Vol. 28 (1997) ACTA PHYSICA POLONICA B No 6

QCD STRUCTURE OF W BOSON AND ELECTRON STRUCTURE FUNCTION* **

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(Received April 21, 1997)

The QCD structure of the W boson is constructed and compared with that of the photon. A new concept of the electron structure function is also defined. The leading order splitting functions of electron into quarks are extracted, showing an important contribution from γ-Z interference. Leading logarithmic QCD evolution equations are constructed and solved in the asymptotic region where $\log^2$ behaviour of the parton densities is observed. Possible applications with clear manifestation of 'resolved' photon and weak bosons are discussed.

PACS numbers: 14.70. Fm, 13.60. -r

At energies much higher than their masses the W and Z bosons can develop QCD structure very much like the 'resolved' photon [1]. In this talk we concentrate first on the quark and gluon content of the W boson. Then we advocate for a new concept in lepton induced hadronic production – the lepton structure function. This function summarizes contributions from all electroweak bosons in a systematic way. We demonstrate new effects appearing in this stucture function and stress differences with the standard approach when a convolution of equivalent bosons and boson structure functions are used.

Let us start with the well known example of the photon. Its anomalous structure has been studied for more than ten years [1]. It is known to dominate at high P^2 (where P^2 is the large momentum scale in the process) over the hadronic (Vector Meson) component and can be measured in lepton–lepton or lepton–hadron scattering [2]. Conceptually this structure emerges

* Presented by J. Szwed at the Cracow Epiphany Conference on W Boson, Cracow, Poland, January 4–6, 1997.

** Work supported by the Polish State Committee for Scientific Research (grant No. 2 P03B 081 09) and the Volkswagen Foundation.

as QCD collinear quark-gluon cascade initiated by a quark–antiquark pair produced by photon. At high momentum transfers, such that $P^2 \gg M_W^2$, analogous cascade can develop from the W boson — this time initiated via weak interactions.

There are several important differences as compared to the photon. First, there are strong polarization asymmetries caused by different couplings to left- and righthanded quarks. Second, the W boson is massive and as such has an on-shell longitudinal component. The trace of it will be discussed below.

One has rather rarely access to the source of real weak intermediate bosons. As in the case of photons, the QCD structure is expected to be seen in the processes where nearly real bosons are emitted by fermions. One may then make use of the equivalent boson approximation known since long for the photons [3] and calculated only recently for the W and Z [4]. "Nearly real" means in this case that the boson momentum squared is negligible as compared to the other scales present in a given process. Whereas this approximation causes little doubts in the case of photons, it requires more attention with the massive weak bosons.

In the following we will find the quark and gluon densities inside W (and Z) bosons for asymptotically large P^2 [5]. We have to find the W splitting functions into quarks, construct evolution equations [6] and finally solve them in the asymptotic region. The first step of this procedure can be performed by calculating any (sub)-process involving colinear quark emissions from the W and extracting the splitting functions by means of the standard factorization prescription [7]. We probe the W boson with an off-shell gluon (G^*) of four momentum squared $p^2 \equiv -P^2$. We choose the gluon as a probe, instead of the photon, to avoid its direct interaction with the W boson. To lowest order in $g \equiv g_{\mathrm{weak}}$ it couples directly to quarks as shown by the diagrams in Fig. 1. To study this interaction in detail we calculate the polarized G-W cross-section

$$G^*_\lambda + W^-_\sigma \to \mathrm{q}_\pm + \bar{\mathrm{q}}_\mp \,. \tag{1}$$

Choosing a frame where G and W collide along z-axis we have following simple parametrization of their 4-momenta and polarization vectors:

$$q^\mu = (q_0, 0, 0, q_z)\,, \tag{2}$$
$$p^\mu = (p_0, 0, 0, p_z)\,, \tag{3}$$

and

$$\varepsilon^\mu_{\mathrm{L}} = \frac{1}{\sqrt{2}}(0, 1, -i, 0)\,, \tag{4}$$

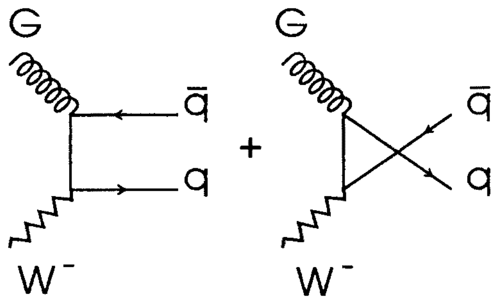

Fig. 1. Lowest order diagrams for $G^* + W^- \to q+\bar{q}$ scattering.

