NONLEPTONIC CHARMED MESON DECAYS: QUARK DIAGRAMS AND FINAL-STATE INTERACTIONS*

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Effects of final-state interactions in nonleptonic decays of charmed mesons are studied in the framework of quark-diagram approach. For the case of u-d-s flavour symmetry we discuss how the inelastic coupled-channel rescattering effects (and, in particular, resonance formation in the final state) modify the input quark-diagram weak amplitudes. It is shown that such inelastic effects lead to the appearance of nonzero relative phases between various quark diagrams, thus invalidating some of the conclusions drawn in the past within the diagrammatical approach. Applicability of quark-diagram approach to the case of SU(3) symmetry-breaking in Cabibbo once-suppressed D^0 decays is also studied in some detail.

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1. Introduction

Various theoretical models of nonleptonic decays of charmed mesons have been developed over the years. The most general and complete one is the diagrammatical approach of Chau and Cheng [1–3]. The factorization method [4] is just a special case of this approach. Another subset of diagrammatical approach is singled out by large-N_c arguments [5]. The basic problem with the diagrammatical approach is the way in which final-state interactions (FSI) and SU(3) breaking are treated. The importance of FSI has been stressed by Lipkin [6], Sorensen [7], Kamal and Cooper [8], Donoghue [9], Chau [10], Chau and Cheng [11], Hinchliffe and Kaeding [12], and others.

Complete descriptions of nonleptonic decays *must* take into account final-state interactions. Since full dynamical calculations of these effects are not

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possible at present, a meaningful comparison of theoretical models with experiment requires at least a phenomenological estimate of FSI. Such an approximate estimate may be obtained using *e.g.* unitarity constraints [13]. Alternatively, one may consider approaches based on approximate flavour-symmetry groups. Their predictions include automatically *all* effects of those FSI which are invariant under these symmetries [6]. The diagrammatical methods of Chau and Cheng provide an approach complementary to that based on flavour-symmetry group [6, 12]. It has been argued [9] that "it is folly to proceed with the quark-diagram approach without considering these (*i.e.* rescattering) effects" since "rescattering can mix up the classification of diagrams". Although the latter statement is obviously true, one should realize that the quark-diagram approach — being complementary to that based on flavour symmetry groups — "deals with effective quark diagrams with all FSI included" [2].

The aim of this paper is:

1. to examine in some detail in what way the introduction of flavour-symmetric FSI in the form of coupled-channel rescattering effects renormalizes the input quark-diagram weak amplitudes (with particular emphasis on resonance formation in the final state) and to compare the results thus obtained with the treatment of FSI adopted in the diagrammatical approach so far (Section 3), and
2. to discuss the applicability of quark-diagram approach to the description of SU(3)-symmetry breaking D^0 decays into $\pi\pi$ and $K\overline{K}$, (Section 4)).

2. General

In this paper we will consider how weak-decay quark-diagram amplitudes are changed when final-state rescattering effects are added. The general approach to the treatment of final-state interactions is reviewed in Refs. [7, 14]. Consider a weak decay of charmed meson D into n two-body channels. Let w be an n-dimensional vector formed by the relevant decay amplitudes in the absence of FSI. We describe strong interactions in the final state by an nxn matrix S_0:

$$S_0 = 1 + 2i\rho^{1/2}A\rho^{1/2}\,, \tag{1}$$

where the diagonal matrix ρ contains the relativistic phase-space factor for the relevant two-body channels

$$\rho_j = \frac{2k_j^{c.m.}}{\sqrt{s}}\Theta(s - s_j^{th})\,, \tag{2}$$

$k_j^{c.m.}$ = centre-of-mass momentum for channel $j(j$=1,2,...$n)$, s_j^{th} = threshold value in channel j. Final-state interactions may include quark-exchange

[9], resonance formation [4], elastic scattering *etc.* Of these it is resonance formation that is in general expected to affect naive approaches most significantly [6–9]. For our purposes we will use the K-matrix parametrization of the strong-interaction matrix A:

$$A = (1 - KM)^{-1}K\,, \tag{3}$$

with $M = M(s)$ being the diagonal matrix given by the kinematic Chew-Mandelstam functions $M_j(s)$ for various two-body channels, and satisfying $M_j(s + i\epsilon) - M_j(s - i\epsilon) = 2i\rho_j(s)$. The K-matrix has no singularities other than isolated poles corresponding to $q\overline{q}$ bound states. The geometric series of Eq. (3) describes then the coupling of such resonances to meson-meson continuum states (see *e.g.* Ref. [15]). Let us denote by W the n-dimensional vector describing weak decay amplitudes corrected for final-state interactions. Vector W must be related to the original input weak decay amplitude vector w by the matrix equation [7, 14]

$$W = Dw\,. \tag{4}$$

Equation (4) admixes into a given decay amplitude contributions from *all* coupled channels.

As discussed in Refs. [7, 14] there are various ambiguities in the determination of D. With our representation of strong S matrix in terms of the K-matrix one has

$$D = (1 - KM)^{-1}\,. \tag{5}$$

Relationship given in Eq. (5) corresponds to the expectation that at high energy the outgoing mesons behave like free particles ($D \to 1$). Some important features of Eqs. (4), (5) are seen already in the one-dimensional case. Let $n = 1$. The Chew-Mandelstam function M has in general nonzero real and imaginary parts. The real part leads to resonance mass shifts induced by coupling to meson-meson continuum. If it may be neglected (compare Refs. [7, 16] or absorbed into the definition of K-matrix (see Ref. [15]) we have $M = i\rho$ and we obtain

$$S = 1 + 2i\frac{\hat{K}}{1 - i\hat{K}} = \exp(2i\delta)\,, \tag{6}$$

with

$$\hat{K} = \rho K = \tan\delta\,. \tag{7}$$

Then Eqs. (4)–(6) yield

$$W = \frac{1}{1 - i\tan\delta}w = \cos(\delta)\exp(i\delta)w \tag{8}$$

in agreement with the general requirements of Watson's theorem [17]. Vanishing of the FSI-corrected amplitude (8) at the position of the resonance ($\delta = 90^o$) should be understood as an effect related to the neglect of a "direct" coupling between the decaying state and the final-state resonance. This is a characteristic (though idealized) feature of "indirect" couplings to FSI resonances [7, 16]. In the case when the real part of the Chew-Mandelstam function is not neglected, Eqs. (5), (8) lead to suppression (instead of vanishing) of the FSI-modified decay amplitude at pole position. Such suppressions would dissappear if "direct" couplings through the weak Hamiltonian were allowed between the D-meson and the resonances (compare the discussion in Refs. [7, 16]). In the factorization approach such "direct" couplings to final-state resonances do vanish. For example, in the case of Cabibbo-allowed D^0 decays the W-exchange amplitudes are argued to be negligible. In the more general quark diagram scheme of Refs. [1–3] there may be some nonvanishing "direct" coupling to a resonance. For the coupled-channel case Eqs. (4), (5) satisfy the requirements of the generalized (many-channel) Watson's theorem [14]. They describe "indirect" final-state interactions proceeding through a formation of a resonance coupled to many meson-meson channels. As we shall see in this paper explicitly, such inelastic rescattering effects have not been included in the diagrammatical approach of Refs. [1–3] so far. For reasons related to the introduction of FSI the diagrammatical approach has been criticised by Donoghue [9], and, very recently, by Hinchcliffe and Kaeding [12]. The question of how the inelastic effects change the overall picture of quark-line approach was first examined in Ref. [9]. Here we will show how the problem gets simplified conceptually, and — in the most important case of final-state $q\overline{q}$ resonance contribution — also computationally, through diagonalization of the K (S) matrix. Diagonalization will be discussed on the example of Cabibbo-allowed parity-conserving decays of D^0 (and D_s^+) into the PV (pseudoscalar meson + vector meson) final states. Results of a similar treatment of the Cabibbo-allowed parity-violating decays D^0, $D_s^+ \to PP$ will also be presented. Finally, we will consider the interesting case of Cabibbo-suppressed parity-violating decays of D^0 into the $\pi\pi$ and $K\overline{K}$ channels where SU(3) breaking must play a significant role. In all of our examples we will neglect complications due to possible presence of coupled channels other than those listed above.

