LONGITUDINALLY ASYMMETRIC $q-\bar{q}$ CONFIGURATIONS IN DEEP INELASTIC LEPTON-ONIUM SCATTERING AT SMALL x_{Bj}

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The contribution of highly asymmetric $q-\bar{q}$ configurations to the onium-onium scattering at high energy is discussed in the framework of Mueller's QCD dipole picture. A modification of Mueller's formula is proposed and applied to deep inelastic lepton-onium scattering.

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1. Onium-onium scattering is a convenient theoretical laboratory for studies of perturbative QCD amplitudes in the limit of very high energy [1, 2]. In this context, an onium is treated as a pair of heavy quark and antiquark with a certain distribution of the light-cone momentum fraction z and their relative transverse distance r. To consider a specific example, the formula for the total onium-onium cross-section can be expressed as [3]

$$\sigma_{\mathrm{tot}} = \int d^2r dz \Phi(z,r;M) \int d^2r' dz' \Phi'(z',r';M) \sigma(r,z;r'z';Y) , \qquad (1)$$

where Φ is the probability distribution for finding a configuration (r,z) inside an onium and M is the mass of the quark and antiquark forming the onium. σ is the total cross-section for scattering of two $q-\bar{q}$ pairs and Y is the total available phase space volume in light-cone momentum fraction of the cascading gluons:

$$Y = y + y' , \qquad (2)$$

where

$$y = \log\left(\frac{2E}{M}\right) ; \quad y' = \log\left(\frac{2E'}{M}\right) . \qquad (3)$$

E and E' are energies of the colliding onia, so that we obtain $Y = \log(s/M^2)$ [3], where $s = 4EE'$ is the total c.m. energy of the collision squared.

The formula for σ was derived by several authors [1, 3, 4]. We give here its integral representation which is convenient for our future argument[1]:

$$\sigma(r,z;r'z';Y) = 2\pi\alpha^2 rr' \int \frac{d\gamma}{2\pi i} e^{\Delta(\gamma)Y} \left(\frac{r}{r'}\right)^{1-\gamma} h(\gamma)\,, \tag{4}$$

where

$$h(\gamma) = \frac{4}{\gamma^2(2-\gamma)^2}\,, \tag{5}$$

and $\Delta(\gamma) = \frac{\alpha N}{\pi}\chi(\gamma)$ with

$$\chi(\gamma) = 2\psi(1) - \psi(\frac{\gamma}{2}) - \psi(1-\frac{\gamma}{2}). \tag{6}$$

α is the strong coupling constant and $N = 3$ is the number of colours.

In absence of high energy beams of onia, to confront these theoretical results with experiment it is necessary to reformulate them in such a way that they are applicable to scattering of virtual photons and hadrons. The Eq. (1) shows that this is –in principle– possible if one knows the distribution of constituents inside a virtual photon and/or a hadron [2].

When a virtual photon dissociates into a $q-\bar{q}$ pair, the distribution of the constituents is given by [6, 7]

$$\Phi^{T,L}(r,z;Q) = \frac{N\alpha_{em}e_f^2}{\pi^2} W^{T,L}(r,z;Q)\,, \tag{7}$$

where

$$W^T(r,z;Q) = \tfrac{1}{2}[z^2+(1-z)^2]\hat{Q}^2 K_1^2(\hat{Q}r) \tag{8}$$

for transverse photons and

$$W^L(r,z;Q) = 2z(1-z)\hat{Q}^2 K_0^2(\hat{Q}r) \tag{9}$$

for longitudinal photons (masses of quarks are neglected, $\hat{Q}^2 = Q^2 z(1-z)$). Once these formulae are introduced into (1) one obtains explicit expressions for total cross-section of a virtual photon.

When we want to implement this program, however, we have to decide how to interpret the Eq. (3). Indeed, for a virtual photon the mass M

[1] A detailed derivation can be found *e.g.* in [5].

[2] For a nucleon target one needs also to assume that its structure can be reasonably approximated by a colour triplet-antitriplet system. To avoid discussion of this problem, in the present note we consider only photon-onium scattering.

of the constituent has a very different meaning than for the onium. The phenomenologically successful proposal of [9–11]

$$Y = y + y' = \log\left(\frac{4EE'}{Q^2}\right) = \log\left(\frac{1}{x_{\rm Bj}}\right) \tag{10}$$

was never given a sound basis and may even seem not to be a natural choice because it introduces an asymmetry between the two colliding systems[3].

At this point it should be emphasized that determination of a unique formula for y and y' goes beyond the possibilities of the leading logarithm approximation (on which the dipole approach is based) and therefore this problem cannot be resolved by solely formal arguments[4]. This does not mean, however, that we cannot bring intuitive physical picture helping (a) to understand the meaning of a given choice and (b) to take into account specific physical effects.

