LAGRANGE TRIANGLE QUARK MODEL OF THE PROTON

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A model of three quarks at the corners of an Lagrange equilateral triangle is used to describe a nucleon. Dipole fits to the electromagnetic form factors are used to require an exponential radial dependence for the wave function. This guide from experiment leads to a linear confining potential using the Dirac equation to describe the dynamics of the model. This model can reproduce the proton magnetic moment, axial charge, Roper resonance energy, and the size of the dipole fits to the electromagnetic form factors, but not all simultaneously. The linear confining potential does not appear in a Schroedinger like second order differential equation involving only the large component of the Dirac equation for the proton ground state.

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1. Introduction

The dipole electromagnetic form factors of the proton have been taken as input in an inversion sequence [1, 2] from the form factors to the hypercentral wave function for three quarks to the quark quark potentials in a three body Schroedinger equation context with central two body potentials. The idea is to use the electric form factor, or dipole fits [3] to it and express the three body Schroedinger equation wave function in terms of an integral over the form factor. The wave function can then be differentiated and used to determine the potentials. In this three body case it is the hypercentral potential that is determined, expressed as a function of the hyperradius. The hypercentral potential is the angular averaged sum of the two body potentials of the three body system. The quark charges of 2/3, 2/3, and -1/3 are the only charges assumed present in the model. The hypercentral configuration assumed for the quark constituents is the $(1/2^+)^3$ positive parity configuration, coupled to the spin of the proton. The quark quark potentials obtained this way do not have confining characteristics at large separations. The purpose of this paper is to show that this may be a consequence from using the Schrodinger equation rather than the Dirac equation for the quark dynamics. The inversion sequence from form factors to hypercentral quark potentials via the three body Dirac equation and its solution is not followed here. The three body Dirac equation, even in hypercentral approximation [4], is not solved here. This involves an eight component radial function that is a solution of eight coupled radial first order differential equations. It is not an easy matter to reduce this to a non-relativistic equation involving only a single component of the composite three body wave function. Derivatives higher than second order appear. Rather an Lagrange equilateral triangle [5] model of three quarks on the corners of a triangle is used. For three identical quarks, the average location of the masses will form an equilateral triangle. Such a equilateral triangle model has been used by Krolikowski [6-8] in modeling three quarks in Dirac equation and Schroedinger equation systems. Barut [9] has used the Lagrange triangle model in a semi-classical approach in trying to understand the proton electron mass ratio. This Lagrange triangle model allows symmetric vibrations or motion of the three quarks that preserves the equilaterial triangle shape, but neglects asymmetric vibrations that would change the triangle shape.

Assuming the three quarks are at the corners of an equilaterial triangle in the overall center of mass frame, reduces the three quark problem to one of four coordinates. The usual three body problem requires six coordinates to locate all three masses in the center of mass frame. The equilateral triangle model constrains two of these coordinates. The four coordinates required to specify the location of all the masses are, θ, ϕ the spherical polar directions specifying the normal of the triangle, ω the angle of the triangle orientation about this normal, with respect to an arbitrary z direction, and r, the radial scale for the distance of a quark from the overall center of mass. This model is not a one body problem as 4 coordinates are needed to locate the system masses, rather than the three coordinates required for a one body problem. With the Lagange equilateral triangle model, the moment of inertia about the triangle normal is independent of the angle ω. The rotational angular momentum about such a symmetry axis must be even, such as zero, 2, *etc.* For any state of total angular momentum 1/2, such as the proton ground state, no rotational angular momentum of ω about the triangle normal is therefore allowed. For this case then the wave function is independent of the coordinate ω. Then a one body Dirac equation can be used to describe the remaining dynamics of the model, with central potentials to be determined. The proton ground state and Roper resonance in this model are taken as obeying such a one body Dirac equation.

Consistent with the dipole electromagnetic form factors, the ground state solution of this Dirac equation is required to be an exponential. The large component of the solution is required to have an exponential fall off with radius. The potential required for this radial dependence is then determined. The non-relativistic reduction that eliminates the small component in favor of a second order differential equation for the large component only is obtained. The confining potential that appeared in the Dirac equation is absent in the non-relativistic Schroedinger like equation. The potential found is the model analog of the three quark hypercentral potential.