3. Cabibbo-allowed decays of D^0 and D_s^+ into PV and PP final states

3.1. Parity conserving PV decays

3.1.1. $D^0 \to PV$ The parity-conserving part of weak interactions induces D^0 decays into eight possible final p-wave PV channels: $K^-\rho^+$, $\overline{K}^0\rho^0$, $\overline{K}^0\omega$, $\eta_s\overline{K}^{*0}$ and $K^{*-}\pi^+$, $\overline{K}^{*0}\pi^0$, $\overline{K}^{*0}\eta_{ns}$, $\phi\overline{K}^0$.

In the first four decay channels (below called $\underline{P}V$) the strange quark from the weak decay of the charmed quark ends up in the pseudoscalar meson P, while in the latter four decay channels (called $\underline{V}P$) this strange quark ends up in the vector meson. The decays proceed through diagrams (a), (b), (c) from Fig. 1.

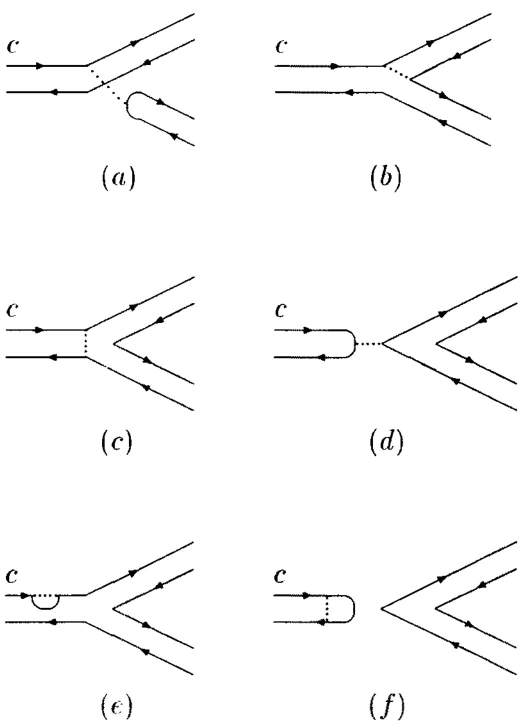

Fig. 1. Quark-line diagrams for weak meson decays: (a), (b) — factorization; (c) — W-exchange; (d) — annihilation; (e) — "horizontal penguin"; (f) — "vertical penguin".

Let us first consider the case when there are no final-state strong interactions ($S = 1$). By a, b, c (a', b', c') we denote reduced matrix elements corresponding to diagrams (a), (b), (c), for $\underline{P}V$ ($\underline{V}P$) channels respectively. (In factorization approach the contribution from diagram (c) is assumed negligible.) Without FSI all these matrix elements are real. Evaluation of contributions from the diagrams of Fig. 1 yields the following weak decay amplitudes

$$\begin{aligned}
\langle(\overline{K}\rho)_{3/2}|w|D^0\rangle &= -\frac{1}{\sqrt{3}}(a+b)\,, \\
\langle(\overline{K}\rho)_{1/2}|w|D^0\rangle &= \frac{1}{\sqrt{6}}(b-2a-3c)\,, \\
\langle\overline{K}^0\omega|w|D^0\rangle &= -\frac{1}{\sqrt{2}}(b+c)\,, \\
\langle\eta_s\overline{K}^{*0}|w|D^0\rangle &= -c\,,
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
\langle(\overline{K}^*\pi)_{3/2}|w|D^0\rangle &= -\frac{1}{\sqrt{3}}(a'+b')\,, \\
\langle(\overline{K}^*\pi)_{1/2}|w|D^0\rangle &= \frac{1}{\sqrt{6}}(b'-2a'-3c')\,, \\
\langle\overline{K}^{*0}\eta_{ns}|w|D^0\rangle &= -\frac{1}{\sqrt{2}}(b'+c')\,, \\
\langle\phi\overline{K}^0|w|D^0\rangle &= -c'\,.
\end{aligned} \tag{10}$$

Subscripts 1/2 and 3/2 denote total isospin of $\overline{K}\rho$ and $\overline{K}^*\pi$ states. Dependence on Cabibbo factors is suppressed both in Eqs. (9), (10) and elsewhere in this paper. That is, all expressions on the right-hand side of Eqs. (9), (10) are to be multiplied by the Cabibbo factor of $\cos^2\Theta_C$: $-(a+b)/\sqrt{3} \to -\cos^2\Theta_C \cdot (a+b)/\sqrt{3}$ *etc.* The normalisation is such that matrix elements a, b, c *etc.* are identical with those used in [1–3].

As already mentioned in the previous Section we are interested in those FSI in which there is an "indirect" $q\bar{q}$ resonance formation, *i.e.* resonances are accessed through an intermediate meson-meson state only. We do not consider a "direct" coupling between the decaying state and the $q\bar{q}$ resonance (such a coupling is zero in the factorization approach). If there is such a coupling it should be included already in Eqs. (9), (10). Later, it should become clear that our *general* conclusions remain valid also when other indirect FSI as well as "direct" coupling to resonances are taken into account. The final-state coupled-channel processes to be considered are visualised in Fig.2.

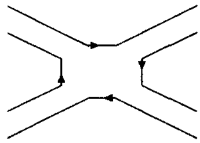

Fig. 2. Resonance contribution to final-state interactions.

Since the intermediate state is a pseudoscalar state (with kaon flavour quantum numbers), the flavour structure of the K matrix may be easily calculated from the product of two F-type ($VP \to P'$) couplings. (In general, one chooses symmetric(antisymmetric) D(F) coupling $\mathrm{Tr}(M_1\{M_2, M_3\})$ ($\mathrm{Tr}(M_1[M_2, M_3])$) when the product of charge conjugation parities of the three mesons is positive (negative).)

This gives the following SU(3)-symmetric strong K matrix for the $\underline{P}V$ subsector:

$$K = \begin{bmatrix} \frac{3}{2} & \frac{\sqrt{3}}{2} & \sqrt{\frac{3}{2}} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} & 1 \end{bmatrix} \cdot \kappa \,, \tag{11}$$

where the channels are ordered $(\overline{K}\rho)_{1/2}$, $\overline{K}^0\omega$, $\eta_s\overline{K}^{*0}$ and "κ" is a function of s, which in the simplest case of a single resonance is of the form $\kappa = g^2/(m^2 - s)$. In the $(\overline{K}\rho)_{3/2}$ sector there are no $q\bar{q}$ poles and the K matrix vanishes. Fixing our attention on this simple case is sufficient for our purposes.

