THE STRUCTURE OF 1s0d- 1p0f-SHELL NUCLEI IN THE COLLECTIVE PAIR APPROXIMATION

E. Kwaśniewicz

Institute of Physics, Pedagogical University
Leśna 16, 25-509 Kielce, Poland

F. Catara

Dipartimento di Fisica, Universita di Catania
Corso Italia 57 - I 95129 Catania, Italy

and M. Sambataro

Istituto Nazionale di Fisica Nucleare, Sezione di Catania
Corso Italia 57 - I 95129 Catania, Italy

(Received July 4, 1996)

The structure of low-lying states of nuclei with four active nucleons in the $1s0d$ and $1p0f$ shells is studied in the framework of the Collective Pair Approximation. The collective pairs determined by diagonalizing the Hamiltonian in the space of two nucleons outside closed shells are considered as building blocks to describe a nucleus with $2n$ valence nucleons in terms of n pairs. It is shown that the low-lying spectrum can be described quite well by considering only a selected subset of all possible collective pairs.

PACS numbers: 21.10. Re

1. Introduction

Features of low-lying spectra of nuclei having several nucleons away from major closed shells have been extensively studied in the framework of the Interacting Boson Model [1, 2] (IBM). The IBM makes use of boson variables which can be interpreted as correlated pairs of nucleons and, consequently, the IBM can be closely related to the spherical shell model.

In the IBM calculations usually only s and d bosons have been incorporated for the description of the basic features of low-lying spectra of collective nuclei [3, 4]. But the calculated spectra are, in general, severely affected by this drastic truncation of the boson space. In order to improve the description of nuclear observables also additional bosons like g boson and excited s' and d' bosons as well as $T = 0$ bosons have been taken into account in some circumstances [5–8].

The Collective Pair Approximation [9–12] (CPA) is an alternative approximation scheme that incorporates the basic philosophy of the IBM in the fermionic space. The CPA can be considered as a truncation scheme with respect to the shell-model calculations. It consists in constructing the spectrum of $2n$ nucleons outside closed shells in a space of n collective pairs. In first approximation, the structure of these pairs can be fixed by diagonalizing the nuclear Hamiltonian in the space of two nucleons outside closed shells. It turns out that a rather good description of the low-lying states can be obtained by including only a (small) selected subset of all possible collective pairs. These play the same role of the bosons in IBM. We stress that the Pauli principle is exactly taken into account in CPA.

In this paper the structure of the low-lying spectra of the $A = 20$ ($T = 0, 1, 2$) and $A = 44$ ($T = 0, 1, 2$) nuclei is studied in the framework of the CPA (see also Ref. [13]). The main objectives of this study are *(i)* to find which collective pairs are essential for a good description of the low-lying spectra of $1s0d$- and $1p0f$-shell nuclei and *(ii)* to verify if calculations performed with effective two-body interactions derived from different methods lead to comparable results.

In Section 2 the formalism is shortly sketched. The details concerning calculations and results are given in Section 3, and 4 while conclusions are outlined in Section 5.

2. Formalism

The operator $A^{\dagger}_{\nu\Omega\Omega'}$ creating a collective pair of multipolarity $\Omega(=J,T)$ and projection $\Omega'(=J', T')$ can be written

$$A^{\dagger}_{\nu\Omega\Omega'} = \sum_{j_1 j_2} C^{\nu}_{\Omega}(j_1 j_2) Z^{\dagger}_{\Omega\Omega'}(j_1 j_2), \tag{1}$$

where

$$Z^{\dagger}_{\Omega\Omega'} = (1+\delta_{j_1 j_2})^{-1/2} [a^{\dagger}_{j_1} a^{\dagger}_{j_2}]_{\Omega\Omega'} \tag{2}$$

creates two nucleons occupying orbitals j_1 and j_2 with total quantum numbers $\Omega\Omega'$. The index ν denotes different collective pairs with the same quantum number Ω. The coefficients $C^{\nu}_{\Omega}(j_1 j_2)$ are obtained from the diagonalization of the Hamiltonian in the complete space spanned by the two-nucleon states $\mid j_1 j_2 \Omega\Omega'\rangle = Z^{\dagger}_{\Omega\Omega'}(j_1 j_2) \mid 0\rangle$.