After the angular integration, the composite particle radial wave function is:

$$\Psi = (1/r)\left\{ \begin{array}{c} F \\ G \end{array} \right\}. \tag{1}$$

This is a composite wave function of the system applying to the simultaneous motion of the three quarks constrained to the corners of an equilateral triangle. There is no center of mass motion problem for this model, as the center of mass of the triangle is fixed in the center of mass frame. The radial Dirac equation becomes:

$$\left\{ \begin{array}{ll} [M+S-E+V]F & +\ [(k/r)-d/dr]G = 0 \\ ((k/r)+d/dr)\,F & +\ [-M-S-E+V]G = 0 \end{array} \right\}. \tag{2}$$

M is three times the quark mass , and E is the rest energy of the proton. S and V are the scalar and fourth component of a vector potential to be determined here. Miller [10] has shown that a scalar, a fourth component of a vector, or a radial component of a tensor potential are all possible for a Dirac equation description of a state of good total angular momentum, and parity. Such a tensor term has been considered [11] but it is not necessary here.

Here F is the large component of the wave function, and k is plus or minus J+1/2, depending on the total angular momentum J and the parity. The wave function normalization is

$$1 = \int_0^\infty [F^2 + G^2]dr\,. \tag{3}$$

The charge and spin of the proton are modeled by assigning appropiate charges to the assumed three quark constituents, and assigning them to a $(1/2^+)^3$ configuration with a total angular momentum of one half. With F and G being the large and small radial components of the Dirac equation, the equilateral triangle model says k must be plus or minus 1, corrosponding to each quark in a $(1/2^+)$ or in a $(1/2^-)$ orbital. We take the k equals minus one, positive parity case. To reproduce the dipole fits to the electromagnetic form factors of the proton, we take the large component to be

$$F = \mathrm{A}r\,\mathrm{e}^{-Lr} \tag{4}$$

and the small component to be:

$$G = \mathrm{B}r^2\,\mathrm{e}^{-Lr}\ . \tag{5}$$

Thus the large component will be an exponential overall, while the small component is r times an exponential. A and B are constants to be determined by the Dirac equation and by the normalization. L is taken at first as a parameter of the model, which will be set to 0.4313 GeV to reproduce the experimental value deduced from the dipole fit [3] to the form factors. One finds that the component constants satisfy B/A= $-(E-M)/3$. With the above exponential forms substituted into the radial Dirac equation with $k=-1$, the potentials are found to be:

$$V + S = L(E-M)r/3 \tag{6}$$

and

$$V - S = E + M - 3L/(E-M)r\,. \tag{7}$$

The potentials are found to be a constant term, an attractive coulomb, and a confining linearly rising term. This is the model version of the hypercentral potential for the three quark system.

The energy is set to the proton rest energy. The model has two parameters, L, the size parameter, and M, taken as three times the quark mass. A charge form factor dominated by the large component of the dirac equation will reproduce the dipole fit to the form factor if L is set to 0.4313 GeV. With L set to this value, the model is said to be size constrained.

The magnetic moment [12, 13] is:

$$\mu = (4kE/(4k^2-1))\int_0^\infty [FrG]dr\,. \tag{8}$$

In figure 1 is shown the size constrained model magnetic moment for the proton versus the assumed quark mass. The maximum value of 2.52 is about 10 per cent less than the experimental value. One could ascribe an anomalous magnetic moment to the quarks of $(\alpha/2\pi)(e/m)$ as from QED, and reproduce the proton experimental magnetic moment with a small quark mass of about 4 MeV.

In figure 2 is shown the model magnetic moment versus L, assuming a quark mass of zero. The magnetic moment is larger for small L as the size of the system is large in this limit. The proton experimental value can be reproduced by the model Dirac magnetic moment with an L of about 0.35 GeV. The axial charge of the proton (1.26) can be reproduced if the large component contribution to the wave function is about 0.817. This

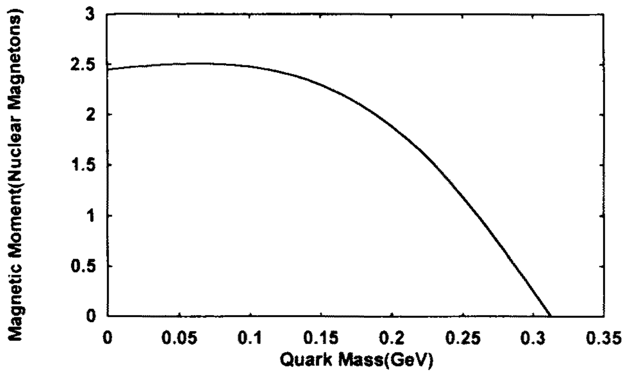

Fig. 1. Proton Magnetic Moment versus quark mass for the size constrained model.