The eigenvalues and eigenvectors of matrix (11) are:

$$\begin{aligned} \lambda_1 &= 3\kappa & |1\rangle &= \frac{1}{\sqrt{6}}(\sqrt{3}(\overline{K}\rho)_{1/2} + (\overline{K}^0\omega) + \sqrt{2}(\eta_s\overline{K}^{*0}))\,, \\ \lambda_2 &= \;\;0 & |2\rangle &= \frac{1}{\sqrt{6}}(-\sqrt{3}(\overline{K}\rho)_{1/2} + (\overline{K}^0\omega) + \sqrt{2}(\eta_s\overline{K}^{*0}))\,, \\ \lambda_3 &= \;\;0 & |3\rangle &= \frac{1}{\sqrt{3}}(\sqrt{2}(\overline{K}^0\omega) - (\eta_s\overline{K}^{*0}))\,. \end{aligned} \tag{12}$$

Equations (9) may be easily rewritten in the new basis (for the $I = 1/2$ sector):

$$\begin{aligned} \langle 1|w|D^0\rangle &= -\frac{1}{\sqrt{3}}(a + 3c)\,, \\ \langle 2|w|D^0\rangle &= \frac{1}{\sqrt{3}}(a - b)\,, \\ \langle 3|w|D^0\rangle &= -\frac{1}{\sqrt{3}}b\,. \end{aligned} \tag{13}$$

Let all the SU(3)-related intermediate meson-meson states are degenerate. Then, we have

$$M_{K\rho} = M_{K\omega} = M_{\eta_s \bar{K}^*} \equiv M \,. \tag{14}$$

In the new basis Eq. (4) reads

$$\langle j|W|D^0\rangle = \frac{1}{1-\lambda_j M}\langle j|w|D^0\rangle \,, \tag{15}$$

with $j = 1, 2, 3$. Eq. (15) is as simple as Eq. (8) of the standard one-channel case. Going back from Eq. (15) to the old basis (and adding the expression for the $I = 3/2$ decay amplitude) one obtains

$$\begin{aligned}
\langle (\overline{K}\rho)_{3/2}|W|D^0\rangle &= -\frac{1}{\sqrt{3}}(A+B)\,, \\
\langle (\overline{K}\rho)_{1/2}|W|D^0\rangle &= \frac{1}{\sqrt{6}}(B-2A-3C)\,, \\
\langle \overline{K}\omega|W|D^0\rangle &= -\frac{1}{\sqrt{2}}(B+C) \\
\langle \eta_s \overline{K}^{*0}|W|D^0\rangle &= -C\,,
\end{aligned} \tag{16}$$

with

$$\begin{aligned}
A &= a\,, \\
B &= b\,, \\
C &= c + \left(c+\frac{a}{3}\right)\left(\frac{1}{1-\lambda_1 M} - 1\right),
\end{aligned} \tag{17}$$

where vanishing of λ_2 and λ_3 has been taken into account.

From Eqs. (16), (17) we see that the case with resonance-induced coupled-channel effects differs from the no-FSI case by a change in the size and phase of the C parameter *only*:

$$C = c \rightarrow C = c + \left(c+\frac{a}{3}\right)\frac{\lambda_1 M}{1-\lambda_1 M}\,. \tag{18}$$

In the case when $M = i\rho$ the complex factor $1/(1-\lambda_1 M)$ in the second term is of the form $\cos(\delta)\exp(i\delta)$ and at the resonance position ($\delta = 90^o$) one obtains a net total contribution to C equal to $-a/3$. While the "indirect" coupling to a resonance affects the second term in Eq. (18), the "direct" coupling contributes to the first term.

In conclusion, after including resonance-induced coupled-channel effects, the reduced matrix elements corresponding to diagrams (a) and (b) remain unchanged and real, while the matrix element of diagram (c) acquires new size and *nonvanishing* (relative to (a) and (b)) *phase*. The coupled-channel effects may generate a sizable nonvanishing effective (c)-type diagram even if the original (c)-type amplitude was negligible as assumed in many papers [4, 7, 9]. This is shown in diagrammatic terms in Fig. 3. Eq. (18) is a mathematical representation of how FSI-modified quark-diagram amplitudes are obtained. When rescattering reactions proceed by quark exchange, the corresponding K matrix has three nonzero eigenvalues. The counterparts of Eqs. (17) are then less transparent and more complicated leading in particular to the appearance of Zweig-rule violating "hairpin" diagrams [18]. In this paper we will not consider these quark-exchange contributions in detail.

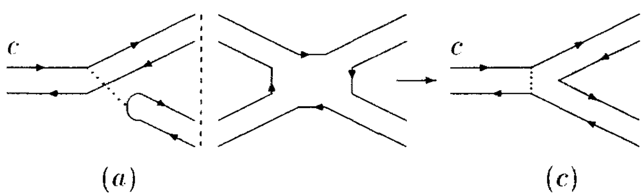

Fig. 3. Generation of (c)-type (exchange) amplitude from (a)-type (factorization) diagram through (indirect) resonance formation

The procedure applied above to the $\underline{P}V$ sector may be repeated in the $\underline{V}P$ sector. In this sector the K matrix has the form given in Eq. (11) (with decay channels ordered $(\overline{K}^*\pi)_{1/2}$, $\overline{K}^{*0}\eta_{ns}$, $\phi\overline{K}^0$) and the same is true for the off-diagonal ($\underline{P}V$ – $\underline{V}P$) part of the total K matrix. Introducing three eigenvectors $|j'\rangle$ ($j' = 1, 2, 3$) of the $\underline{V}P$ sector (given by formulae (12) with $(\overline{K}\rho)_{1/2}$, $\overline{K}^0\omega$, $\eta_s\overline{K}^{*0}$ replaced by $(\overline{K}^*\pi)_{1/2}$, $\overline{K}^{*0}\eta_{ns}$, $\phi\overline{K}^0$ respectively one obtains the total K matrix connecting all eight PV channels:

$$K = \begin{pmatrix} 3 & 3 & 0 & \dots \\ 3 & 3 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ . & . & . & \dots \\ . & . & . & \dots \end{pmatrix} \cdot \kappa \,, \tag{19}$$

with rows and columns ordered $|1\rangle, |1'\rangle, |2\rangle, |2'\rangle$ ... . Diagonalizing the total K matrix and repeating the procedure outlined earlier one obtains — in addition to Eqs. (16) — the following expressions for the FSI-modified $\underline{V}P$ decays:

$$\langle(\overline{K}^*\pi)_{3/2}|W|D^0\rangle = -\frac{1}{\sqrt{3}}(A'+B'),$$
$$\langle(\overline{K}^*\pi)_{1/2}|W|D^0\rangle = \frac{1}{\sqrt{6}}(B'-2A'-3C'),$$
$$\langle\overline{K}^{*0}\eta_{ns}|W|D^0\rangle = -\frac{1}{\sqrt{2}}(B'+C'),$$
$$\langle\phi\overline{K}^0|W|D^0\rangle = -C'. \qquad (20)$$

The reduced matrix elements of Eqs. (16), (20) are now given by

$$A = a,$$
$$A' = a',$$
$$B = b,$$
$$B' = b',$$
$$C - C' = c - c',$$
$$C + C' = c + c' + (\cos\delta\exp i\delta - 1)\,(c + c' + \frac{a+a'}{3}), \qquad (21)$$

where $\tan\delta = 6\kappa\rho$ and we have already assumed $M = i\rho$. Since this simplifying assumption does not affect our argument (which rests solely on *complexity* of $1/(1-\lambda_j M)$ factors) we will assume $M = i\rho$ in the remaining part of this paper. In conclusion, including resonance-induced coupled-channel effects in the SU(3)-symmetric case results in a change of size and phase of matrix elements of diagrams (c) *only*. Since in the diagrammatical approach the size of (c)-type amplitudes was treated as a free parameter [1–3] anyway, the only observable effect of FSI is the appearance of (in general different) nonzero phases of C and C'.