A four-nucleon state can be expressed as a linear combination of states built from two collective pairs of Eq. (1)

$$\mid \Omega_1\nu_1\Omega_2\nu_2; \Lambda\Lambda'\rangle = [A^{\dagger}_{\Omega_1\nu_1} A^{\dagger}_{\Omega_2\nu_2}]_{\Lambda\Lambda'} \mid 0\rangle = \sum_{\Omega_1'\Omega_2' j_1 j_2 j_3 j_4} (\Omega_1\Omega_1'\Omega_2\Omega_2' \mid \Lambda\Lambda')$$

$$\times C^{\nu_1}_{\Omega_1}(j_1 j_2) C^{\nu_2}_{\Omega_2}(j_3 j_4) Z^{\dagger}_{\Omega_1\Omega_1'}(j_1 j_2) Z^{\dagger}_{\Omega_2\Omega_2'}(j_3 j_4) \mid 0\rangle\,, \tag{3}$$

where $(\Omega_1\Omega_1'\Omega_2\Omega_2' \mid \Lambda\Lambda')$ stands for the product of the spin $(J_1J_1'J_2J_2' \mid JJ')$ and isospin $(T_1T_1'T_2T_2' \mid TT')$ Clebsch–Gordan coefficients. In the case of identical pairs $(\nu_1=\nu_2, \Omega_1=\Omega_2)$ the spin and isospin angular momenta $\Lambda(=JT)$ have to fulfil the condition $(-1)^{\Omega_1+\Omega_2-\Lambda} = 1$. States (3) are neither normalized nor linearly independent. In order to find a set of orthonormal and linearly independent states we proceede as follows.

Taking into consideration the completeness of the states $\mid n, \beta\Gamma\Gamma'\rangle$ spanning the shell-model space, the unit operator $\hat{I}(n)$ can be defined

$$\hat{I}(n) = \sum_{\beta\Gamma\Gamma'} \mid n, \beta\Gamma\Gamma'\rangle\langle n, \beta\Gamma\Gamma' \mid, \tag{4}$$

where n is the number of nucleons in the active orbits, $\Gamma\Gamma'$ define total spin and isospin angular momenta of the n nucleons and their projections, and β gives a set of additional quantum numbers to distinguish states with the same $n\Gamma\Gamma'$ quantum numbers. By inserting the $\hat{I}(n = 2)$ and $\hat{I}(n = 4)$ into Eq. (3), employing the Wigner–Eckart theorem and utilizing the orthonormality conditions of the Clebsch–Gordan coefficients, Eq. (3) can be expressed as

$$\mid \Omega_1\nu_1\Omega_2\nu_2; \Lambda\Lambda'\rangle = \sum_{\beta} B(\beta\Omega_1\nu_1\Omega_2\nu_2\Lambda) \mid 4, \beta\Lambda\Lambda'\rangle, \tag{5}$$

where

$$\begin{aligned} B(\beta\Omega_1\nu_1\Omega_2\nu_2\Lambda) &= (-1)^{(3\Omega_1+\Omega_2-\Lambda)}(2\Lambda-1)^{-\frac{1}{2}} \\ &\times \sum_{j_1j_2j_3j_4} C^{\nu_1}_{\Omega_1}(j_1j_2)C^{\nu_2}_{\Omega_2}(j_3j_4)\langle 4, \beta\Lambda \mid\mid Z^{\dagger}_{\Omega_1}(j_1j_2) \mid\mid 2, \Omega_2(j_3j_4)\rangle. \end{aligned} \tag{6}$$

States $\mid \Omega_1\nu_1\Omega_2\nu_2; \Lambda\Lambda'\rangle$ are neither normalized nor linearly independent. In order to obtain a new set of orthonormal states the eigenvalue problem of the overlap matrix

$$O(\Omega_1\nu_1\Omega_2\nu_2\Omega_3\nu_3\Omega_4\nu_4; \Lambda) = \langle \Omega_1\nu_1\Omega_2\nu_2; \Lambda\Lambda' \mid \Omega_3\nu_3\Omega_4\nu_4; \Lambda\Lambda'\rangle \tag{7}$$

has to be solved.