$$\varepsilon^{\mu}_{\mathrm{R}} = \frac{1}{\sqrt{2}}(0, 1, +i, 0)\,, \tag{5}$$

$$\varepsilon^{\mu}_{||}(p) = \sqrt{\frac{1}{|p^2|}}\,(p_z, 0, 0, p_0)\,, \tag{6}$$

$$\varepsilon^{\mu}_{||}(q) = \frac{1}{M_W}\,(q_z, 0, 0, q_0)\,. \tag{7}$$

The cross-section summed over the final quark polarizations reads:

$$\sigma(G^*_\lambda W^-_\sigma) = 2\pi\alpha_{\mathrm{s}}\alpha_{\mathrm{em}}\frac{x}{P^2}\,F_{\lambda\sigma}\left(x, \log\frac{P^2}{M_W^2}\right)\sum_{q=1}^{n_{\mathrm{f}}} w_{\mathrm{q}}\,, \tag{8}$$

where α_{em} and α_{s} are electromagnetic and strong coupling constants, respectively and $F_{\lambda\sigma}$ can be found in Table I. The coupling w_{q} reads:

$$w_{\mathrm{q}} = \frac{1}{2\sin^2\theta_W}\,. \tag{9}$$

where θ_W is the Weinberg angle. Within leading-log approximation we will parametrize α_{s} as

$$\alpha_{\mathrm{s}}(t) = \frac{2\pi}{bt}\,, \tag{10}$$

with $t = \ln(P^2/\Lambda^2_{\mathrm{QCD}})$ and $b = 11/2 - n_{\mathrm{f}}/3$ for n_{f} flavours.

TABLE I

Coefficients $F_{\lambda\sigma}(x, \mathcal{L})$, of Eq.(8)

	$\lambda = -$	$\lambda = +$	$\lambda = \|\|$
$\sigma = -$	$(1-x)^2(\mathcal{L}-2)$	$x^2(\mathcal{L}-2)$	$2x(1-x)$
$\sigma = +$	$x^2(\mathcal{L}-2)$	$(1-x)^2(\mathcal{L}-2)$	$2x(1-x)$
$\sigma = \|\|$	$2x(1-x)$	$2x(1-x)$	0

Let us observe that the splitting functions of the longitudinally polarized weak intermediate bosons ($\sigma = ||$) vanish in the lowest α_s order. That means that for $P^2 \to \infty$ the longitudinal $W^{\pm}$ will contain negligible amounts of quarks and gluons as compared to the transverse bosons. Keeping this in mind we present the following results for transverse weak intermediate bosons only, defined as $W_{\perp} = (W_+ + W_-)/2$. This choice is also convenient for the comparison with the photon. Extracting the coefficients at logs in the Eq. (8) we obtain the spin-dependent splitting functions of weak intermediate bosons. All non-zero splitting functions read

$$P_{\mathrm{d}_- W_{\perp}^-}(x) = P_{\bar{\mathrm{u}}_+ W_{\perp}^-}(x) = \frac{3}{2\sin^2\theta_W}\, s(x)\,, \tag{11}$$

with $s(x) = [x^2 + (1-x)^2]/2$.

With these functions at hand we are able to construct the evolution equations for quarks of any flavour and gluons. Instead of writing them down for each polarization separately we prefer to use the unpolarized ($q = q_+ + q_-$) and polarized ($\Delta q = q_+ - q_-$) densities. In this notation the evolution equations for the $W_{\perp}^- = (W_-^- + W_+^-)/2$ read:

$$\begin{aligned}
\frac{dq(x,t)}{dt} &= \frac{\alpha_{\mathrm{em}}}{2\pi} P_{\mathrm{q}W_{\perp}^-}(x) + \frac{\alpha_s(t)}{2\pi} P_{\mathrm{qq}} \otimes q(t) + \frac{\alpha_s(t)}{2\pi} P_{\mathrm{qG}} \otimes G(t)\,,\\
\frac{d\bar{q}(x,t)}{dt} &= \frac{\alpha_{\mathrm{em}}}{2\pi} P_{\bar{\mathrm{q}}W_{\perp}^-}(x) + \frac{\alpha_s(t)}{2\pi} P_{\mathrm{qq}} \otimes \bar{q}(t) + \frac{\alpha_s(t)}{2\pi} P_{\mathrm{qG}} \otimes G(t)\,,\\
\frac{dG(x,t)}{dt} &= \frac{\alpha_s(t)}{2\pi} \sum_{\mathrm{q}} P_G\mathrm{Gq} \otimes [q(t) + \bar{q}(t)] + \frac{\alpha_s(t)}{2\pi} P_{\mathrm{GG}} \otimes G(t)
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
\frac{d\Delta q(x,t)}{dt} &= \frac{\alpha_{\mathrm{em}}}{2\pi} \Delta P_{\mathrm{q}W_{\perp}^-}(x) + \frac{\alpha_s(t)}{2\pi} \Delta P_{\mathrm{qq}} \otimes \Delta q(t) + \frac{\alpha_s(t)}{2\pi} \Delta P_{\mathrm{qG}} \otimes \Delta G(t)\,,\\
\frac{d\Delta\bar{q}(x,t)}{dt} &= \frac{\alpha_{\mathrm{em}}}{2\pi} \Delta P_{\bar{\mathrm{q}}W_{\perp}^-}(x) + \frac{\alpha_s(t)}{2\pi} \Delta P_{\mathrm{qq}} \otimes \Delta\bar{q}(t) + \frac{\alpha_s(t)}{2\pi} \Delta P_{\mathrm{qG}} \otimes \Delta G(t)\,,\\
\frac{d\Delta G(x,t)}{dt} &= \frac{\alpha_s(t)}{2\pi} \sum_{\mathrm{q}} \Delta P_{\mathrm{Gq}} \otimes [\Delta q(t) + \Delta\bar{q}(t)] + \frac{\alpha_s(t)}{2\pi} \Delta P_{\mathrm{GG}} \otimes \Delta G(t)\,,
\end{aligned} \tag{13}$$