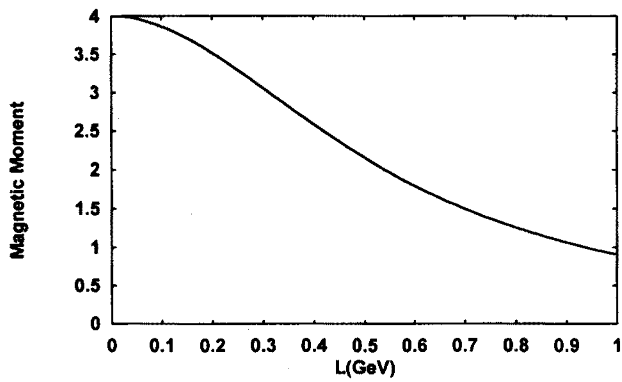

Fig. 2. Proton Magnetic Moment for zero mass quarks versus size parameter L. Each quark is assumed to be in a $(1/2^+)$ state.

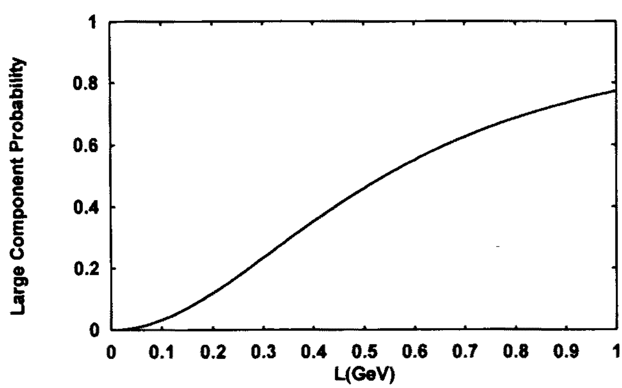

Fig. 3. Large component contribution to the Normalization versus the size parameter L. The quark mass is assumed to be zero, with an exponential radial dependence for the large component.

occurs for zero mass quarks when L is about 1.2 GeV. The large component contribution to the normalization can be seen in figure 3 for massless quarks. The experimental magnetic moment, axial charge, and size of charge and current distributions are thus not all simultaneously reproduced with this equilateral triangle quark model. Quark distributions in this model must be smaller than the charge distributions to reproduce the axial charge. This is consistent with the idea of charged pions on the proton periphery contributing to the charge and current distributions.

2. Non relativistic reduction

We consider the solution found for the Dirac equation with $k = -1$. We write it as:

$$\left\{ \begin{array}{lcl} [M - E + ar]F & - & [(1/r) + d/dr]G = 0 \\ ((-1/r) + d/dr)F & + & [-M - E + c - b/r]G = 0 \end{array} \right\} . \tag{9}$$

Here we have defined the potential constants as

$$a = (E - M)L/3, b = 3L/(E - M). c = E + M . \tag{10}$$

The second of the above equations is differentiated, and eliminate G' and G to obtain a second order equation involving the upper component only. It is

$$-F'' - F'/r + [1/r^2 + (E + M - c + b/r)(M - E + ar)]F = 0 . \tag{11}$$

Eliminating the potential constants in favor of L, one has for the ground state:

$$-F'' - F'/r + [1/r^2 + L^2 - 3L/r]F = 0 . \tag{12}$$

One can eliminate the first derivative term by the change of

$$F = \chi/r^{1/2} \tag{13}$$

to obtain

$$-\chi'' + [3/4r^2 - 3L/r + L^2]\chi = 0 . \tag{14}$$

In either of these equations for the ground state, the linear confining potential does not appear. This Schroedinger like reduction results in a combination of a coulombic attraction and a $1/r^2$ term, that with the appropiate coefficients will reproduce an exponential wave function suggested by the experiments. But there is no rising quark confining potential term in these

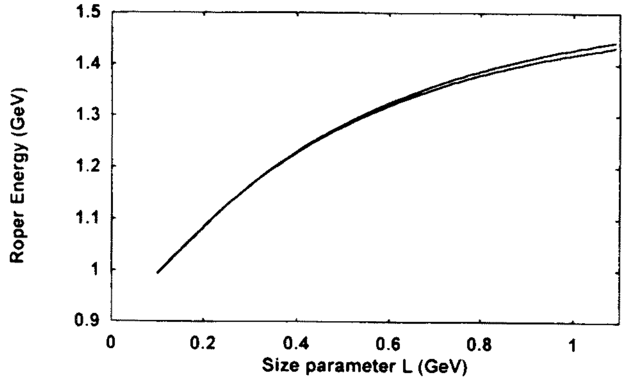

Fig. 4. Roper resonance energy versus the size parameter L, in Gev. The upper curve is for massless quarks. The lower curve is for quark masses of 10 MeV.

non-relativistic Schroedinger like equations for the large component of the composite wave function only. The interpertation by Ferraris *et al.* [1] that deconfinment due to a meson cloud at large separations may not be necessary. Confining potentials may well result from their inversion sequence when the Dirac equation is used rather than the Schroedinger equation.