From the solution of the eigenvalue equation

$$\begin{aligned} &\sum{}_{\Omega_3\nu_3\Omega_4\nu_4}\langle \Omega_1\nu_1\Omega_2\nu_2; \Lambda\Lambda' \mid \Omega_3\nu_3\Omega_4\nu_4; \Lambda\Lambda'\rangle D(\alpha\Omega_3\nu_3\Omega_4\nu_4\Lambda) \\ &= N(\alpha\Lambda)D(\alpha\Omega_1\nu_1\Omega_2\nu_2\Lambda) \end{aligned} \tag{8}$$

one obtains a new set of orthonormal, linearly independent states spanning the subspace of two collective pairs which can be written as

$$
\begin{aligned}
| \alpha \Lambda \Lambda' \rangle &= (N(\alpha\Lambda))^{-\frac{1}{2}} \sum_{\Omega_1 \nu_1 \Omega_2 \nu_2} D(\alpha \Omega_1 \nu_1 \Omega_2 \nu_2 \Lambda) \mid \Omega_1 \nu_1 \Omega_2 \nu_2; \Lambda \Lambda' \rangle \\
&= (N(\alpha\Lambda))^{-\frac{1}{2}} \sum_{\Omega_1 \nu_1 \Omega_2 \nu_2 \beta} D(\alpha \Omega_1 \nu_1 \Omega_2 \nu_2 \Lambda) B(\beta \Omega_1 \nu_1 \Omega_2 \nu_2 \Lambda) \mid 4 \beta \Lambda \Lambda' \rangle. \quad (9)
\end{aligned}
$$

In Eq. (9) the coefficients $N(\alpha\Lambda)$ and $D(\alpha\Omega_1\nu_1\Omega_2\nu_2\Lambda)$ are obtained from the solution of Eq. (8). The number of states (9) is equal to the number of coefficients $N(\alpha\Lambda) > 0$, which in some cases is less than the dimension of the overlap matrix (7).

In order to solve the eigenvalue problem of the shell-model Hamiltonian in the space spanned by two collective pairs let us first express that Hamiltonian as follows

$$
\hat{H} = \sum_{\alpha\Lambda} \mid \Psi_{n\alpha\Lambda} \rangle E_{n\alpha\Lambda} \langle \Psi_{n\alpha\Lambda} \mid, \tag{10}
$$

where the eigenvectors $\mid \Psi_{n\alpha\Lambda}\rangle$ correspond to the eigenvalues $E_{n\alpha\Lambda}$ ($\hat{H} \mid \Psi_{n\alpha\Lambda}\rangle = E_{n\alpha\Lambda} \mid \Psi_{n\alpha\Lambda}\rangle$) and the index α distinguishes the eigenvectors and eigenvalues that belong to the same $n\Lambda$. Noticing that the eigenvectors $\mid \Psi_{n\alpha\Lambda}\rangle$ can be expanded in the complete shell-model basis of Eq. (4), *i.e.*

$$
\mid \Psi_{n\alpha\Lambda} \rangle = \sum_{\beta} A(\alpha\beta\Lambda) \mid n\beta\Lambda \rangle, \tag{11}
$$

the matrix representation of the shell-model Hamiltonian (10) expressed in the subspace of states (9) can finally be written as

$$
\begin{aligned}
\langle \alpha_1 \Lambda \| \hat{H} \| \alpha_2 \Lambda \rangle = & \sum_{\beta_1 \beta_2 \alpha \Omega_1 \nu_1 \Omega_2 \nu_2 \Omega_3 \nu_3 \Omega_4 \nu_4} B(\beta_1 \Omega_1 \nu_1 \Omega_2 \nu_2 \Lambda) \\
& \times D(\alpha_1 \Omega_1 \nu_1 \Omega_2 \nu_2 \Lambda) A(n\beta_1 \alpha \Lambda) E_{n\alpha\Lambda} A(n\beta_2 \alpha \Lambda) \\
& \times D(\alpha_2 \Omega_3 \nu_3 \Omega_4 \nu_4 \Lambda) B(\beta_2 \Omega_3 \nu_3 \Omega_4 \nu_4 \Lambda). \quad (12)
\end{aligned}
$$

Thus in order to solve the eigenvalue problem of the Hamiltonian in the space spanned by two collective pairs, first the eigenvalue problem of the Hamiltonian in the complete shell-model space has to be solved, and the matrix elements of the two-nucleon transfer operators in the same space have to be calculated. Both these calculations can be done with the aid of standard shell-model programs, *e.g.* RITSSCHIL [14].