where

$$(P \otimes f)(x) \equiv \int dx_1\, dx_2\, P(x_1)\, f(x_2)\, \delta(x - x_1 x_2)\,. \tag{14}$$

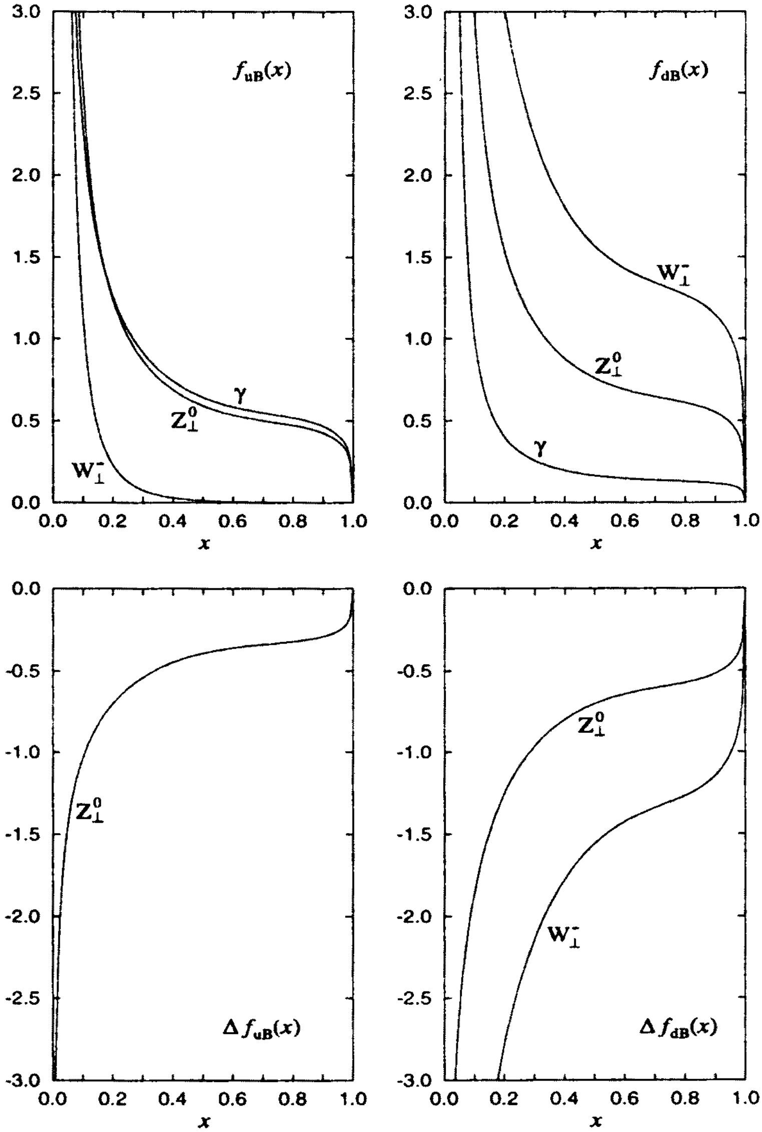

Fig. 2. u-type and d-type quark distribution functions in γ, $W^-_\perp$ and $Z_\perp$

and

$$\begin{aligned} P_{dW^-_\perp}(x) &= -\Delta P_{dW^-_\perp}(x) = \tfrac{1}{2} w_q a(x)\,, \\ P_{uW^-_\perp}(x) &= \Delta P_{uW^-_\perp}(x) = 0\,, \\ P_{\bar{d}W^-_\perp}(x) &= \Delta P_{\bar{d}W^-_\perp}(x) = 0\,, \end{aligned} \tag{15}$$

$$P_{\bar{u}W^-_\perp}(x) \; = \Delta P_{\bar{u}W^-_\perp}(x) \; = \tfrac{1}{2} w_q a(x) \,.$$

