} - \mu_u^{(\mathrm{u})}\right) + \frac{1}{3}\mu_u^{(\mathrm{u})} \end{array}\right\} = \mu_q^{(\mathrm{u})} \overset{\alpha^{(\mathrm{u})}\to 2}{\longrightarrow} \mp\frac{\sqrt{2}}{2m} \tag{43}$$

(if $e^{(\mathrm{u})} > 0$ or $< 0$, respectively) and

$$\mu_p = \frac{2}{3}(2\mu_u - \mu_d) + \frac{1}{3}\mu_d \simeq \frac{\sqrt{\alpha}}{2M_q} \simeq 3\frac{\sqrt{\alpha}}{2M_N},$$
$$\mu_n = \frac{2}{3}(2\mu_d - \mu_u) + \frac{1}{3}\mu_u \simeq -\frac{2}{3}\frac{\sqrt{\alpha}}{2M_q} \simeq -2\frac{\sqrt{\alpha}}{2M_N}. \tag{44}$$

We can see that the nucleon magnetic moments as evaluated in Eq. (44) are consistent with their experimental values $\mu_p^{\rm exp} = 2.8\sqrt{\alpha}/2m_N$ or $\mu_n^{\rm exp} = -1.9\sqrt{\alpha}/2m_N$. On the other hand, from Eqs. (43) and (44) it follows that

$$|\mu_p^{(\mathrm{u})}| : \mu_p \overset{\alpha^{(\mathrm{u})}\to 2}{\longrightarrow} \sqrt{\frac{2}{\alpha}}\,(M_q : m) = O(10^{-2}) \text{ to } O(10^{-3}), \tag{45}$$

if the preon mass $m = O(1\text{ TeV})$ to $O(10\text{ TeV})$, what gives the reasonable quark size $1/m = O(10^{-16}\,\text{cm})$ to $O(10^{-17}\,\text{cm})$. Then,

$$\left(3e_{\rm eff}^{(\mathrm{u})}\right)^2 \simeq (3e)^2\left(\mu_p^{(\mathrm{u})\,2} : \mu_p^2\right) \overset{\alpha^{(\mathrm{u})}\to 2}{\longrightarrow} O(10^{-6}) \text{ to } O(10^{-8}) \tag{46}$$

(where $\mu_N^{(\mathrm{u})} = \mu_q^{(\mathrm{u})} \equiv e_{\rm eff}^{(\mathrm{u})}/2M_q \simeq 3e_{\rm eff}^{(\mathrm{u})}/2M_N$ both for $N = p,\, n$). From the experimental viewpoint [4], these values do not seem to be hopelessly small.

In particular, the classic radiofrequency experiments, determining $H_2$ rotational levels in external magnetic field [9], measured hfs effects in $H_2$

molecules fully consistent with the ordinary magnetic dipole-dipole interactions of two protons involved. This sets an upper limit on the hypothetical ultramagnetic dipole-dipole interaction between two protons:

$$-\frac{\mu_p^{(\mathrm{u})\,2}}{r^3}\left[3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})-\vec{\sigma}_1\cdot\vec{\sigma}_2\right] , \tag{47}$$

where due to Eq. (43) $\mu_p^{(\mathrm{u})} \simeq 3\epsilon_{\mathrm{eff}}^{(\mathrm{u})}/2M_N \to \mp\sqrt{2}/2m$ when $\alpha^{(\mathrm{u})} \to 2$. From an analysis in Ref. [10] it follows that the experiments of Ref. [9] leave the margin

$$\mu_p^{(\mathrm{u})\,2}\langle\frac{1}{r^3}\rangle_{\mathrm{H}_2} < 3\times 10^{-19}\ \mathrm{MeV} \tag{48}$$

with

$$\langle\frac{1}{r^3}\rangle_{\mathrm{H}_2} = \left(0.74\times 10^{-8}\mathrm{cm}\right)^{-3} . \tag{49}$$

Hence, $\left(3\epsilon_{\mathrm{eff}}^{(\mathrm{u})}\right)^2 < 2\times 10^{-7}$, what is not inconsistent with our rough estimation (46).