3. Results and discussion

The method outlined in Section 2 has been applied to describe the low-lying spectra of the $A = 20 (T = 0, 1, 2)$ and $A = 44 (T = 0, 1, 2)$ nuclei by employing the same effective interactions in the shell-model and in the CPA calculations. For each nucleus we have performed a series of calculations, by including a smaller and smaller set of collective pairs. In this way we have searched for a "minimum set" of pairs necessary in order to have a good reproduction of the shell-model results for the low-lying spectrum.

3.1. The 1s0d shell

In the *1s0d*-shell there are 28 two-nucleon eigenstates of the shell-model Hamiltonian which can be considered as building blocks to describe nuclei with a larger number of active nucleons. Their spectrum obtained with three interactions is shown in Table I. Following the standard notation we will denote as s, p, d, f, g,... the $T = 1$ pairs with $J = 0, 1, 2, ...$, while the $T = 0$ pairs will be denoted as Θ_J. Of course, the spectrum obtained by diagonalizing the Hamiltonian in the space spanned by the 4-nucleon states built in terms of all possible two pair states is identical to the shell-model one. Our aim is to investigate to which extent a selected subset of collective pairs is sufficient to give a good reproduction of the low-lying 4-nucleon shell-model spectrum. Starting from the set including all pairs and then removing step by step less important ones, we single out the "minimum set".

TABLE I

The low-lying shell-model spectra of two nucleons in the $1s0d$-shell. Calculations are performed with the Wildenthal [15] (E_W), Sussex [16] (E_S) and Bonn B [17] (E_B) interactions. Eigenenergies are in MeV.

1s0d shell							
T=0				T=1			
J	E_W	E_S	E_B	J	E_W	E_S	E_B
1	0.0	0.0	0.0	0	0.0	0.0	0.0
3	1.37	0.75	2.04	2	2.18	1.73	2.12
5	1.46	1.17	2.50	4	3.78	3.00	3.62
2	4.31	2.78	4.08	0	4.32	4.38	4.19
3	4.51	4.24	5.78	2	4.44	3.48	4.23
1	4.91	4.04	5.01	3	5.73	4.63	5.06
1	6.71	5.36	6.99	4	8.75	7.38	8.21
4	6.78	5.74	7.15				
2	7.43	7.29	7.86				

Calculations have been performed for three different two-body effective interactions in the *1s0d* shell, *i.e.* for *(i)* Wildenthal's model independent interaction [15] determined from fits to experimental energies of a selected set of normal parity states in $1s0d$-shell nuclei, *(ii)* Sussex interaction [16] deduced from the experimental nucleon-nucleon scattering phase shifts, *(iii)* Bonn B interaction [17] obtained from the G-matrix folded diagram method and the nucleon-nucleon Bonn potential. The single-particle energies for $0d_{\frac{5}{2}}$, $1s_{\frac{1}{2}}$ and $0d_{\frac{3}{2}}$ shells have been in case *(i)* adopted from Ref. [15] while in cases *(ii)* and *(iii)* they have been determined from the experimental lowest-lying levels of the ^{17}O nucleus.

For the sake of brevity only the most representative $T = 0$ case is illustrated. In Tables II(a)–(c) selected results for the lowest levels of ^{20}Ne obtained with the use of three above cited interactions are reported. In the first two columns the spins and shell-model energies of ^{20}Ne are presented. In the other columns the energies obtained with different subsets of pairs are reported together with the overlap of the appropriate CPA wavefunction with the corresponding shell-model one. In addition we also report for each calculation the quantities

$$\sigma_1 = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(E_i^{\mathrm{SM}} - E_i^{\mathrm{CPA}})^2}, \tag{13}$$

$$\sigma_2 = \sqrt{\frac{1}{N-1}\sum_{i=2}^{N}[(E_i^{\mathrm{SM}} - E_1^{\mathrm{SM}}) - (E_i^{\mathrm{CPA}} - E_1^{\mathrm{CPA}})]^2}, \tag{14}$$

$$\sigma_3 = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \mid \langle \Psi_i^{\mathrm{SM}} \mid \Psi_i^{\mathrm{CPA}} \rangle \mid^2}, \tag{15}$$

