# $W^-$ PAIR PRODUCTION IN $e^-e^-$ COLLISIONS* **

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The discovery potential of the $e^-e^-$ option of a new linear collider is discussed. The standard model processes of $W^-W^-$, $W^-Z^0$ and $Z^0Z^0$ production in $e^-e^-$ collisions are studied as potential sources of background to 'new physics' reactions, in particular, to the $W^-$ pair production through the lepton number violating reaction $e^-e^- \rightarrow W^-W^-$. The possibility of probing anomalous quartic vector boson couplings in $e^-e^-$ collisions with highly polarized beams is demonstrated.

PACS numbers: 12.15. Ji, 14.60. St, 14.70. Fm, 14.70. Hp

## 1. Introduction

One of great advantages of linear collider projects which have been discussed in world-wide collaborations [1] for the past several years is that the facility could relatively easy be operated in other modes than the basic $e^+e^-$ mode. Colliding electrons against electrons is one of the options discussed in this context. Because of non zero electric charge and lepton number of the $e^-e^-$ initial state, only Møller scattering is possible to lowest order of the standard model (SM). This fact, which makes the $e^-e^-$ mode rather unsuitable for performing the SM tests, is an advantage when search for 'new physics' is concerned. The clean experimental environment which can be obtained with the $e^-e^-$ initial state gives excellent prospects for studying such new physics issues as Majorana neutrinos, supersymmetry, dileptons, leptoquarks, diquarks, the $Z'$ boson, anomalous couplings of vector bosons

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and Higgs bosons [2,3]. In particular, one becomes very sensitive to lepton number violating reactions such as $W^-$ pair production

$$e^- e^- \to W^- W^-, \tag{1}$$

creation of selectrons or charginos, and $\mu^-$ pair production through the exchange of a dilepton or the double-charged member of Higgs triplet in the $s$-channel.

Reaction (1), which may occur through the exchange of a heavy Majorana neutrino in the $t$- and $u$-channel and the exchange of a double-charged Higgs boson in the $s$-channel, has been the most often discussed in the literature [4–10]. While experimental prospects for observation of reaction (1) reported in Refs [4–6] are rather poor, predictions for the nonstandard $W^- W^-$ production rates in $e^- e^-$ collisions presented in Refs [8] and [9] are more promising. The authors of Ref. [8] estimated the cross section of reaction (1) in the centre of mass (CM) energy region $2m_W < \sqrt{s} < m_N$, where the electrons interact dominantly through the exchange of heavy Majorana neutrino singlets with masses in the 1 TeV range. Such heavy neutrino singlets are naturally incorporated, $e.g.$, in the framework of $SO(10)$ GUT. Their existence would be of crucial importance for understanding of the very light masses of the three known neutrino states. Assuming purely left-handed $W^-$ couplings and taking into account constraints on neutrino masses and mixings from low energy data, as well as unitarity considerations, the total unpolarized cross section of reaction (1) has been estimated to be

$$\sigma^N \leq \begin{cases} 0.01 - 4 \text{ fb} & \text{at } \sqrt{s} = 0.5 \text{ TeV}, \\ 0.16 - 64 \text{ fb} & \text{at } \sqrt{s} = 1 \text{ TeV}. \end{cases} \tag{2}$$

Another interesting theoretical analysis has been performed in Ref. [9], where reaction (1) has been studied in the framework of extended SM with additional right-handed heavy neutrino singlets. In particular, the question of how large the cross section of (1) could be without violating limits on neutrino masses and mixing parameters, which can be derived from the precision LEP result on the number of light neutrino species and nonobservation of the neutrinoless double-$\beta$ decay, has been addressed. Also some general theoretical constraints, namely unitarity of the mixing matrix in the lepton sector and absence of Higgs triplets, in order to prevent mass terms for the left-handed neutrinos in the model, have been imposed. The maximum cross section of reaction (1) is obtained in a CP conserving model with three right-handed neutrino singlet states $N_l$, $l = 1, 2, 3$ with purely complex CP parities, $\eta_{CP}(N_1) = \eta_{CP}(N_1) = -\eta_{CP}(N_1) = i$, and masses satisfying the relationships

$$\frac{m_{N_2}}{m_{N_1}} \gg 1, \qquad \frac{m_{N_3}}{m_{N_1}} \sim 2 - 10. \tag{3}$$

The assumption about CP parities allows to relax the constraint on masses of heavy neutrinos and their mixing parameters with the electron. In this rather optimistic scenario, the cross section estimates at the same order of magnitude as those of Eq. (2) are obtained.