In Refs. [1–3] an attempt to include the effects of FSI has been made. To permit easy comparison with expressions derived above we rewrite below a few formulae from Table I of Ref. [3] (SU(3)-symmetry case):

$$\langle(\overline{K}\rho)_{3/2}|W|D^0\rangle = \frac{1}{\sqrt{3}}(\mathcal{A}+\mathcal{B})\exp i\delta^{K\rho}_{3/2},$$
$$\langle(\overline{K}\rho)_{1/2}|W|D^0\rangle = \frac{1}{\sqrt{6}}(2\mathcal{A}-\mathcal{B}+3\mathcal{C})\exp i\delta^{K\rho}_{1/2},$$
$$\langle(\overline{K}^*\pi)_{3/2}|W|D^0\rangle = \frac{1}{\sqrt{3}}(\mathcal{A}'+\mathcal{B}')\exp i\delta^{K^*\pi}_{3/2},$$
$$\langle(\overline{K}^*\pi)_{1/2}|W|D^0\rangle = \frac{1}{\sqrt{6}}(2\mathcal{A}'-\mathcal{B}'+3\mathcal{C}')\exp i\delta^{K^*\pi}_{1/2},$$

$$\begin{aligned} \langle \phi \overline{K}^0 | W | D^0 \rangle &= \mathcal{C}' \exp i\delta^{\phi K}\,, \\ \langle \overline{K}^{*0} \eta_{ns} | W | D^0 \rangle &= (\text{mixture})\,. \end{aligned} \tag{22}$$

In papers [1–3] amplitudes $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, *etc.* (Eq. (22)) were real while the phase factors were allowed both real and imaginary parts in the hope of taking into account all inelasticities and phase shifts possible. After comparing Eq. (22) with Eqs. (16), (20), (21) we see that resonance contribution does *not* follow the ansatz of [2, 3]: In Eq. (22) the *relative* phases of contributions from diagrams (a),(b),(c) are *zero*, while explicit consideration of resonance-induced inelastic coupled-channel effects leaves relative phases of A, B, A', B' zero, but adds an important *nonzero* phase to C and C'. Thus, analysis of Ref. [2, 3] does *not* take into account effects due to the possible formation of resonances in the final state *even in the case of SU(3)-symmetry* (though elastic scattering is taken care of). (See also the paper of Hinchcliffe and Kaeding [12] for a general comment on the inclusion of FSI in the diagrammatic approach.)

3.1.2. $D_s^+ \to PV$. In this case one obtains the following FSI-modified expressions:

$$\begin{aligned} \langle (\rho\pi)_2 | W | D_s^+ \rangle &= 0\,, \\ \langle (\rho\pi)_1 | W | D_s^+ \rangle &= D - D'\,, \\ \langle \omega\pi^+ | W | D_s^+ \rangle &= -\frac{1}{\sqrt{2}}(D + D')\,, \\ \langle \phi\pi^+ | W | D_s^+ \rangle &= -A'\,, \\ \langle \rho^+\eta_{ns} | W | D_s^+ \rangle &= -\frac{1}{\sqrt{2}}(D + D') \\ \langle \rho^+\eta_s | W | D_s^+ \rangle &= -A\,, \\ \langle K^{*}\overline{K}^0 | W | D_s^+ \rangle &= -B - D'\,, \\ \langle \overline{K}^{*0} K^+ | W | D_s^+ \rangle &= -B' - D\,, \end{aligned} \tag{23}$$

with

$$\begin{aligned} A &= a\,, \\ A' &= a'\,, \\ B &= b\,, \\ B' &= b'\,, \\ D + D' &= d + d'\,, \\ D - D' &= d - d' + (\cos\delta \exp i\delta - 1)(d - d' - \frac{1}{3}(b - b'))\,, \end{aligned} \tag{24}$$

with appropriate δ. From Eqs. (23), (24) we see that, as before, matrix elements corresponding to diagrams (a), (b) are not affected by FSI, while for (d)-type diagrams it is only the difference $D-D'$ that gets modified. Even if one starts with $d-d'=0$ (expected on the basis of flavour $u \leftrightarrow \bar{d}$ symmetry, see Fig. 1), the coupled channel effects generate a complex, nonvanishing, effective $D-D'$ proportional to $b-b'$.

3.2. Parity-violating PP decays

In the parity-violating s-wave decays $D^0, D_s^+ \to PP$ the final-state interactions are most probably dominated by formation of scalar resonances [19]. We accept here that in the $S=-1$ sector under discussion scalar resonances are susceptible to a simple $q\bar{q}$ description (see [15]) so that the treatment of the previous section may be applied. Proceeding as before one can then derive:

$$\begin{aligned}
\langle(\overline{K}\pi)_{3/2}|W|D^0\rangle &= -\frac{1}{\sqrt{3}}(A+B)\,, \\
\langle(\overline{K}\pi)_{1/2}|W|D^0\rangle &= \frac{1}{\sqrt{6}}(B-2A-3C)\,, \\
\langle\overline{K}^0\eta_{ns}|W|D^0\rangle &= -\frac{1}{\sqrt{2}}(B+C)\,, \\
\langle\eta_s\overline{K}^0|W|D^0\rangle &= -C\,,
\end{aligned} \tag{25}$$

with $A=a$, $B=b$ and $C=c+(c+a/3)(\cos\delta\exp i\delta-1)$ and appropriate δ . As noted in Ref. [7] one may justify the neglect of "direct" coupling to scalar resonances also by the fact that the overlap of P-wave $q\bar{q}$ resonant state with pointlike W-exchange is expected to be small. Even if the input quark-model (c)-type amplitude is negligible [7], the effective W-exchange amplitude may be significant.

Similarly, for D_s^+ decays we get

$$\begin{aligned}
\langle\pi^+\pi^0|W|D_s^+\rangle &= 0\,, \\
\langle\pi^+\eta_{ns}|W|D_s^+\rangle &= -\sqrt{2}D\,, \\
\langle\pi^+\eta_s|W|D_s^+\rangle &= -A\,, \\
\langle K^+\overline{K}^0|W|D_s^+\rangle &= -(B+D)\,,
\end{aligned} \tag{26}$$

with $A=a$, $B=b$ and $D=d+(\cos\delta\exp\delta-1)(d+b/3)$ and appropriate δ.