With the potential constants a, b, and c chosen to reproduce the proton energy, equation (11) above can be solved for the first radially excited state [14–17]. This breathing mode state is identified as the Roper resonance energy. Dividing by L squared, this equation can be written in dimensionless form, using the dimensionless separation, $y = Lr$, as:

$$-\chi'' + [V]\chi = E_0\chi\,, \tag{15}$$

where

$$V = 3/4y^2 - 3(E-M)/y(E_p - M) + 1 + (E - E_p)(E_p - M)y/3L^2 \tag{16}$$

and

$$E_0 = (E-M)(E-E_p)/L^2\,. \tag{17}$$

The energy of this state is shown in figure 4 versus the size parameter L. The Roper resonance energy can be reproduced in this Lagrange equilateral triangle model for size parameters, L, of about 1 GeV. The use of the one body Dirac equation to describe the Lagrange equilateral triangle model for three quarks allows the Roper resonance excited state energy to be easily obtained. Whereas the ground state wave function has an exponential dependence, the excited state has an asymptotic Airy function dependence. For the excited states, the coefficient of the linear term of the potential in equation (16) is positive. This potential is energy dependent. It is displayed

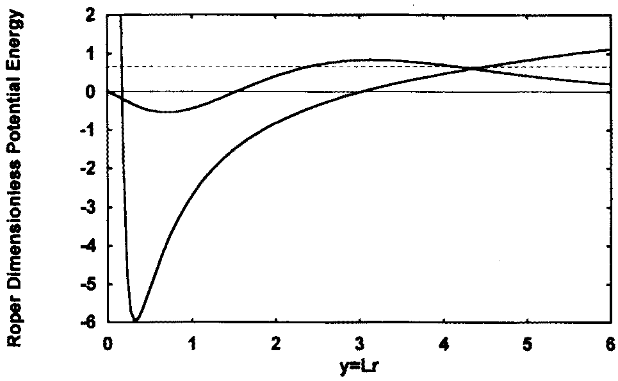

Fig. 5. Dimensionless Roper potential energy versus the scaled radial distance, Lr. The large component of the wave function has a maximum at about $y = 3$, and then asymptotically goes to zero monotonically. The dashed line is the value of the dimensionless quanty E_o.

in figure 5 for the Roper resonance energy versus a dimensionless radius, $y = Lr$. The potential has a short ranged angular momentum barrier, an attractive coulomb term, and a linear confining term asymptotically for the excited state. Also shown in figure 5 is the excited state solution corresponding to the Roper resonance energy, assuming massless quarks. The ground state potential has no linear confining potential term in the reduced one component Schroedinger like equation.

3. Conclusions

It is shown that an equilateral triangle model of three quarks in a nucleon provides a partial description of the proton charge and current distributions. When the Dirac equation is solved requiring an exponential radial dependence for the large component, a confining linearly rising potential is required. The magnetic moment of the size constrained model is about 2.52 nuclear magnetons for quark masses of less than 0.15 GeV. One could ascribe an anomalous magnetic moment to the quarks of $(\alpha/2\pi)(e/m)$ as from QED, and reproduce the proton experimental magnetic moment with a small quark mass. The confining potential does not appear in the reduction of the Dirac equation to a second order differential equation involving only the large component of the composite wave function for the proton. This Schroedinger like reduction results in a combination of a coulombic attraction and a $1/r^2$ term but no confining potential term. With the appropiate coefficients, this potential will reproduce an exponential wave function as suggested by experiment. For the Roper resonance excited state, the Dirac equation of the model is the same as for the proton state, only the energy is increased. The reduction to a single second order differential equation involving only the large component has an energy dependent linear confining term for the excited state. The wave function consequently is not exponential, but asymptotically of the Airy type for the excited state. Confining potentials and exponential or other bound wave functions can result when the Dirac equation is used rather than the Schroedinger equation to describe the dynamics.

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