Several authors [10, 11] have argued that the optimistic predictions of Refs [8] and [9] are in contradiction to the existing limits on the neutrinoless double-$\beta$ decay which already exclude a successful observation of reaction (1). Their argumentation has been based on the observation that reaction (1) can be considered as the inverse neutrinoless double-$\beta$ decay which, however, seems to be too direct an analogy, as it has been recently pointed out in Ref. [12].

Another advantage of the $e^- e^-$ mode is that it offers a possibility of colliding highly polarized beams. The electron polarizations exceeding 80% seem to be achievable at the moment, whereas it is not yet clear whether polarized positron beams may ever be obtained at all [3]. Thus, three independent experiments could be performed with differently polarized initial electron beams. The use of polarized beams increases the resolving power of experiments, especially if search for new physics is concerned: some novel effects present for a specific beam polarizations can be turned off when one or both incoming polarizations are inverted. For example, the cross section of reaction (1) with purely left-handed $W^-$ couplings to electrons becomes a factor four bigger than the estimate of Eq. (2) if both beams are fully left-handed polarized and it equals zero in any other initial polarization case. Also the sensitivity to anomalous couplings of vector bosons increases if the polarized initial beams are used.

In spite of some technical problems related to the effects specific for colliding particle bunches of the same electric charge, integrated luminosities in the range $10 - 100$ fb$^{-1}$ per year for a linac operating in the $e^- e^-$ mode at the CM energies 0.5–1.0 TeV seem to be realistic at present [13]. With these luminosities and highly polarized electron beams, reaction (1) may either be detected at the linear collider, or its nonobservation will deliver new limits on the masses and couplings of heavy Majorana neutrinos.

The pair production of $W^-$ bosons in the SM as well as other SM backgrounds to reaction (1) are discussed in Section 2. How the processes of pair production of weak bosons can be used for testing anomalous quartic couplings is demonstrated in Section 3. Finally, the conclusions are given in Section 4.

## 2. Pair production of $W^-$ bosons in the SM

In order to fully exploit the discovery potential of high energy $e^- e^-$ collisions, it is necessary to understand the SM background. One of the

dominant background processes is

$$e^- e^- \to W^- W^- \nu_e \nu_e, \tag{4}$$

which mimics the final state of the new physics reaction (1). Also other SM reactions of such as

$$e^- e^- \to W^- Z^0 e^- \nu_e \tag{5}$$

and

$$e^- e^- \to Z^0 Z^0 e^- e^- \tag{6}$$

become potential sources of background to reaction (1), in particular, if the $Z^0$ bosons decay into jets and the primary electrons are lost in the beam pipe.

There are 66, 88 and 86 Feynman diagrams of reactions (4), (5) and (6), respectively, already in the lowest order of SM. The typical topologies are the double bremsstrahlung diagrams, the diagrams containing one or two triple gauge boson couplings, two gauge-Higgs boson couplings, or the quartic coupling of gauge bosons. The latter is present only in the SM reaction (4). There are no diagrams involving Higgs boson coupling to electrons, because the limit of zero electron mass has been assumed in the calculation. In this limit the actual number of Feynman diagrams which contribute to a specific channel depends on the polarization of the initial state.

The large number of diagrams precludes the use of trace technique in the calculation of squared matrix elements of reactions (4)–(6). Instead of that, the polarized matrix elements are calculated using the helicity amplitude method described in Ref. [14] and then squared and averaged over spins, if necessary. The integration over the four-particle phase space is performed with the Monte Carlo method after the strongest collinear peaks have been smoothed by appropriate changes of integration variables. The reader interested in details of the calculation is referred to original papers [15] and [16].