Examples of Cabibbo-allowed D^0 and D_s^+ decays studied in this and previous subsections exhibit the way in which quark-line-diagram approach is affected when (indirect) $q\bar{q}$ resonance formation in the final state is taken into account. In phenomenological approaches in which sizes of matrix elements constitute free parameters, the only observable effect of such contribution from resonances in the final state is the appearance of *non-zero phases of the* (*c*) *and* (*d*)-type amplitudes. (Clearly, if "direct" coupling to resonances is allowed the (c), (d) amplitudes are complex already at the input level). Thus, the quark-diagram version of the general statement that "predictions based on flavour-symmetry groups automatically include *all* effects of those FSI which are invariant under these symmetries" [6] is that the amplitudes corresponding to individual diagrams should be allowed *independent nonzero phases*. Depending on what types of FSI are considered one can then have various conditions imposed upon these phases.

4. Cabibbo-suppressed D^0 decays and SU(3) symmetry breaking

In this section we will analyze in some detail the application of the quark-diagram approach to the case of Cabibbo-once-forbidden D^0 decays (where SU(3) symmetry breaking must play an important role - as evidenced by the $\Gamma(D^0 \to K^+K^-)/\ \Gamma(D^0 \to \pi^+\pi^-)$ ratio). In these decays there are 9 possible s-wave PP final states: $\pi^+\pi^-$, $\pi^0\pi^0$, K^-K^+, $K^0\overline{K}^0$, $\eta_8\pi^0$, $\eta_1\pi^0$, $\eta_8\eta_8$, $\eta_8\eta_1$, $\eta_1\eta_1$. Each of these can be reached from the initial D^0 state by appropriate linear combination of six reduced matrix elements corresponding to diagrams (a)-(f) of Fig. 1. Omitting the Cabibbo factor of $\sin\Theta_C\cos\Theta_C$ (see comment after Eqs. (9), (10)) we obtain the following expressions for FSI-uncorrected weak decays of D^0:

$$
\begin{aligned}
\langle\pi^+\pi^-|w|D^0\rangle &= -(a+c-e-2f)\,,\\
\langle\pi^0\pi^0|w|D^0\rangle &= -\frac{1}{\sqrt{2}}(b-c+e+2f)\,,\\
\langle K^-K^+|w|D^0\rangle &= -(\tilde{a}+\tilde{c}+e+2f)\,,\\
\langle K^0\overline{K}^0|w|D^0\rangle &= (-c+\tilde{c}+2f)\,,\\
\langle\pi^0\eta_8|w|D^0\rangle &= \frac{1}{\sqrt{3}}(\tilde{b}-c-e)\,,\\
\langle\pi^0\eta_1|w|D^0\rangle &= -\frac{1}{\sqrt{6}}(\tilde{b}+2c+2e)\,,\\
\langle\eta_8\eta_8|w|D^0\rangle &= \frac{1}{\sqrt{2}}(b-c-\frac{1}{3}e-2f)+\frac{2}{3\sqrt{2}}(2(c-\tilde{c})+(\tilde{b}-b))\,,\\
\langle\eta_8\eta_1|w|D^0\rangle &= \frac{1}{\sqrt{2}}(b+2c-\frac{2}{3}e)+\frac{1}{3\sqrt{2}}(-4(c-\tilde{c})+(\tilde{b}-b))\,,\\
\langle\eta_1\eta_1|w|D^0\rangle &= -\sqrt{2}(\frac{1}{3}e+f)+\frac{\sqrt{2}}{3}(c-\tilde{c}+b-\tilde{b})\,.
\end{aligned}
\tag{27}
$$

In Eq. (27) matrix elements with (without) a tilde correspond to a strange (nonstrange) pair emitted in the original weak interaction process.

In standard approaches the (FSI-unmodified) contribution from diagrams (e) and (f) is negligible [11] (it vanishes in the SU(3) limit). Accordingly, we will neglect (e), (f)-type amplitudes in FSI-uncorrected D^0 decays. In the SU(3) limit we also have $\tilde{a} = a$, $\tilde{b} = b$, $\tilde{c} = c$. Our aim is to study if and how final-state interactions reintroduce (e)- and (f)- type amplitudes and lift equalities $\tilde{a} = a$, $\tilde{b} = b$, $\tilde{c} = c$. (As discussed in Section 3.2 amplitudes c and $\tilde{c}$ are expected to be small).

We will consider the contribution of PP coupled-channel effects only. The two-meson s-wave PP state may interact strongly through formation of neutral scalar resonances S. Assuming these belong to a $q\bar{q}$ nonet (see discussion later on and Ref. [15]) we consider three resonances with flavour quantum numbers of π^0, η_8, and η_1. Their couplings to the PP-state are of D-type.

The K-matrix splits block-diagonally into three submatrices in the isospin $I = 2, 1, 0$ sectors respectively:

1. sector $I = 2$: , $K(I=2) = 0$
   for the $(\pi\pi)_{I=2}$ state only.
2. sector $I = 1$

$$K(I=1) = 2\begin{bmatrix} 1 & \sqrt{\frac{2}{3}} & \frac{2}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} & \frac{2}{3} & \frac{2\sqrt{2}}{3} \\ \frac{2}{\sqrt{3}} & \frac{2\sqrt{2}}{3} & \frac{4}{3} \end{bmatrix} \cdot \kappa_S^{I=1} \tag{28}$$

with rows (columns) corresponding to states $(K\overline{K})_{I=1}$, $\pi^0\eta_8$, $\pi^0\eta_1$. The eigenvalues and their corresponding eigenvectors are:

$$\begin{aligned}
\lambda_1^{I=1} &= 6\kappa_S^{I=1} \\
&|1^{I=1}\rangle = \frac{1}{3}(\sqrt{3}(K\overline{K})_{I=1} + \sqrt{2}(\pi^0\eta_8) + 2(\pi^0\eta_1)) \\
\lambda_2^{I=1} &= 0 \\
&|2^{I=1}\rangle = \frac{1}{3}(\sqrt{6}(K\overline{K})_{I=1} - (\pi^0\eta_8) - \sqrt{2}(\pi^0\eta_1)) \\
\lambda_3^{I=1} &= 0 \\
&|3^{I=1}\rangle = -\frac{1}{\sqrt{3}}(\sqrt{2}(\pi^0\eta_8) - (\pi^0\eta_1)) \,.
\end{aligned} \tag{29}$$

Vanishing of $\lambda_2^{I=1}$ results from the assumed nonet symmetry of couplings, which relates the couplings of $(K\overline{K})_{I=1}$, $(\pi^0\eta_8)$, and $(\pi^0\eta_1)$ in such a way that linear combination $|2^{I=1}\rangle$ decouples from the scalar $I=1$, $I_z=0$ meson (hereafter denoted π^0_S). State $|2^{I=1}\rangle$ would couple to π^0_S if SU(3)-breaking coupling $\mathrm{Tr}(M_{\pi^0_S}\{M_2\lambda_8 M_3 + M_3\lambda_8 M_2\})$ were introduced. Vanishing of $\lambda_3^{I=1}$ corresponds to the absence of hairpin diagrams ($|3^{I=1}\rangle = \pi^0\eta_s$).