where σ_1 and σ_2 represent the RMS deviations in the absolute energies and in the excitation energies, while σ_3 is the RMS of the overlaps between shell-model and CPA wavefunctions. In the first presented CPA calculation we have included the lowest five $T = 1$ and six $T = 0$ pairs out of the 28 total number of pairs (see Table I). As can be seen from the third column of Tables II(a)–(c), the overall agreement is quite good as testified also by the values of the three σ's. In the next two columns we show the results obtained when the three highest $T = 0$ pairs have been removed. In such a case we observe that energy of the first 0^+ state is essentially unaffected (in the case of Wildenthal interaction [15] both 0^+ states are almost unaffected) while the others are shifted upwards by about $0.8 \div 1.0$ MeV for all three considered interactions. When all the $T = 0$ pairs are removed, the excitation energy spectrum as well as the overlaps are still quite good, while the absolute energies deviate appreciably from the shell model ones. Therefore, we can conclude that the exclusion of all the $T = 0$ pairs leads to an almost uniform shift of the spectrum without appreciably affecting the overlaps. Finally, from the last two columns, one sees that the removal of the second 0^+ $T = 1$

TABLE II(a)

The results of shell-model and selected CPA calculations for the ^{20}Ne nucleus with the following sets of pairs: 1) s, s', d, d', g, Θ_1, Θ_1', Θ_2, Θ_3, Θ_3', Θ_5, 2) s, s', d, d', g, $\Theta_1, \Theta_3, \Theta_5$, 3) s, s', d, d', g and 4) s, d, d', g. The symbol O stands for overlap between CPA and shell-model wavefunctions. Calculations are performed with the Wildenthal [15] interaction. For details see text.

J	E_{SM}	E	O	E	O	E	O	E	O
		(1)		(2)		(3)		(4)	
0	-41.41	-41.34	0.999	-41.11	0.993	-39.85	0.949	-39.39	0.928
0	-34.42	-34.36	0.998	-34.03	0.990	-33.39	0.961	-30.20	0.739
2	-39.50	-39.21	0.992	-38.55	0.973	-38.02	0.949	-37.76	0.936
2	-33.87	-33.47	0.981	-32.86	0.956	-32.47	0.937	-32.23	0.905
4	-37.00	-36.05	0.961	-35.47	0.934	-34.86	0.884	-34.84	0.885
6	-32.40	-32.24	0.994	-31.43	0.946	-31.14	0.915	-31.14	0.915
σ_1		0.44		0.96		1.52		2.38	
σ_2		0.44		0.77		0.39		1.07	
σ_3		0.99		0.97		0.93		0.89	

TABLE II(b)

The same as in Table II(a) but for the Sussex [16] interaction.

J	E_{SM}	E	O	E	O	E	O	E	O
		(1)		(2)		(3)		(4)	
0	-39.66	-39.49	0.996	-39.34	0.992	-37.97	0.940	-37.43	0.917
0	-32.34	-32.30	0.999	-31.77	0.981	-30.99	0.939	-29.05	0.779
2	-38.26	-37.86	0.989	-37.34	0.972	-36.45	0.931	-36.16	0.916
2	-31.88	-31.54	0.989	-30.94	0.957	-30.56	0.938	-30.44	0.921
4	-35.94	-35.24	0.976	-34.47	0.939	-33.61	0.878	-33.56	0.879
6	-31.87	-31.50	0.986	-30.72	0.939	-30.16	0.874	-30.16	0.874
σ_1		0.40		0.97		1.74		2.27	
σ_2		0.29		0.75		0.37		0.64	
σ_3		0.99		0.96		0.92		0.88	

TABLE II(c)

The same as in Table II(a) but for the Bonn B [17] interaction.

J	E_{SM}	E	O	E	O	E	O	E	O
		(1)		(2)		(3)		(4)	
0	-40.55	-40.43	0.998	-40.17	0.991	-38.17	0.914	-37.67	0.893
0	-33.60	-33.55	0.999	-32.62	0.968	-31.93	0.930	-29.24	0.743
2	-38.56	-38.16	0.989	-37.25	0.958	-36.41	0.919	-36.05	0.899
2	-32.08	-31.59	0.981	-30.96	0.939	-30.41	0.915	-30.07	0.871
4	-36.06	-35.26	0.971	-34.49	0.937	-33.30	0.855	-33.25	0.857
6	-30.86	-30.41	0.982	-29.78	0.943	-29.30	0.880	-29.30	0.879
σ_1		0.46		1.13		2.08		2.83	
σ_2		0.40		0.86		0.61		1.07	
σ_3		0.99		0.96		0.90		0.86	

pair seriously deteriorates the quality of the results. Any further reduction of the CPA subspace leads to worse and worse results. Similar conclusions can be drawn for the $T = 1$ four-nucleon states. Of course, for the $T = 2$ states, only the $T = 1$ pairs contribute. Also in this case we found that the s, s', d, d', g pairs are the "minimum set" necessary to get a reasonable agreement between CPA and shell-model calculations. Besides, it has been verified that the use of the three above cited interactions leads to similar conclusions.