TABLE I

Total cross sections in femtobarns for various beam polarizations at $\sqrt{s} = 500$ GeV.

| $e^- e^- \to$ | $e^- e^- \to W^- W^- \nu_e \nu_e$ | $e^- e^- \to W^- Z^0 e^- \nu_e$ | $e^- e^- \to Z^0 Z^0 e^- e^-$ |
|---|---|---|---|
| LL | 9.87 | 23.49 | 1.30 |
| LR | – | 6.95 | 1.13 |
| RR | – | – | 0.57 |
| Unpolarized | 2.47 | 9.35 | 1.03 |

Results for total cross sections of reactions (4), (5) and (6) for different combinations of the left-handed (L) and right-handed (R) polarizations of initial electrons at the CM energy of 500 GeV are given in Table I [16]. The results depend in a relevant way on the value of the running fine structure constant $\alpha(Q^2)$ which enters in the fourth power. There is some ambiguity in the choice of the scale $Q^2$, which is the problem typical for $t$-channel processes. The scale is fixed at $Q^2 = m_Z^2$ and the value $\alpha(m_Z^2) = 1/128.87$ is assumed in Table I and in the following figures. Another ambiguity is related to the collinear singularity. Whereas the electron mass can be safely neglected in reaction (4), it is kept non-zero in the denominator of the photon propagator which is coupled to an on-shell electron line in reactions (5) and (6) in order to regularize the collinear singularity. Only the leading collinear logarithms are taken into account in this way.
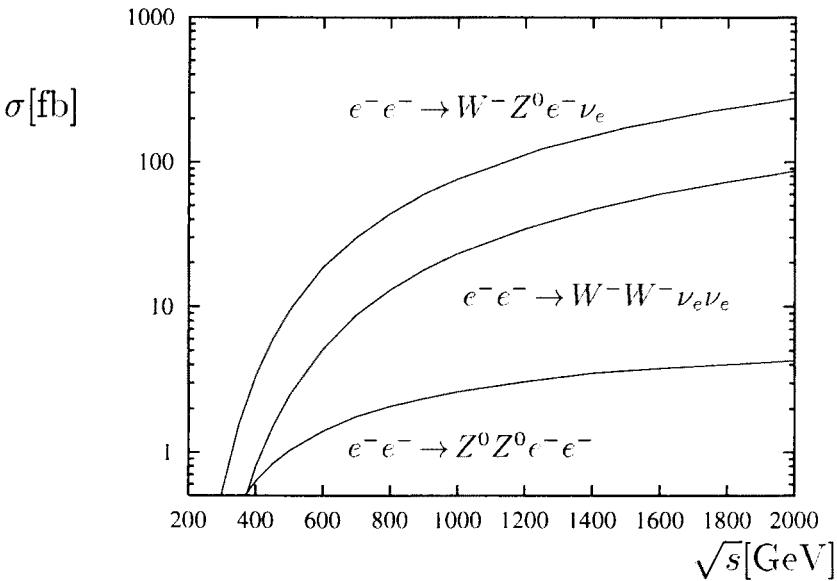


Fig. 1. Total cross sections for gauge boson pair production with unpolarized $e^-$ beams.

The energy dependence of the total unpolarized cross sections of reactions (4)–(6) is plotted in Fig. 1 [16]. In the energy region above 1 TeV, the cross sections of reaction (4) and (5) become quite sizeable and could be measured in the next generation of linear colliders. This would allow to study the subtle $SU(2)_L \times U(1)_Y$ gauge cancellations between diagrams involving different couplings mentioned above, which at the CM energy of 2 TeV amount to 3 orders of magnitude. In case of reaction (4), the quartic SM coupling of $W$ bosons could be tested.