3. sector $I=0$

$$K(I=0) = 2\begin{bmatrix} 3 & -\sqrt{3} & 1 & 2 & 0 \\ -\sqrt{3} & 3 & -\sqrt{3} & 0 & 0 \\ 1 & -\sqrt{3} & 1 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \kappa_S^{I=0} \tag{30}$$

with rows (columns) corresponding to states (in that order): $(K\overline{K})_{I=0}$, $(\pi\pi)_{I=0}$, $\eta_{ns}\eta_{ns}$, $\eta_s\eta_s$, $\eta_{ns}\eta_s$.
The eigenvalues and eigenvectors are:

$$\begin{aligned}
\lambda_1^{I=0} &= 12\kappa_S^{I=0} \\
&|1^{I=0}\rangle = \frac{1}{3}(-2(K\overline{K})_{I=0} - (\eta_s\eta_s) + \sqrt{3}(\pi\pi)_{I=0} - (\eta_{ns}\eta_{ns})) \\
\lambda_2^{I=0} &= 6\kappa_S^{I=0} \\
&|2^{I=0}\rangle = \frac{1}{3}((K\overline{K})_{I=0} + 2(\eta_s\eta_s) + \sqrt{3}(\pi\pi)_{I=0} - (\eta_{ns}\eta_{ns})) \\
\lambda_3^{I=0} &= 0 \\
&|3^{I=0}\rangle = \frac{1}{3}(2(K\overline{K})_{I=0} - 2(\eta_s\eta_s) + \frac{\sqrt{3}}{2}(\pi\pi)_{I=0} - \frac{1}{2}(\eta_{ns}\eta_{ns})) \\
\lambda_4^{I=0} &= 0 \\
&|4^{I=0}\rangle = \frac{1}{2}((\pi\pi)_{I=0} + \sqrt{3}(\eta_{ns}\eta_{ns})) \\
\lambda_5^{I=0} &= 0 \\
&|5^{I=0}\rangle = \eta_{ns}\eta_s
\end{aligned} \tag{31}$$

State $|1^{I=0}\rangle$ is SU(3) singlet, while $|2^{I=0}\rangle$ is SU(3) octet. The couplings of the remaining three states are zero as a result of the quark-level nonet symmetry of D-type coupling. For example, $\lambda_4^{I=0}$ may be nonzero if $(\pi\pi)_{I=0}$ and $(\eta_{ns}\eta_{ns})$ couplings to σ_{ns} (*i.e.* to $(u\overline{u}+d\overline{d})/\sqrt{2}$) are *not* related as specified by D-type coupling. Similarly, deviation of the scale of couplings involving strange quarks from those in which strange quarks are absent would result in nonvanishing of the coupling between the $|3^{I=0}\rangle$ state and the $I=0$ mesons. Coupling of state $|5^{I=0}\rangle$ to $q\overline{q}$ mesons would be nonzero if hairpin diagrams were allowed.

Let us reexpress the general FSI-modified formulae of type (27) in the K-matrix approach:
a) sector $I = 2$

$$\langle(\pi\pi)_{I=2}|W|D^0\rangle = -\frac{1}{\sqrt{3}}(A+B) = \langle(\pi\pi)_{I=2}|w|D^0\rangle = (-\frac{1}{\sqrt{3}})(a+b) \quad (32)$$

b) sector $I = 1$

$$\begin{aligned}\langle(K\overline{K})_{I=1}|W|D^0\rangle &= -\frac{1}{\sqrt{2}}(\tilde{A}+C+E)\\ &= \frac{1}{\sqrt{3}}\cdot T_1^{(1)}\langle 1^{I=1}|w|D^0\rangle + \langle(K\overline{K})_{I=1}|w|D^0\rangle\\ \langle\pi^0\eta_8|W|D^0\rangle &= \frac{1}{\sqrt{3}}(\tilde{B}-C-E)\\ &= \frac{\sqrt{2}}{3}\cdot T_1^{(1)}\langle 1^{I=1}|w|D^0\rangle + \langle\pi^0\eta_8|w|D^0\rangle\\ \langle\pi^0\eta_1|W|D^0\rangle &= -\frac{1}{\sqrt{6}}(\tilde{B}+2C+2E)\\ &= \frac{2}{3}\cdot T_1^{(1)}\langle 1^{I=1}|w|D^0\rangle + \langle\pi^0\eta_1|w|D^0\rangle\,, \end{aligned} \quad (33)$$

where

$$\langle 1^{I=1}|w|D^0\rangle = -\frac{1}{\sqrt{6}}(\tilde{a}+3c+3e) \overset{SU(3)}{\rightarrow} -\frac{1}{\sqrt{6}}(a+3c) \quad (34)$$

$$\langle(K\overline{K})_{I=1}|w|D^0\rangle = -\frac{1}{\sqrt{2}}(\tilde{a}+c+e) \overset{SU(3)}{\rightarrow} -\frac{1}{\sqrt{2}}(a+c)\,, \quad (35)$$

and $T_1^{(1)}$ is a complex factor which in the idealized case is ($\delta_1^{I=1}$ corresponds to $\lambda_1^{I=1}$):

$$T_1^{(1)} \equiv \cos\delta_1^{I=1}\exp i\delta_1^{I=1} - 1\,. \quad (36)$$

Solution of Eqs. (33) for SU(3)-symmetric input amplitudes is

$$\begin{aligned}\tilde{A} &= a\,,\\ \tilde{B} &= b\,,\\ C+E &= c+T_1^{(1)}(c+\frac{a}{3})\,,\end{aligned} \quad (37)$$

c) <u>sector $I = 0$</u>

$$\begin{aligned}
\langle (K\overline{K})_{I=0}|W|D^0\rangle &= \frac{1}{\sqrt{2}}(-\tilde{A} + C - 2\tilde{C} - E - 4F) \\
&= \frac{1}{3}(-2T_1^{(0)}\langle 1^{I=0}|w|D^0\rangle + T_2^{(0)}\langle 2^{I=0}|w|D^0\rangle) \\
&\quad +\langle (K\overline{K})_{I=0}|w|D^0\rangle \\
\langle (\pi\pi)_{I=0}|W|D^0\rangle &= \sqrt{\frac{2}{3}}(-A + \frac{1}{2}B - \frac{3}{2}C + \frac{3}{2}E + 3F) \\
&= \frac{1}{\sqrt{3}}(T_1^{(0)}\langle 1^{I=0}|w|D^0\rangle + T_2^{(0)}\langle 2^{I=0}|w|D^0\rangle) \\
&\quad +\langle (\pi\pi)_{I=0}|w|D^0\rangle \\
\langle \eta_8\eta_8|W|D^0\rangle &= \frac{1}{\sqrt{2}}(B - C - \frac{1}{3}E - 2F) + \frac{2}{3\sqrt{2}}(2(C - \tilde{C}) + (\tilde{B} - B)) \\
&= \frac{1}{3}(-T_1^{(0)}\langle 1^{I=0}|w|D^0\rangle + T_2^{(0)}\langle 2^{I=0}|w|D^0\rangle) \\
&\quad +\langle \eta_8\eta_8|w|D^0\rangle \\
\langle \eta_1\eta_1|W|D^0\rangle &= -\frac{\sqrt{2}}{3}(-C + \tilde{C} - B + \tilde{B} + E + 3F) \\
&= -\frac{1}{3}T_1^{(0)}\langle 1^{I=0}|w|D^0\rangle + \langle \eta_1\eta_1|w|D^0\rangle \\
\langle \eta_1\eta_8|W|D^0\rangle &= \frac{1}{\sqrt{2}}(B + 2C - \frac{2}{3}E + \frac{4}{3}(\tilde{C} - C) + \frac{1}{3}(\tilde{B} - B)) \\
&= -\frac{2}{3}T_2^{(0)}\langle 2^{I=0}|w|D^0\rangle + \langle \eta_1\eta_8|w|D^0\rangle\,, \qquad (38)
\end{aligned}$$

where

$$\begin{aligned}
\langle 1^{I=0}|w|D^0\rangle &= -\frac{\sqrt{2}}{3}(a - \tilde{a} + 3(c - \tilde{c}) - 3e - 9f) \\
\langle 2^{I=0}|w|D^0\rangle &= -\frac{1}{\sqrt{2}}(\frac{1}{3}(\tilde{a} + 2a) + c + 2\tilde{c} - e) \qquad (39)
\end{aligned}$$

and, in the idealized case,

$$T_j^{(0)} = \cos\delta_j^{I=0}\exp\delta_j^{I=0} - 1 \qquad (40)$$

with $j = 1, ...5$ (in Eq. (38) we have used $T_{3,4,5}^{(0)} = 0$).