3.2. The 1p0f shell

The same procedure has been applied to the $A = 44$ nuclei using the FPD6 two-body interaction and single particle energies from Ref. [18]. In the Table III, the spectrum of the lowest two-nucleon eigenenergies of the shell-model Hamiltonian is shown for both $T = 0$ and $T = 1$. In Table IV, we report the results of the calculations for $T = 0$ four-nucleon states. The first two columns of the table are reserved to spins and shell-model energies of the lowest levels of ^{44}Ti. For reasons of space we present only a few selected CPA results. The first one is reported in the third and fourth columns and includes the lowest nine $T = 1$ and four $T = 0$ pairs out of the 60 total number of pairs. The overal agreement with the shell-model results is good and characterized by a rather uniform shift up in energy of all levels, less evident only for the ground state. Indeed, the RMS deviation in the absolute energies is $\sigma_1 = 0.85$ MeV while that in the excitation energies is $\sigma_2 = 0.54$ MeV. It is worthwile noticing that the overlaps between shell model and CPA wavefunctions for the considered states are rather good, as shown also by the value $\sigma_3 = 0.94$. In the next two columns, we show results obtained by only removing the three $T = 1$ pairs s', f, h from the previous CPA space. This operation essentially affects the 0_2^+ state which moves up by about 700 KeV while the other states remain stable within 200 KeV. This indicates the minor role of these pairs for the low-lying states.

TABLE III

The low-lying shell-model spectrum of two nucleons in the $1p0f$-shell obtained with the FPD6 [18] interaction. Eigenenergies are in MeV.

1p0f shell			
T=0		T=1	
J	E	J	E
7	0.00	0	0.00
1	0.26	2	1.78
5	0.60	4	2.71
3	0.79	6	3.15
5	2.42	2	4.11
3	2.85	4	4.92
2	3.34	3	5.15
4	3.59	5	5.56
1	3.87	0	5.96

In the seventh and eighth columns, we report the calculations obtained by eliminating all the $T = 0$ pairs from the last CPA space and, therefore, only leaving the set formed by the s, d, d', g, g', i pairs. This spectrum exhibits a shift up in energy with respect to the shell-model results, which does not alter significantly the excitation energy, as it can also be observed by looking at σ_1 and σ_2. The value $\sigma_3 = 0.87$ testifies that the overlaps are still quite good. Finally, in the last two columns, we display the results obtained by removing also the g' pair. In spite of the fact that the excitation spectrum looks still good, $\sigma_2 = 0.53$ MeV, one observes that the overlaps for the 0_2^+ and 4_2^+ states are very poor, which shows that the g' pair plays an important role. It is intriguing that the importance of the g' pair is less pronounced when $T = 0$ pairs are included. Indeed, the results obtained in a calculation with s, d, d', g, i and Θ_1, Θ_3 Θ_5, Θ_7 pairs are very similar to those refering to the case 3).

TABLE IV

The results of shell-model and selected CPA calculations for the ^{44}Ti nucleus with the following sets of pairs: 1) s, s', d, d', f, g, g', h, i, Θ_1, Θ_3, Θ_5, Θ_7, 2) s, d, d', g, g', i, Θ_1, Θ_3, Θ_5, Θ_7, 3) s, d, d', g, g', i and 4) s, d, d', g, i . The symbol O stands for overlap between CPA and shell-model wavefunctions. For details see text.