The question of whether reactions (4)–(6) may obscure a possible observation of reaction (1) at the linear collider can be answered by analysing differential distributions. To illustrate this, consider the energy distribution of a $W^-$ boson pair produced in reaction (4) at the CM energy of 500 GeV represented by the dotted line in Fig. 2. In contrary to what is expected in the new physics reaction (1), where, neglecting the radiation effects, the $W^-$ boson pair would carry the whole CM energy, the energy distribution has its maximum at about 250 GeV and it dies out completely when energy approaches 500 GeV. Thus, a simple energy cut should almost completely eliminate the background resulting from reaction (4). The same argumentation holds for the background coming from the SM reactions (5) and (6).
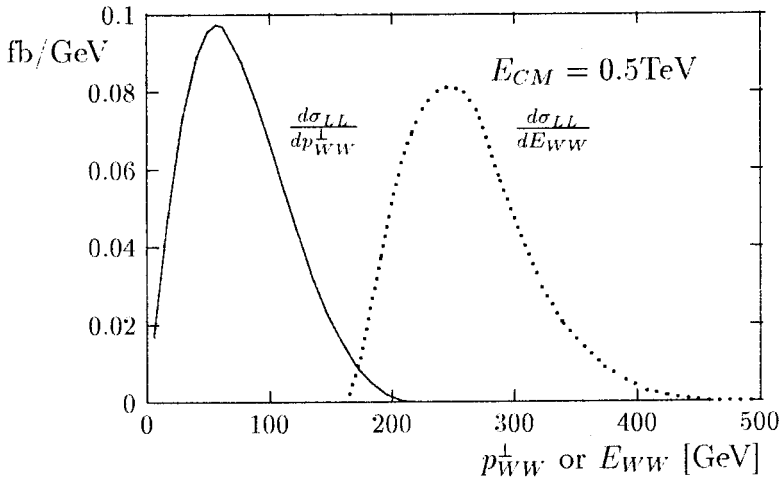


Fig. 2. Transverse momentum and energy distributions of the $W^-W^-$ pair.

The number of background events expected from processes (4)–(6) to different new physics reactions has been estimated in Ref. [16]. Most of the background events can be easily eliminated by imposing some simple kinematical cuts without reducing substantially the signal from new physics reactions.

## 3. Probing anomalous quartic couplings

The high energy behaviour of the cross sections of reactions (4)–(6) is guaranteed by the $SU(2)_L \times U(1)_Y$ gauge cancellations which, as it has been already mentioned, reduce contributions of individual Feynman diagrams by a few orders of magnitude. Any departure of the vector boson couplings from the values predicted by the gauge theory will obviously spoil these subtle cancellations and, in the consequence, violate unitarity. Quadrilinear

couplings of gauge bosons are an essential ingredient of the SM. Whereas the trilinear couplings have already been tested indirectly through loop effects at LEP I experiments and their direct measurement is anticipated at LEP II, the exploration of quartic couplings will be only possible at colliders of the next generation.

Anomalous quadrilinear couplings of vector bosons are often introduced in order to parametrize new physics effects. Their presence usually implies modifications of the trilinear couplings, too. Since the latter will be probably well constrained by the LEP II data, it is natural to restrict the discussion to the quartic couplings which do not induce non-standard trilinear couplings and correspond to the operators of the lowest dimension four [17]. In the presence of such couplings the SM lagrangian is modified by the following extra pieces [18], [19]

$$\mathcal{L}_0 = g_0 g_W^2 \left( W^{+\mu} W_\mu^- W^{+\nu} W_\nu^- + \frac{1}{\cos^2 \theta_w} W^{+\mu} W_\mu^- Z^\nu Z_\nu \right. \tag{7}$$

$$\left. + \frac{1}{4 \cos^4 \theta_w} Z^\mu Z_\mu Z^\nu Z_\nu \right) .$$

$$\mathcal{L}_c = g_c g_W^2 \left( \frac{1}{2} (W^{+\mu} W_\mu^- W^{+\nu} W_\nu^- + W^{+\mu} W_\mu^+ W^{-\nu} W_\nu^-) \right. \tag{8}$$