Eqs. (38) are much simplified when one accepts the SU(3)-limit for input weak decays:

$$\begin{aligned}\langle 1^{I=0}|w|D^0\rangle &\stackrel{\mathrm{SU(3)}}{\to} 0\,,\\ \langle 2^{I=0}|w|D^0\rangle &\stackrel{\mathrm{SU(3)}}{\to} -\frac{1}{\sqrt{2}}(a+3c)\,.\end{aligned}\tag{41}$$

From Eqs. (41) it follows that SU(3)-singlet resonances do not affect the final formulae since in the SU(3) limit the SU(3)-singlet state is not produced through an FSI-unmodified weak process. Solution of Eqs. (38) for SU(3)-symmetric input amplitudes is then

$$\begin{aligned}\tilde{B} &= b\,,\\ A-2B &= a-2b\\ \tilde{A}+B &= a+b\,,\\ B+C-\tilde{C}-E-3F &= b\,,\\ \tilde{C}+F &= c+T_2^{(0)}(c+\frac{a}{3})\,.\end{aligned}\tag{42}$$

A look at how resonance-induced FSI (Fig. 3 and its counterparts) generate FSI-modified diagrams from the input (a), (b), (c) amplitudes confirms that diagrams of type (a) and (b) cannot actually be generated. Thus, we must have

$$\begin{aligned}A=\tilde{A} &= a\,,\\ B=\tilde{B} &= b\,,\\ C-E-2F &= \tilde{C}+F\,.\end{aligned}\tag{43}$$

From Eqs. (37), (42), (43) we see that in the case when strong interactions exhibit SU(3) symmetry, *i.e.* when the intermediate $I=1$ and $I=0$ octet scalar resonances are degenerate and couple to PP with the same strength we have $\kappa_S^{I=0}=\kappa_S^{I=1}$ so that

$$T_2^{(0)} = T_1^{(1)} \equiv T\,,\tag{44}$$

and one obtains

$$\begin{aligned}E+F &= \frac{1}{2}(T_1^{(1)}-T_2^{(0)})(c+\frac{a}{3})=0\,,\\ C+E=\tilde{C}+F &= c+T(c+\frac{a}{3})\,.\end{aligned}\tag{45}$$

Thus, physically measurable amplitudes *may* be described with vanishing effective penguin amplitudes E, F and SU(3)-symmetric exchange amplitudes $\tilde{C} = C$.

In reality, in the energy region below 1500 MeV the scalar resonance sector exhibits peculiar SU(3)-symmetry breaking [15, 20]. The masses and couplings of physical resonances are known to be different from those expected when these resonances are assigned a simple $q\overline{q}$ structure. Descriptions of these resonances as $qq\overline{q}\overline{q}$ states have been proposed. Despite years of intensive efforts the questions related to the nature of these states have not been settled as yet. The strangeness $S = 0$ sector under consideration in this section is particularly troublesome as exhibited by conflicting interpretations of the $a_0(980)$, $f_0(980)$, and $f_0(1300)$ states [21]. One should therefore expect that in the D-mass region around 1870 MeV the situation is also complicated (see also [19]). A detailed analysis of the nature of resonances affecting two-meson interactions at this energy is clearly far beyond the scope of this paper. For our purposes the crucial point is the relative size of amplitudes corresponding to diagrams of Fig. 2 and Fig. 4.

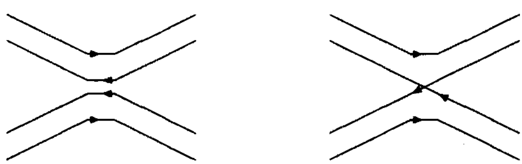

Fig. 4. Contributions of four-quark intermediate states to FSI.

If diagrams of Fig. 4 do not contribute significantly to the mechanism of physical resonance formation, as is the case in the unitarised quark model (UQM) of Törnqvist [15, 20], one should expect that the values of amplitudes A, B may be taken from the $S = -1$ sector of D^0 decays or from D^+ decays, and subsequently used in the $S = 0$ sector. On the other hand, the remaining parameters of the quark-line approach needed for the description of the $D^0 \to \pi\pi, K\overline{K}$ *etc.* decays may exhibit significant SU(3)-breaking within and between the I=0 and I=1 subsectors of the S=0 sector (and, of course, between the S=0 and S=-1 sectors) [20]. In the analysis of Törnqvist [15] the scalar meson sector exhibits energy-dependent mixing of the two $I = 0$ resonances. The value of the mixing angle undergoes a fairly rapid change in the vicinity of the $K\overline{K}$ threshold. Below 900 MeV the mixing is nearly ideal while above 1.1 GeV one has nearly pure SU(3) eigenstates. The $f_0(1300)$ appears then as a near-octet resonance. Although the analysis of

[15] stops at 1.6 GeV there are no reasons to expect a qualitative change in the mixing angle when energy changes from 1.6 to 1.87 GeV: The majority of PP thresholds lie well below 1.6 GeV (see also [7]). Despite significant SU(3) breaking incorporated into the UQM (*i.e.* through realistic positions of thresholds) this model confirms essentially that our use of pure SU(3) eigenstates at D^0 energy is justified. SU(3)-breaking enters the K matrix through the difference in the real parts of the vacuum polarization functions $\Pi(s)$ [15] which are different for each of the $I = 1$ and two $I = 0$ states. Consequently, the $I = 0$ and $I = 1$ octet channels are affected differently corresponding to strong SU(3) breaking between $a_0(980)$ and $f_0(1300)$. In our simplified treatment this means that the sizes of K-matrix elements in the $I = 1$ and $I = 0$ octet channels are different: $\kappa_S^{I=0} \neq \kappa_S^{I=1}$. As a result we should treat the isospin amplitudes in the $I = 0$ and $I = 1$ sectors as *independent* free parameters.