In the present paper we discuss this problem in a little more detail. In particular, we propose another formula which can replace (3) and (10). We believe that it takes better into account the longitudinal momentum distribution of quarks in the colliding onia.

Before going into derivation, let me quote the final result: The formula (4) for the total cross-section of two $q - \bar{q}$ pairs remains valid provided the definition of y in (3) is changed into

$$y = \log\left(\frac{cp^+ z_< r^2}{\tau_{\rm int}}\right), \tag{11}$$

where p^+ is the light-cone momentum of the incident onium, $z_<$ is the smaller of light-cone momentum fractions of the onium constituents:

$$z_< = z \quad \text{if} \quad z \leq \tfrac{1}{2}; \qquad z_< = 1 - z \quad \text{if} \quad z \geq \tfrac{1}{2}\,. \tag{12}$$

and $\tau_{\rm int} = \tau_{\rm int}(r, r')$ is a characteristic time of the collision. The constant c is arbitrary. It remains undetermined because the leading logarithm character of the calculation implies that it is always allowed to change y by an arbitrary additive constant.

Analogous formulae are valid for y', so that we obtain

$$Y = y + y' = \log\left(\frac{csz_< z'_< r^2 r'^2}{\tau^2_{\rm int}}\right), \tag{13}$$

[3] To illustrate this point, consider *e.g.* photon-photon scattering where a more symmetric formula $Y = \log(4EE'/QQ')$ was recently suggested [12, 13].

[4] I would like to thank R. Peschanski for very illuminating correspondence concerning this problem.

where $s = p^+p^{+'}$ is the total c.m. energy of the collision squared. $\tau_{\rm int}$ has the dimension of length and is in general a symmetric function of r and r'. As remarked above, this function cannot be exactly determined by formal arguments. Intuitively, $\tau_{\rm int}$ is the time needed for the exchanged gluons to travel the necessary distance in the transverse space. Therefore we find it natural to take

$$\tau_{\rm int} = {\rm const}\ \ r_> \,, \tag{14}$$

where $r_>$ is the larger of r and r'. We show below that (14) gives the result which is rather close to (10) (apart from corrections related to z-dependence). Thus our result may be considered as a justification and generalization of the asymmetric, phenomenologically successful, choice (10).

We also find that to obtain a formula which is close to the one advocated in [12, 13] for photon-photon scattering, one has to take

$$\tau^2 = {\rm const}\ \ rr' \,. \tag{15}$$

We were unable, however, to construct an intuitive argument which would justify this choice.

In the next section we give arguments in favour of Eqs. (13) and (14).

2. To discuss Y we have to go back to the derivation of (4) given in [2, 4]. It starts from the formula

$$\sigma(r,z;r'z';Y) = \int \frac{dx}{x}\frac{dx'}{x'}\hat{\sigma}(x,x')d^2s\ n(r,x,y,s)n(r',x',y',b-s)d^2b \,, \tag{16}$$

where $n(r,x,y,s)$ is the density of the QCD dipoles of transverse size x at the transverse distance s from the center of the $q-\bar{q}$ system of transverse size r. $\sigma(x,x')$ is (energy independent) cross-section for scattering of two dipoles. The dipole density inside the high-energy $q-\bar{q}$ system arises from a cascade process whose length is denoted by y. It can be shown that the result depends only on the sum $y+y'=Y$.

At this point we would like to observe that in the argument leading to Eq. (3) and(4) it was implicitly assumed [2, 4] that the z-distribution of quarks in an "onium" is peaked around $z=\frac{1}{2}$, so that the rapidity of the colliding onia (given by (3)) are not substantially different from that of their constituents. A possible finite difference is neglected in comparison to the large incident rapidity (one should remember that the Eq. (4) is an asymptotic formula for very high energies). In this case the length y of the cascade does not depend on z at all. Although this may be not a bad approximation for an onium made of two heavy quarks, it seems doubtful for distributions (8) and (9) of quarks and antiquarks in the virtual photon. Particularly for transverse photons the contributions from the region $z \approx 0$, *i.e.* highly asymmetric pairs, is very important. It is therefore necessary to consider this region in more detail.

A closer look at the derivation [2] shows that the length of the cascade is given by

$$y = \log\left(\frac{z_<}{z_0}\right) , \tag{17}$$

where $z_<$ is given by (12) and z_0 is the minimal light-cone momentum fraction of the emitted gluon

$$z_0 = \frac{p_0^+}{p^+} . \tag{18}$$

The factor $z_<$ in Eq. (17) explicitly shows that y is connected with the energy of the *slower* of the two onium constituents rather than with the total energy of the incident $q - \bar{q}$ system. The reason is that in the leading logarithm approximation all emitted gluons (of which the dipoles are formed) are required to carry a negligible fraction of the energy of *both* quark and antiquark [2].