J	E_{SM}	E	O	E	O	E	O	E	O
		(1)		(2)		(3)		(4)	
0	-48.45	-48.07	0.986	-48.03	0.983	-46.63	0.909	-46.38	0.887
0	-43.09	-42.01	0.933	-41.29	0.845	-40.81	0.786	-40.11	0.685
2	-47.15	-46.29	0.955	-46.14	0.945	-45.40	0.898	-45.15	0.868
2	-44.02	-43.32	0.953	-43.12	0.935	-42.85	0.914	-42.67	0.887
4	-45.91	-44.92	0.933	-44.73	0.917	-44.20	0.882	-43.68	0.791
4	-42.89	-41.99	0.889	-41.87	0.855	-41.59	0.822	-41.01	0.517
σ_1		0.85		1.13		1.71		2.14	
σ_2		0.54		0.83		0.43		0.53	
σ_3		0.94		0.91		0.87		0.78	

4. The truncated Hamiltonian

The results of Section 3 indicate that the low-lying spectra of *1s0d-* and *1p0f*-shell nuclei can satisfactorily be reproduced by a "minimum set" of pairs selected from the 28 or 60 pairs allowed in the *1s0d-* or *1p0f*-shell, respectively. Aim of this section is to show that the low-lying shell-model spectra are better reproduced by the CPA with the fixed "minimum set" of pairs if the shell-model Hamiltonian to be used within CPA is replaced by its projection onto the subspace spanned by the low-lying shell model states. This procedure was introduced by Bonatsos *et al.* [19] as a way to simulate the effective Hamiltonian in the truncated shell-model space.

Let us consider a separation of the four-nucleon space into two orthonormal spaces. One of these subspaces consists of the eigenstates of the Hamiltonian which lie lower in energy and the other of the rest of the eigenstates. The projection operator onto the subspace of the low-lying spectrum can be written as

$$\hat{P} = \sum_{\alpha'\Lambda'} | \Psi_{n\alpha'\Lambda'}\rangle\langle\Psi_{n\alpha'\Lambda'} |, \tag{16}$$

where the meaning of the $n\alpha'\Lambda'$ is the same as in Section 3 and the summation is only over the low-lying eigenstates $| \Psi_{n\alpha'\Lambda'}\rangle$. The part of the shell-model Hamiltonian that produces the low-lying spectrum can be expressed as

$$\hat{P}\hat{H}\hat{P} = \sum_{\alpha'\Lambda'} \mid \Psi_{n\alpha'\Lambda'}\rangle E_{n\alpha'\Lambda'} \langle \Psi_{n\alpha'\Lambda'} \mid . \tag{17}$$

The next steps of the procedure to deal with are similar to that considered in Section 3. The only difference is that the overlap matrix of Eq. (7) is replaced by the matrix

$$O'(\Omega_1\nu_1\Omega_1\nu_2\Omega_3\nu_3\Omega_4\nu_4\Lambda) = \langle \Omega_1\nu_1\Omega_2\nu_2; \Lambda\Lambda' \mid \hat{P} \mid \Omega_3\nu_3\Omega_4\nu_4; \Lambda\Lambda' \rangle \tag{18}$$

and the shell-model Hamiltonian (10) is replaced in Eq. (12) by its part (17) corresponding to the low-lying spectrum. In order to illustrate how much the low-lying CPA spectra studied in Section 3 come close to the shell-model spectra we have performed calculations for ^{20}Ne with the "minimum set" of pairs in the reduced four-nucleon spaces down up to 10% of the size of the full shell-model space. The RMS deviations in the absolute energies σ_1 (Eq. (13)) and RMS of the overlaps between shell-model and CPA wavefunctions σ_3 (Eq. (15)) are presented in Fig. 1. It is seen that the first six lowest-lying shell-model states of ^{20}Ne are perfectly reproduced by the "minimum set" of pairs *(s, s', d, d', g)* if the size of the shell-model space is reduced down up to 10%. (In the case of ^{44}Ti its six lowes shell-model states are very well