The above equations can be solved in the asymptotic region. In analogy to the photon case, the solutions have the form

$$\begin{aligned} q(x,t) &\simeq \frac{\alpha_{\rm em}}{2\pi} q^{\rm as}(x)\, t \,, \\ \bar{q}(x,t) &\simeq \frac{\alpha_{\rm em}}{2\pi} \bar{q}^{\rm as}(x)\, t \,, \\ G(x,t) &\simeq \frac{\alpha_{\rm em}}{2\pi} G^{\rm as}(x)\, t \end{aligned} \tag{16}$$

where $q^{\rm as}$ $\bar{q}^{\rm as}$ and $G^{\rm as}$ fulfil following equations:

$$\begin{aligned} d^{\rm as}(x) &= \tfrac{1}{2} w_q a(x) + \tfrac{1}{b} P_{\rm qq} \otimes d^{\rm as} + \tfrac{1}{b} P_{\rm qG} \otimes G^{\rm as} \,, \\ u^{\rm as}(x) &= \tfrac{1}{b} P_{\rm qq} \otimes u^{\rm as} + \tfrac{1}{b} P_{\rm qG} \otimes G^{\rm as} \,, \\ \bar{d}^{\rm as}(x) &= \tfrac{1}{b} P_{\rm qq} \otimes \bar{d}^{\rm as} + \tfrac{1}{b} P_{\rm qG} \otimes G^{\rm as} \,, \\ \bar{u}^{\rm as}(x) &= \tfrac{1}{2} w_q a(x) + \tfrac{1}{b} P_{\rm qq} \otimes \bar{u}^{\rm as} + \tfrac{1}{b} P_{\rm qG} \otimes G^{\rm as} \,, \\ G^{\rm as}(x) &= \tfrac{1}{b} \sum_{\text{d-, u-type}} P_{\rm Gq} \otimes (d^{\rm as} + u^{\rm as} + \bar{d}^{\rm as} + \bar{u}^{\rm as}) + \tfrac{1}{b} P_{\rm GG} \otimes G^{\rm as} \,. \end{aligned} \tag{17}$$

We solve the above equations numerically. The resulting unpolarized densities for u and d quarks and gluons are shown in Fig. 2–3 (together withthe photon and Z^0 results. At the energies under consideration we take into account six flavours. The c and t quarks distributions follow that of the u quark; the s and b ones – that of the d. Comparison with the hadronic content of the photon shows the W boson structure slightly richer than that of the photon.

As already mentioned one has rarely real weak bosons at one's disposal. In physical processes, wher the structure of electroweak bosons may contribute significantly, it is the lepton, initiating the process which is the source of the intermediate bosons. The standard procedure applied in such cases is to use the equivalent photon approximation (extended also to the case of weak bosons) and, as a next step, to convolute the obtained boson distributions with parton densities inside the bosons. For example the parton k density of the electron $F_k^{e^-}$ would read:

$$F_k^{e^-}(z, \hat{Q}^2, P^2) = \sum_B F_B^{e^-}(\hat{Q}^2) \otimes F_k^B(P^2) \,. \tag{18}$$

where $(F \otimes G)(z) \equiv \int dx\, dy\, \delta(z - xy) F(y) G(x)$, $\hat{Q}^2$ is the maximum allowed virtuality of the boson, P^2 is the hard process scale and z — the momentum

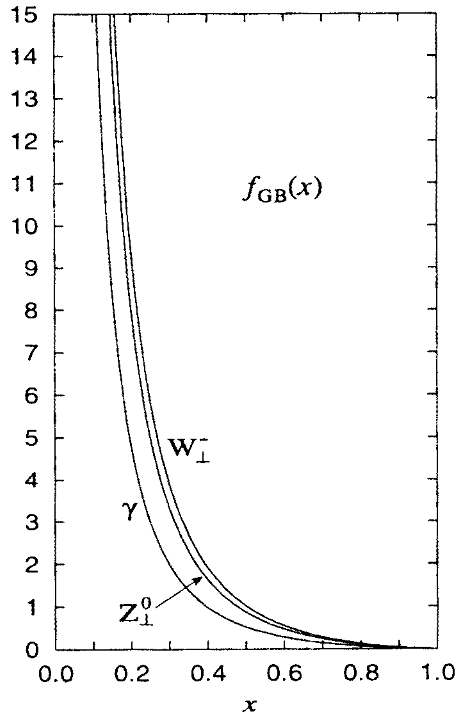

Fig. 3. Gluon distribution functions in γ, $W_\perp^-$ and $Z_\perp$

fraction of the parton k with respect to the electron (detailed definitions follow). In such an approximation several questions arise: how far off-shell can the intermediate bosons be, how large are their interference effects, what are the relations between the energy scales governing the consecutive steps, is the convolution Eq. (18) correct in general? We think that it is more precise to answer the direct question: what is the quark and gluon content of the incoming lepton or, in other words, what is the lepton structure function? The answer to the above question [8] brings several corrections to the standard procedure.

The construction of the electron structure function can be divided into two steps. In the first we calculate the splitting of the electron into a $q\bar{q}$ pair, in the second the quark-gluon cascade is resummed with the use of the evolution equations. To obtain the electron splitting functions let us consider inclusive scattering of a virtual gluon off an electron. In the lowest order in the electromagnetic and strong coupling constants (α and α_s) the electron couples to $q\bar{q}$ pair as shown in Fig. 4.