Now, the crucial point is that p_0^+ cannot be arbitrary small for the Eq. (16) to be valid. Indeed, as is clearly seen from its form, Eq. (16) was derived under the assumption that the fluctuations of the onium wave function which are effective in the collision have long enough life time so that one can separate the gluons in the wave function (which form the dipoles in one onium) from the gluons which are exchanged between the dipoles from the colliding onia (and which are thus responsible for interaction) [4]. In short, the minimal life time of the fluctuation $\tau_{\rm min} = \tau_{\rm min}(r, p_0^+)$ must be larger than the characteristic time of the interaction $\tau_{\rm int} = \tau_{\rm int}(r, r')$. This sets the limit

$$\tau_{\rm min} \equiv \frac{p_0^+}{< k_t >^2} = \tau_{\rm int}(r, r') , \tag{19}$$

where

$$< k_t >= \frac{\rm const}{r} \tag{20}$$

is the average transverse momentum of the emitted gluons. Thus we have

$$p_0^+ = \frac{{\rm const}\ \tau_{\rm int}(r, r')}{r^2} . \tag{21}$$

Using (17),(18) and (21) we finally obtain

$$Y = y + y' = \log\left(\frac{{\rm const}\ z_< z'_< p^+ p^{+'} r^2 r'^2}{\tau_{\rm int}^2(r, r')}\right) . \tag{22}$$

It remains to determine $\tau_{\rm int}(r,r')$. This we do by observing that –on the average– the time necessary for exchange of two gluons between the projectile and target cannot be smaller than the transverse size of *larger* of the two colliding objects. The simplest way to implement this idea is to write[5]

$$\tau_{\rm int}(r,r') = {\rm const}\; r_> \,. \tag{23}$$

In the next section we show that, when inserted into the formulae (1) and (16) for the total cross-section, Eqs. (22) and (23) imply –for large Q^2– the prescription which is close to the one suggested in [9–11]. We also show that this result is substantially different from that used in [12, 13] for photon-photon cross-section.

3. The formulae (13), (14) differ from the hitherto employed prescription [9–11] given by (10). To compare explicitly the practical consequences of the new and old approach we apply (13) and (14) to calculation of the total cross-section of a highly virtual photon scattered off a $q-\bar{q}$ system at fixed configuration $r'=r_0$ and $z'=z_0$. We obtain

$$\begin{aligned}\sigma_{\gamma^*}^{T,L}(Q^2;r_0,z_0;s) &= \int d^2r dz \Phi^{T,L}(z,r;Q)\sigma(r,z;r_0,z_0;Y)\\ &= \frac{2\alpha^2 N\alpha_{em}e_f^2}{\pi}\int\frac{d\gamma}{2\pi i} r_0^{\gamma}(csz_0)^{\Delta(\gamma)}H^{T,L}(\gamma,Q)h(\gamma)\,,\end{aligned} \tag{24}$$

where[6]

$$H^{T,L}(\gamma,Q) = 2\int_0^{1/2} dz z^{\Delta(\gamma)}\int_0^{\infty} d^2r r^{2-\gamma+2\Delta(\gamma)}W^{T,L}(r,z;Q)\,. \tag{25}$$

The integrals over d^2r can be done with the help of the identity [8]

$$\int_0^{\infty} t^{-\lambda}dtK_{\mu}^2(t) = \frac{2^{-2-\lambda}}{\Gamma(1-\lambda)}\Gamma\left(\frac{1-\lambda}{2}+\mu\right)\Gamma\left(\frac{1-\lambda}{2}-\mu\right)\Gamma^2\left(\frac{1-\lambda}{2}\right). \tag{26}$$

This gives

$$H^{T,L} = \pi\left(\frac{2}{Q}\right)^{2-\gamma'} S^{T,L}(\gamma)Z^{T,L}(\gamma)\,, \tag{27}$$

[5] Another interesting possibility is $\tau_{\rm int}^2 = r^2 + r'^2$. It leads , however, to more complicated algebra and therefore we do not consider it here.

[6] We assume that r_0 is large enough to justify the integration over dr^2 from 0 to ∞.

where

$$S^T(\gamma) = \frac{4-\gamma'}{2-\gamma'}S^L(\gamma); \quad S^L(\gamma) = \frac{\Gamma^4(2-\frac{\gamma'}{2})}{\Gamma(4-\gamma')}, \tag{28}$$

$$Z^T(\gamma) = \int_0^{1/2} dz[z^2+(1-z)^2]z^{\frac{\gamma}{2}-1}(1-z)^{\frac{\gamma'}{2}-1}, \tag{29}$$

$$Z^L(\gamma) = 4\int_0^{1/2} dz z^{\frac{\gamma}{2}}(1-z)^{\frac{\gamma'}{2}}, \tag{30}$$

and

$$\gamma' = \gamma - 2\Delta(\gamma). \tag{31}$$

Introducing this into (24) we finally obtain

$$\sigma_{\gamma *}^{T,L}(Q^2;r_0,z_0;s) = 2\alpha^2 N\alpha_{em}e_f^2 r_0^2 \int \frac{d\gamma}{2\pi i}\left(\frac{2}{Qr_0}\right)^{2-\gamma}\left(\frac{4csz_0}{Q^2}\right)^{\Delta(\gamma)} h(\gamma)S^{T,L}(\gamma)Z^{T,L}(\gamma). \tag{32}$$