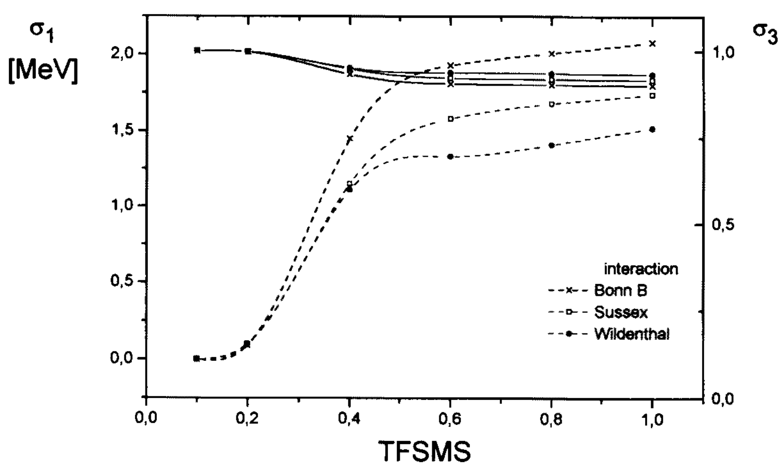

Fig. 1. The RMS deviation in the absolute energies (σ_1) and the RMS of the overlaps between shell-model and CPA wavefunctions (σ_3) versus the truncation factor of the shell-model space (TFSMS) for six lowest-lying states of ^{20}Ne. The CPA calculations were performed with Wildenthal [15], Sussex [16] and Bonn B [17] interactions for *s, s', d, d', g* pairs.

reproduced by the "minimum set" of pairs *(s, d, d', g, g', i)* if the shell-model space is truncated up to 6%).

From the results of Section 3 and 4 one can conclude that the lowest-lying states of $1s0d$- and $1p0f$-shell nuclei can be quite good described by diagonalizing the shell-model Hamiltonian in the CPA space built from several pairs (from "minimum set"). On the other side a drastic improvement of the CPA results can be attained by diagonalizing only that part of the shell-model Hamiltonian which produces low-lying spectrum in the same CPA space as above.

5. Conclusions

A comparison has been reported between shell model and Collective Pair Approximation (CPA) for nuclei with four valence nucleons in the *0d1s* and *0f1p* shells. From the quality of the results obtained with a substantial truncation in the space of the collective pairs taken into account we can conclude that the CPA seems to be a flexible and promising tool for the study of the low lying spectrum of even-even nuclei. The analysis extended to nuclei with six active nucleons would be interesting and valuable.

E. K. would like to thank the INFN and the Department of Physics of the University of Catania for the hospitality and financial support.

REFERENCES

[1] A. Arima, F. Iachello, *Adv. Nucl. Phys.* **Vol.13**, Eds. J.W. Negele and E. Vogt, Plenum, New York 1984.
[2] F. Iachello, I. Talmi, *Rev. Mod. Phys.* **59**, 33 (1987).
[3] A. Arima, F. Iachello, *Phys. Rev. Lett.* **35**, 1069 (1975).
[4] A. Arima, F. Iachello, *Ann. Phys. (N.Y.)* **99**, 253 (1976).
[5] H.C. Wu, *Phys. Lett.* **B110**, 1 (1982).
[6] Y. Akiyama, *Nucl. Phys.* **A433**, 369 (1985).
[7] N. Yoshinaga, Y. Akiyama, A. Arima, *Phys. Rev. Lett.* **56**, 1116 (1986).
[8] P. Navratil, J. Dobes, *Phys. Rev.* **C40**, 2371 (1989).
[9] E. Maglione, F. Catara, A. Insolia, A. Vitturi, *Nucl. Phys.* **A397**, 102 (1983).
[10] E. Maglione, A. Vitturi, F. Catara, A. Insolia, *Nucl. Phys.* **A411**, 181 (1983).
[11] K. Allaart, E. Boeker, G. Bonsignori, M. Savoia, Y.K. Gambhir, *Phys. Rep.* **169**, 169 (1988).
[12] M. Sambataro, A. Insolia, *Phys. Lett.* **B166**, 259 (1986).
[13] E. Kwaśniewicz, F. Catara, M. Sambataro, *New Perspectives in Nucl. Structure*, ed. A. Covello, World Scientific, Singapore 1996, pp 207-214.

[14] D. Zwarts, *Comput. Phys. Commun.* **38**, 365 (1985).

[15] B.A. Brown, W.A. Richter, R.E. Julies, B.H. Wildenthal, *Ann. Phys. (N.Y.)* **182**, 191 (1988).

[16] L.D. Skouras, H. Muther, *Nucl. Phys.* **A515**, 93 (1990).

[17] M.F. Jiang, R. Machleidt, D.B. Stout, T.T.S. Kuo, *Phys. Rev.* **C46**, 910 (1982).

[18] W.A. Richter, M.G. Van Merve, R.E. Julies, B.A. Brown, *Nucl. Phys.* **A523**, 325 (1991).

[19] D. Bonatsos, L.D. Skouras, P. Van Isacker, M.A. Nagarajan, *J. Phys.* **G17**, 1803 (1991).