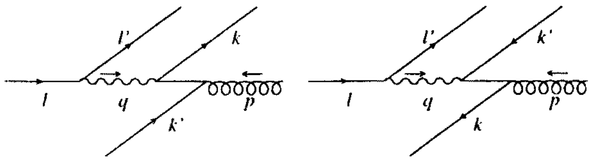

Fig. 4. Lowest order graphs contributing to the process: $e + G^* \to \ell' + q + \bar{q}$.

The incoming electron e carries 4-momentum l and the off-shell gluon G^* of 4-momentum p with large $P^2 \equiv -p^2$, serves here as a probe of the electron. In the final state we have a massless quark q and antiquark $\bar{q}$ of 4-momenta k and k' and lepton ℓ' (electron or neutrino) of 4-momentum l'. The exchanged boson $B = \{\gamma, Z, W\}$ carries 4-momentum q ($Q^2 \equiv -q^2$).

The current matrix element squared for an unpolarized electron reads:

$$\begin{aligned} &\mathcal{J}(e^- G^* \to \ell' q_\eta \bar{q}_{-\eta}) = \mathcal{J}^\eta_{\mu\nu}(l,p) \\ &= \frac{1}{4\pi} \int d\Gamma_{l'} d\Gamma_k d\Gamma_{k'} (2\pi)^4 \delta_4(k + k' + l' - p - l) \\ &\times \langle e^- | J^\dagger_\mu(0) | \ell' q_\eta \bar{q}_{-\eta} \rangle \langle \ell' q_\eta \bar{q}_{-\eta} | J_\nu(0) | e^- \rangle \,, \end{aligned} \tag{19}$$

where

$$d\Gamma_k = \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^2) \,. \tag{20}$$

For massless quarks we can decompose the current in the helicity basis:

$$\mathcal{J}^\eta_\sigma(l,p) = \varepsilon^{\mu *}_{(\sigma)}(p)\, \mathcal{J}^\eta_{\mu\nu}(l,p)\, \varepsilon^\nu_{(\sigma)}(p) \,. \tag{21}$$

In a frame where $\vec{q}$ is antiparallel to $\vec{p}$ contributions from different helicities of exchanged bosons do not mix and the current reads:

$$\begin{aligned} \mathcal{J}^\eta_\sigma(p,l) = &\frac{\alpha_s \alpha^2}{2\pi} \sum_{A,B,\rho} g^{Aq}_\eta g^{Bq}_\eta \int \frac{dy}{y} P^{e^-}_{A_\rho B_\rho}(y) \\ &\times \int\limits_{Q^2_{\min}}^{Q^2_{\max}} \frac{Q^2 dQ^2}{(Q^2 + M_A^2)(Q^2 + M_B^2)} H^\eta_{\rho\sigma}(x, Q^2) \,, \end{aligned} \tag{22}$$

where $P^{e^-}_{A_\rho B_\rho}(y)$ describes weak boson emission from the electron and $H^\eta_{\rho\sigma}(x, Q^2)$ — $q\bar{q}$ pair production by virtual gluon and electroweak boson. g^{Aq}_η is the boson A to quark q_η coupling in the units of proton charge e. The sum runs over the electroweak bosons ($A, B = \gamma, W^-, Z$) and their polarizations $\rho = \pm 1, 0$. Note that although the sum is diagonal in the polarization index ρ it is not in the boson type A, B. The off-diagonal terms in the sum arise from the γ-Z interference. As demonstrated below their contribution is substantial.

To answer our main problem of 'an electron splitting into a quark' we take the limit $Q^2 \ll P^2$ and keep the leading terms only. Within this approximation the kinematic variables x, y, z read

$$y = \frac{pq}{pl}, \quad z = xy = \frac{-p^2}{2pl}, \tag{23}$$

aquiring the parton model interpretation of the quark momentum fraction (z) and of boson momentum fraction (y), both with respect to the parent electron. The leading term of the hadronic part does not depend on the quark helicity η:

$$H_{\pm\mp}(x, Q^2) = x^2 \log\frac{P^2}{Q^2}, \tag{24}$$

$$H_{\pm\pm}(x, Q^2) = (1-x)^2 \log\frac{P^2}{Q^2}, \tag{25}$$

with other components finite for $P^2/Q^2 \to 0$.

We also recognize $P^{e^-}_{A_\rho B_\rho}(y)$ as a generalization of the splitting functions of an electron into bosons [5]:

$$P^{e^-}_{A_\rho B_\rho}(y) = \tfrac{1}{2}(g_-^{Ae^-} g_-^{Be^-} yY_{-\rho} + g_+^{Ae^-} g_+^{Be^-} yY_\rho) \tag{26}$$

with

$$Y_+(y) = \frac{1}{y^2}, \tag{27}$$

$$Y_-(y) = \frac{(1-y)^2}{y^2}, \tag{28}$$

$$Y_0(y) = \frac{2(1-y)}{y^2}, \tag{29}$$

where $g_\pm^{Ae^-}$ is the electron to boson A_ρ coupling in the units of proton charge e.