Eq. (32) can be transformed into formula for the "structure function" of the target $q-\bar{q}$ pair by means of the identity

$$F_2 = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_{\gamma *}. \tag{33}$$

This gives

$$F_2^{T,L}(x,Q^2;r_0,z_0) = \frac{2\alpha^2 Ne_f^2}{\pi^2}\int \frac{d\gamma}{2\pi i}\left(\frac{Qr_0}{2}\right)^{\gamma}\left(\frac{4cz_0}{x_{\mathrm{Bj}}}\right)^{\Delta(\gamma)} h(\gamma)S^{T,L}(\gamma)Z^{T,L}(\gamma), \tag{34}$$

where $x_{\mathrm{Bj}} = Q^2/s$.

Eq. (34) is to be compared with the analogous formula derived from (1) and (4) using (10) which reads

$$F_{2(\mathrm{old})}^{T,L}(x,Q^2;r_0,z_0) = \frac{\alpha^2 Ne_f^2}{2\pi^2}\int \frac{d\gamma}{2\pi i}\left(\frac{Qr_0}{2}\right)^{\gamma}\left(\frac{c}{x_{\mathrm{Bj}}}\right)^{\Delta(\gamma)} h(\gamma)G^{T,L}(\gamma), \tag{35}$$

where

$$G^T(\gamma) = \frac{(2+\gamma)(4-\gamma)}{2\gamma(2-\gamma)} G^L(\gamma);$$
$$G^L(\gamma) = 2\frac{\Gamma^4(2-\frac{\gamma}{2})\Gamma(1+\frac{\gamma}{2})}{\Gamma(4-\gamma)\Gamma(2+\gamma)}. \quad (36)$$

A quick look on these two formulae shows that the differences between them are not dramatic. In the saddle point approximation they simply reduce to a change in normalization. It seems therefore likely that our new formula (34) should also describe correctly the data (with somewhat different parameters than those given by [10]).

At this point it is also interesting to compare this result with the one following from the symmetric choice (15) in which case a calculation analogous to the one presented above gives

$$F_2^{T,L}(x,Q^2;r_0,z_0) = \frac{2\alpha^2 N e_f^2}{\pi^2}\int \frac{d\gamma}{2\pi i}\left(\frac{Qr_0}{2}\right)^{\gamma}\left(\frac{cQr_0z_0}{x_{\rm Bj}}\right)^{\Delta(\gamma)} h(\gamma)\hat{S}^{T,L}(\gamma)\hat{Z}^{T,L}(\gamma) \quad (37)$$

with

$$\hat{S}^T(\gamma) = \frac{4-\gamma_-}{2-\gamma_-}\hat{S}^L(\gamma);\ \ \hat{S}^L(\gamma) = \frac{\Gamma^4(2-\frac{\gamma_-}{2})}{\Gamma(4-\gamma_-)}, \quad (38)$$

$$\hat{Z}^T(\gamma) = \int_0^{1/2} dz[z^2+(1-z)^2]z^{\frac{\gamma_+}{2}-1}(1-z)^{\frac{\gamma_-}{2}-1}, \quad (39)$$

$$\hat{Z}^L(\gamma) = 4\int_0^{1/2} dz z^{\frac{\gamma_+}{2}}(1-z)^{\frac{\gamma_-}{2}}, \quad (40)$$

and

$$\gamma_\pm = \gamma \pm \Delta(\gamma). \quad (41)$$

Comparing (34) and (37) we notice a substantial difference in the dependence on Q: one finds in (37) an extra factor $Q^{\Delta(\gamma)} \approx Q^{\Delta_P}$. It thus seems unlikely that these two formulae can be reconciled by adequate adjustment of the parameters. A detailed analysis goes beyond the scope of this paper, however.

4. To summarize, we propose a modification of the Mueller formulation of onium-onium scattering which is more suitable for application to realistic processes. Our approach takes explicitly into account the dependence of the amplitudes on the longitudinal momentum fractions carried by the constituents of the "onium". It can thus account for the contributions of highly asymmetric pairs, known to be important in collisions with transverse photons. Application to virtual photon-onium scattering shows that the new approach affects mostly the normalization and does not change substantially dependence on x_{Bj} and Q^2. It shall, however, affect the parameters of the fit to data.

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