$$\left. + \frac{1}{\cos^2 \theta_w} Z^\mu W_\mu^+ Z^\nu W_\nu^- + \frac{1}{4 \cos^4 \theta_w} Z^\mu Z_\mu Z^\nu Z_\nu \right) .$$

where $g_0$ and $g_c$ are the anomalous couplings and the remaining notation is obvious. The number $n(g_0, g_c)$ of events expected for each of reactions (4), (5) and (6) in the presence of anomalous couplings of Eqs. (7) and (8) deviates from the corresponding number $n_{SM}$ of events expected in the SM. The results for the $e^- e^-$ collider operating at the CM energy 500 GeV with an integrated luminosity of 10 fb$^{-1}$ are shown in Fig. 3 as the 95% confidence level contours in the $(g_0, g_c)$ plane of the function

$$\chi^2 = \left( \frac{n(g_0, g_c) - n_{SM}}{\Delta n} \right)^2 . \tag{9}$$

where $\Delta n$ is the combined statistical and systematical error [17]. It has been assumed that both vector bosons decay with no missing energy and the acceptance cut of $10^o \leq \theta_e \leq 170^o$ on the polar angle of primary electrons in reactions (5) and (6) has been imposed. In this way, it should be possible to reconstruct the final state of each of reactions (4)–(6). The corresponding limit on the anomalous couplings $g_0$ and $g_c$ which can be obtained in the $e^+ e^-$ mode of the linear collider [19] are indicated in Fig. 3 by the thin line.
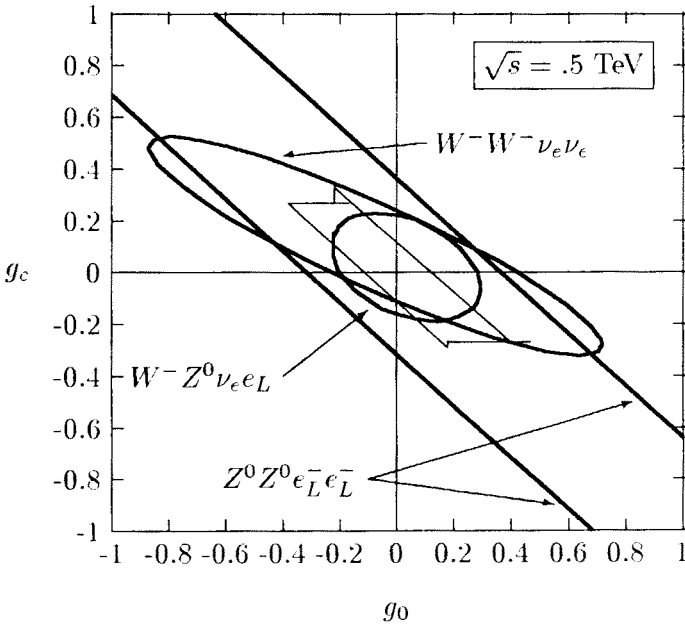
Fig. 3. Contours of observability at 95% confidence level of the anomalous quartic gauge couplings $g_0$ and $g_c$. The CM energy 500 GeV and 10 $fb^{-1}$ of accumulated luminosity are assumed. The limits which can be obtained under similar conditions in the $e^+e^-$ mode of the same collider [19] are indicated by the thin line.

## 4. Conclusions

The discovery potential of the $e^-e^-$ mode of the new linear collider has been discussed. The electron-electron collisions are very sensitive to lepton-number violating processes and could be very suitable for studying the important question of neutrino masses.

The production rates of weak boson pairs through reactions (4), (5) and (6) are known to the lowest order of SM. Typical topologies of the final states can be predicted by studying differential distributions. The background to new physics reactions, in particular to $e^-e^- \rightarrow W^-W^-$, can be eliminated by imposing appropriate cuts.

In the 1 TeV energy region the cross sections of reactions (4) and (5) become big enough to be measured at a linear collider of the new generation. The $SU(2)_L \times U(1)_Y$ gauge cancellations including the quartic coupling of gauge bosons could be tested in this way and new limits on possible anomalous quartic couplings may be obtained.

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