From kinematics $y \in [z, 1 - O(m_e^2/P^2)]$ and the integration limits for Q^2 read

$$Q^2_{\min} = m_e^2 \frac{y^2}{1-y}, \quad Q^2_{\max} = P^2 \frac{z + y - zy}{z}, \tag{30}$$

with m_e being the electron mass. Although smaller than the already neglected quark masses, it is the electron mass which must be kept finite in order to regularize collinear divergencies. The upper limit of integration requires particular attention. In general it is a function of P^2, however integration up to the maximum kinematically allowed value $Q^2_{\max}$ would violate the condition $Q^2/P^2 \ll 1$. For our approximation to work we integrate over Q^2 up to $\hat{Q}^2_{\max} = \epsilon P^2$ where $\varepsilon \ll 1$ and generally depends on y and z. A similar condition is in fact used in phenomenological applications of the equivalent photon approximation [9]. Note that the maximum virtuality $\hat{Q}^2_{\max}$ can also be kept independent of P^2 when special kinematical cuts are arranged in experiment. We concentrate here on the fully inclusive case with the maximum virtuality being P^2-dependent. Integrating Eq. (22) over Q^2 within such limits and keeping only leading-logarithmic terms leads to

$$\mathcal{J}^\eta_\sigma(p,l) = \frac{\alpha_s\alpha}{6} \sum_{A,B,\rho} g^{Aq}_\eta g^{Bq}_\eta F^{e^-}_{A_\rho B_\rho}(P^2)$$
$$\otimes[P^\rho_{q_\eta}\delta_{\eta,-\sigma} + P^\rho_{\bar{q}_{-\eta}}\delta_{\eta,\sigma}]\log P^2 \,. \tag{31}$$

In the above equation $P^\rho_{q_\eta}(x)$ and $P^\rho_{\bar{q}_\eta}(x)$ are boson-quark (-antiquark) splitting functions [1, 5]

$$P^\pm_{q_\pm}(x) = P^\pm_{\bar{q}_\pm}(x) = 3x^2,\ P^\mp_{q_\pm}(x) = P^\mp_{\bar{q}_\pm}(x) = 3(1-x)^2 \tag{32}$$

and $F^{e^-}_{A_\rho B_\rho}(y, P^2)$ is the density matrix of polarized bosons inside electron. Its transverse components read

$$F^{e^-}_{\gamma_\pm\gamma_\pm}(y) = \frac{\alpha}{2\pi}\frac{(1-y)^2+1}{2y}\log\mu_0\,, \tag{33a}$$

$$F^{e^-}_{Z_+Z_+}(y) = \frac{\alpha}{2\pi}\tan^2\theta_W\,\frac{\rho^2_W(1-y)^2+1}{2y}\log\mu_Z\,, \tag{33b}$$

$$F^{e^-}_{Z_-Z_-}(y) = \frac{\alpha}{2\pi}\tan^2\theta_W\,\frac{\rho^2_W+(1-y)^2}{2y}\log\mu_Z\,, \tag{33c}$$

$$F^{e^-}_{\gamma_+Z_+}(y) = \frac{\alpha}{2\pi}\tan\theta_W\,\frac{\rho_W(1-y)^2-1}{2y}\log\mu_Z\,, \tag{33d}$$

$$F^{e^-}_{\gamma_-Z_-}(y) = \frac{\alpha}{2\pi}\tan\theta_W\,\frac{\rho_W-(1-y)^2}{2y}\log\mu_Z\,, \tag{33e}$$

$$F^{e^-}_{W_+W_+}(y) = \frac{\alpha}{2\pi}\frac{1}{4\sin^2\theta_W}\frac{(1-y)^2}{y}\log\mu_W\,, \tag{33f}$$

$$F^{e^-}_{W_-W_-}(y) = \frac{\alpha}{2\pi}\frac{1}{4\sin^2\theta_W}\frac{1}{y}\log\mu_W\,, \tag{33g}$$

where

$$\rho_W = \frac{1}{2\sin^2\theta_W} - 1 \tag{34}$$

and

$$\log \mu_0 = \log \frac{\varepsilon P^2}{m_e^2}, \; \log \mu_B = \log \frac{\varepsilon P^2 + M_B^2}{M_B^2}. \tag{35}$$

All other density matrix elements (containing at least one longitudinal boson) do not develop logarithmic behaviour.