It should be emphasized, however, that in our argument the ultraphoton rest mass is zero exactly (*i.e.*, the ultraphoton is the gauge boson of the new unbroken U(1) local symmetry generated by ultracharge). The experimental upper limit $\left(3\epsilon_{\mathrm{eff}}^{(\mathrm{u})}\right)^2 < \mathrm{O}(10^{-7})$ might increase drastically if the ultraphoton developed a nonnegligible rest mass $m_\Gamma$ (in a process of breaking our U(1) local symmetry), what would introduce to Eq. (47) the Yukawa exponent $\exp(-m_\Gamma r)$. For instance, in the case of $m_\Gamma =$ (5 to 25) keV one would get the upper limit $\left(3\epsilon_{\mathrm{eff}}^{(\mathrm{u})}\right)^2 < 0.0004$ to 1 (*cf.* Ref. [10]).

## 5. Final remarks

Eventually, we would like to point out that the model of composite quarks considered in this paper exploits (in a particular way) the notion of "real" compositeness *i.e.*, the compositness in physical space. Such a notion ought to be sharply contrasted with the notion of (pure) algebraic compositness that was applied recently to the problem of three lepton and quark generations [11], and gave, jointly with a new idea of the intrinsic exclusion principle, a *consistent* explanation of this puzzling phenomenon.

In the present (alternative) paper, the puzzle of existing more than one fermion generation is correlated rather with intergenerational cancellation of chiral anomalies in the spatially composite quark model, but then the number of generations is not (uniquely) determined to be three. Of course, the ultracharge formula $Q^{(u)} = (3/2)[(1/2)\lambda_3+(1/2\sqrt{3})\lambda_8]$ mentioned in Section

1 as an example, might suggest a broken horizontal-SU(3)-group structure for three generations of ultraquarks, ultrascalars and leptons (though the last have *zero ultracharge*, they are *needed* to cancel the Standard Model chiral anomalies in each of three fermion generations). In this case, such a broken horizontal SU(3) group should play some role in developing masses of these three generations for ultraquarks and ultrascalars (either in a spontaneous[4] or explicit way).

I am indebted to Sławomir Wycech for a helpful remark.

Note added in proof: quite recently ZEUS and H1 collaborations at HERA have announced an excess of observed neutral-current and charged-current candidate events at very high $Q^2$ and large $x$ in $e^+p$ collisions over Standard Model expectations (preprints of February 15 and February 13, 1997).

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[4] Perhaps through a horizontal octet of new nonchiral Higgs bosons $\varphi^a$ $(a = 1,\ldots,8)$ with $\langle\varphi^3\rangle \neq 0$ and $\langle\varphi^8\rangle \neq 0$. Then, the horizontal gauge vector bosons $V^a$ (mentioned in Section 1) become massive for $a \neq 3, 8$, while $V^3$ and $V^8$ (and their two important linear combinations) remain massless. Thus, the horizontal SU(3) gauge group is spontaneously broken into U(1)×U(1), where the two U(1) gauge groups are generated by the ultracharge and the second diagonal horizontal charge (it may be denoted by $Q'^{(u)}$, while the corresponding gauge vector boson — by $\Gamma'$). The $\Gamma'$ is active only for the second and third generations, where it collaborates with $\Gamma$. In particular, within bound states $q_2 = (Q_2\, S_2)$ and $q_3 = (Q_3\, S_3)$ the gauge bosons $\Gamma$ and $\Gamma'$ produce together the Coulombic attractive potentials equal to $-(1/4 + 3/4)\alpha^{(u)}/r = -\alpha^{(u)}/r$, thus the same as the Coulombic potential acting within $q_1 = (Q_1\, S_1)$, caused by $\Gamma$ only. In contrast, for the pairs $Q_2\, S_3$ and $Q_3\, S_2$ the repulsive Coulombic potentials equal to $(-1/4+3/4)\alpha^{(u)}/r = (1/2)\alpha^{(u)}/r$ result. Similarly, for the pairs $Q_3 S_1$ and $Q_1 S_3$ the potentials are $(1/2)\alpha^{(u)}/r$, while the pairs $Q_i \bar{S}_j$ $(i \neq j)$ can be now bound by the potentials $-(1/2)\alpha^{(u)}/r$.

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