At this point we are able to define the splitting functions of an electron into a quark at the momentum scale P^2 as

$$\mathcal{P}^{e^-}_{q_\eta}(P^2) = \sum_{AB} g_\eta^{Aq} g_\eta^{Bq} \sum_\rho F^{e^-}_{A_\rho B_\rho}(P^2) \otimes P^\rho_{q_\eta}. \tag{36}$$

The expicit expressions for quarks read

$$\begin{aligned} \mathcal{P}^{e^-}_{q_+}(z, P^2) &= \frac{3\alpha}{4\pi}\{e_q^2\,[\Phi_+(z) + \Phi_-(z)]\log\mu_0 \\ +e_q^2 &\quad \tan^4\theta_W \left[\Phi_+(z) + \rho_W^2\Phi_-(z)\right]\log\mu_Z \\ -2e_q^2 &\quad \tan^2\theta_W\,[-\Phi_+(z) + \rho_W\Phi_-(z)]\log\mu_Z\}\,, \end{aligned} \tag{37a}$$

$$\begin{aligned} \mathcal{P}^{e^-}_{q_-}(z, P^2) &= \frac{3\alpha}{4\pi}\{e_q^2\,[\Phi_+(z) + \Phi_-(z)]\log\mu_0 \\ +z_q^2 &\quad \tan^4\theta_W \left[\Phi_-(z) + \rho_W^2\Phi_+(z)\right]\log\mu_Z \\ +2e_q z_q &\quad \tan^2\theta_W\,[-\Phi_-(z) + \rho_W\Phi_+(z)]\log\mu_Z \\ + &\quad (1+\rho_W)^2\Phi_+(z)\delta_{qd}\log\mu_W\}\,, \end{aligned} \tag{37b}$$

where

$$\Phi_+(z) = \frac{1-z}{3z}(2 + 11z + 2z^2) + 2(1+z)\log z\,, \tag{38}$$

$$\Phi_-(z) = \frac{2(1-z)^3}{3z}. \tag{39}$$

and

$$z_q = \frac{T_3^q}{\sin^2\theta_W} - e_q\,, \tag{40}$$

with e_q and T_3^q being the quark charge and 3-rd weak isospin component, respectively. The splitting functions for antiquark of opposite helicity can be obtained from Eq. (37) by interchanging Φ_+ with Φ_-.

The splitting functions introduced above show two new features. The first one, already mentioned before, is the contribution from the interference of electroweak bosons (γ and Z only). The second is their P^2 dependence, which arises from the upper integration limit $\hat{Q}^2_{\max}$.

Having completed the first step of our procedure — the calculation of the splitting functions, we can proceed to the resumation of the QCD cascade using the evolution equations. We consider the evolution equations in the first order in electroweak couplings and leading logarithmic in QCD. Remembering that there is no direct coupling of the electroweak sector to gluons we can write:

$$\frac{dF^{e^-}_{q_\eta}(t)}{dt} = \frac{\alpha}{2\pi}\mathcal{P}^{e^-}_{q_\eta}(t) + \frac{\alpha_s(t)}{2\pi}\sum_{k,\rho} P^{k_\rho}_{q_\eta} \otimes F^{e^-}_{k_\rho}(t)\,, \tag{41a}$$

$$\frac{dF^{e^-}_{G_\lambda}(t)}{dt} = \frac{\alpha_s(t)}{2\pi}\sum_{k,\rho} P^{k_\rho}_{G_\lambda} \otimes F^{e^-}_{k_\rho}(t). \tag{41b}$$

We stress that the convolution of the equivalent boson distributions and boson-quark splitting functions, Eq. (36), occurs at the level of splitting functions. It is not equivalent to the usually performed convolution of the distribution functions because of the P^2 dependence of the boson distribution functions Eq. (33). Only in the case when the upper limit of integration $\hat{Q}^2_{\max}$ is kept fixed (P^2-independent), e.g. by special experimental cuts, are the convolutions equivalent at both levels.

The equations Eq. (41) can be solved in the asymptotic t region. The solution to Eqs. (41) for the parton k of polarization ρ can be now parametrized as

$$F^{e^-}_{k_\rho}(z,t) \simeq \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2 f^{\rm as}_{k_\rho}(z)\, t^2 \tag{42}$$

resulting in purely integral equations

$$f^{\rm as}_{i_\rho} = \hat{\mathcal{P}}^{e^-}_{i_\rho} + \frac{1}{2b}\sum_{k,\lambda} P^{k_\lambda}_{i_\rho} \otimes f^{\rm as}_{k_\lambda}\,, \tag{43}$$

where $\hat{\mathcal{P}}^{e^-}_{i_\rho}(z)$ are equal to $P^{e^-}_{i_\rho}(z)$ (Eqs.(37)) divided by $\alpha/2\pi$ and with all $\log \mu_A \equiv 1$.

Numerical solutions to the above equations with the method described in Ref. [5] are presented in Fig. 5 for the unpolarized quark and gluon distributions. One notices significant contribution from the W intermediate state in the d-type quark density. The most surprising however is the γ-Z interference contribution which cannot be neglected, as it is comparable to the Z term. It violates the standard probabilistic approach where only diagonal terms are taken into account. This also stresses the necessity of introducing the concept of electron structure function in which all contributions from intermediate bosons are properly summed up. Due to the nature of weak

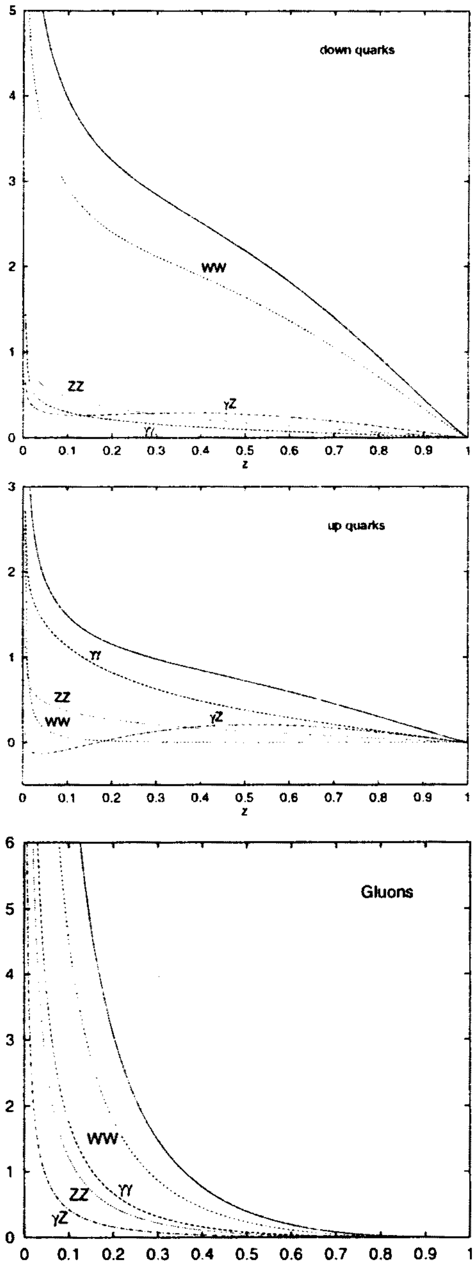

Fig. 5. Unpolarized quark and gluon distributions $zf^{as}(z)$ inside the electron — solid line. The other lines show contributions from single electroweak bosons.

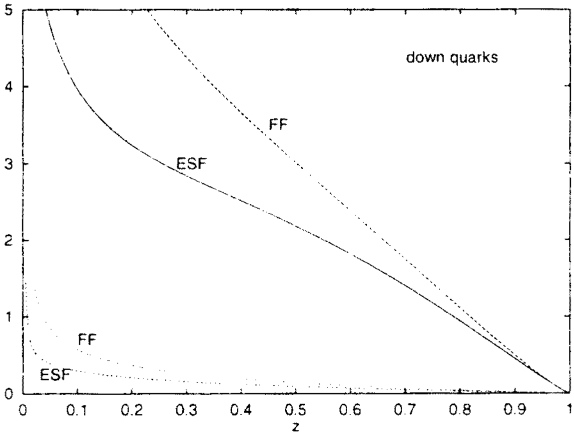

Fig. 6. Comparison of unpolarized d-quark distributions $zf^{as}(z)$ in the electron calculated by ESF and FF methods. The upper two lines result from contributions from all electroweak bosons while the lower two from $\gamma\gamma$ only.

couplings they turn out to be nonzero, even in the case of gluon distributions. Again the γ-Z interference term is important and the W contribution dominates in the asymptotic region.

One should keep in mind that at finite t the logarithms multiplying the photon contribution differ from the remaining ones (Eq. (35)). Being scaled by m_e, they lead to the photon domination at presently available P^2. The importance of the interference term remains constant relative to the Z contribution, as they are both governed by the same logarithm. But even at presently available momenta, where the 'resolved' photon dominates, one can see how the correct treatment of the scales changes the evolution in the fully inclusive case. In Fig. 6 we present the asymptotic solutions of the evolution equations following from our procedure (ESF) compared to those following from the application of the convolution Eq. (18) (FF). The difference which can be noticed vanishes only in case one puts an experimental cut on $\hat{Q}^2_{\max}$.

To summarize we have presented a construction of weak bosons structure functions and compared them with that of the photon. Significant spin and flavour dependence can be observed in the quark–gluon contentof the W and Z bosons. We argued that the concept of the electron structure function is the correct approach to lepton induced processes when the QCD partons are collinear with the lepton. The calculation has been done in the leading logarihtmic approximation to QCD and leading order in electroweak interactions. One can definitely improve this approximation, in particular, treating the electroweak sector more precisely. The γ-Z interference, contrary to naive expectations, turns out to be important. Direct calculation of the splitting functions of an electron into quarks allows for precise control of the momentum scales entering the evolution. It also shows that the convolution of leptons, electroweak bosons and quarks should be made at the level of splitting functions rather than distribution functions. Unless forced otherwise by the experimental cuts, the electron splitting functions depend on the external scale P^2 and influence significantly the parton evolution.

Phenomenological applications of the above analysis require very high momentum scales in order to see the weak boson and interference contributions. Possible processes where these effects could show up include heavy flavour, large $p_\perp$ jet and Higgs boson production in lepton induced processes. At presently available momenta, where the photons dominate, the use of the electron structure function allows to treat correctly the parton evolution. It seems also more plausible to use one phenomenological function of the 're-solved' electron instead of parametrizing the equivalent bosons' and off-shell, 'resolved' bosons' spectra separately.

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