In terms of amplitudes with definite isospin the amplitudes of the four measured $D^0 \to K\overline{K}, \pi\pi$ decays are given by

$$\begin{aligned}
\langle K^+K^-|W|D^0\rangle &= \frac{1}{\sqrt{2}}(\mathcal{K}_1 + \mathcal{K}_0)\,, \\
\langle K^0\overline{K}^0|W|D^0\rangle &= \frac{1}{\sqrt{2}}(\mathcal{K}_1 - \mathcal{K}_0)\,, \\
\langle \pi^+\pi^-|W|D^0\rangle &= \frac{1}{\sqrt{3}}(\mathcal{P}_2 + \sqrt{2}\mathcal{P}_0)\,, \\
\langle \pi^0\pi^0|W|D^0\rangle &= \frac{1}{\sqrt{3}}(\sqrt{2}\mathcal{P}_2 - \mathcal{P}_0)\,,
\end{aligned} \tag{46}$$

where the amplitudes of definite isospin (specified by subscript) can be expressed in terms of quark-line amplitudes as follows

$$\begin{aligned}
\mathcal{K}_1 &= -\frac{1}{\sqrt{2}}(\tilde{A} + C + E)\,, \\
\mathcal{K}_0 &= -\frac{1}{\sqrt{2}}(\tilde{A} + \tilde{C} + F)\,, \\
\mathcal{P}_2 &= -\frac{1}{\sqrt{3}}(A + B)\,, \\
\mathcal{P}_0 &= \sqrt{\frac{2}{3}}(-A + \frac{B}{2} - \frac{3}{2}(\tilde{C} + F)) = \frac{1}{\sqrt{6}}(A + B) + \sqrt{3}\mathcal{K}_0\,.
\end{aligned} \tag{47}$$

Note that (apart from the contribution of the FSI-unmodified (a)- and (b)-type amplitudes) the isospin $I = 0$ (1) amplitude may be put in a one-to-one correspondence with the $\tilde{C} + F$ ($C + E$) combination of quark-diagram amplitudes respectively. Defining $Y = \frac{1}{\sqrt{2}}(\mathcal{K}_1 - \mathcal{K}_0)$ and $X = \sqrt{2}\mathcal{K}_0$ the amplitudes of the four considered decays acquire simple form given in Table I, where the corresponding experimental branching ratios taken from [22] are also displayed.

TABLE I

Theoretical amplitudes and branching ratios of the four $D^0 \to KK$, $\pi\pi$ decays measured.

decay	amplitude	branching ratio in %
$K^+ K^-$	$X+Y$	0.454 ± 0.029
$K^0 \overline{K}^0$	Y	0.11 ± 0.04
$\pi^+\pi^-$	X	0.159 ± 0.012
$\pi^0\pi^0$	$-\frac{1}{\sqrt{2}}(A+B+X)$	0.088 ± 0.023

The data of Table 1 permit us to establish that

$$\begin{aligned} |X+Y| &= (4.51 \pm 0.14) * 10^{-6}\ \text{GeV}\,, \\ |Y| &= (2.21 \pm 0.40) * 10^{-6}\ \text{GeV}\,, \\ |X| &= (2.47 \pm 0.09) * 10^{-6}\ \text{GeV} \\ |A+B+X| &= (2.60 \pm 0.3) * 10^{-6}\ \text{GeV}\,. \end{aligned} \tag{48}$$

From the branching ratio of the $D^+ \to \pi^+\overline{K}^0$ decay (equal to $(2.74 \pm 0.29) * 10^{-2}$ [22]) described by amplitude $A+B$ one infers that

$$|A+B| = (1.35 \pm 0.07) * 10^{-6}\ \text{GeV}\,, \tag{49}$$

Equations (48) and (49) show that, contrary to the conclusions of Ref. [11], it is still possible to *keep* SU(3)-symmetry in factorization amplitudes A, B provided *one takes into account coupled channel effects in the final state interactions* (SU(3)-symmetry of factorization amplitudes was used when writing the last equality in Eq. (47)). In particular, even if the data were consistent with $Y = 0$ (and $|X+Y| = |X| \approx 2.5 * 10^{-6}$ GeV), *i.e.* if the amplitudes were SU(3)-symmetric, we would still have to conclude from the values of $|A+B|$, $|X|$, and $|A+B+X|$ that the relative phase of $A+B$ and X must be close to 90°. Since in the quark-diagram approach $X = -A-\tilde{C}-F$, it follows that the relative phase of $\tilde{C}+F$ and A, B must be significant, in agreement with the message of this paper. Nonzero phase of $\tilde{C}+F$ is a direct result of inelasticity in FSI.

The SU(3)-breaking Y amplitude is expressed through quark-diagram amplitudes as $Y = -\frac{1}{2}(C+E-\tilde{C}-F)$. Large observed size of Y means that, when interpreted in terms of quark-diagram amplitudes, the data can be described either by a strong breaking of SU(3)-symmetry in W-exchange amplitude ($C \neq \tilde{C}$ and $E - F \approx 0$) or by a large contribution from effective long-range penguins ($E - F \neq 0$), or both. One has to keep in mind, however, that in the SU(3)-symmetry breaking case the combination $C + E$ (determined from the $I = 1$ sector) cannot be used in the $I = 0$ sector (*i.e.* in Eq. (43)): The diagrammatic approach suggests more symmetry between the $I = 0$ and $I = 1$ sectors than *is* actually present.

5. Conclusions

We have studied in some detail how inelastic coupled-channel rescattering effects (and, in particular, indirect $q\bar{q}$ resonance formation in the final state) modify the input weak amplitudes of the quark-line diagrammatical approach. Through an explicit calculation it has been demonstrated that such coupled-channel effects lead to the appearance of nonzero relative phases between various quark diagrams, thus invalidating the way in which final-state interactions were incorporated into the diagrammatical approach in the past. The case of SU(3)-symmetry breaking in Cabibbo once-forbidden D^0 decays has been also discussed. It has been shown that data may be described when inelastic final-state interactions (which must be SU(3)-breaking as well) are introduced. On the other hand, contrary to statements in literature, the data do not *require* SU(3)-symmetry breaking in factorization amplitudes. Significant breaking of SU(3) symmetry in FSI in charmed meson decays (in particular through the sector of scalar resonances) and the possible importance of FSI-induced hairpin-like diagrams make the usefulness of the strict quark-diagram approach to charmed meson decays quite questionable. I expect, however, that the diagrammatical approach with coupled channel effects included through the appearance of relative phases between various quarks diagrams should be quite applicable to charmed baryon decays where various SU(3) symmetry breaking effects typical of the meson sector are either absent or much smaller.

REFERENCES

[1] L.-L. Chau, H.-Y. Cheng, *Phys. Rev. Lett.* **56**, 1655 (1986).
[2] L.-L. Chau, H.-Y. Cheng, *Phys. Rev.* **D36**, 137, (1987).
[3] L.-L. Chau, H.-Y. Cheng, *Phys. Lett.* **B222**, 285 (1989).
[4] M.Bauer, B. Stech, M. Wirbel, *Z. Phys.* **C34**, 103 (1987).
[5] D. Lee, *Phys. Lett.* **B275**, 469 (1992).
[6] H.J. Lipkin, *Phys. Rev. Lett.* **44**, 710 (1980); *Phys. Rev. Lett.* **46**, 1307 (1981).
[7] C. Sorensen, *Phys. Rev.* **D23**, 2618 (1981).
[8] A.N. Kamal, E.D. Cooper, *Z. Phys.* **C8**, 67 (1981).

[9] J.F. Donoghue, *Phys. Rev.* **D33**, 1516 (1986).
[10] L.-L. Chau, *Phys. Rep.* **95**, 1 (1983).
[11] L.-L. Chau, H.-Y. Cheng, *Phys. Lett.* **B280**, 281 (1992).
[12] I. Hinchcliffe, Th.A. Kaeding, *Phys. Rev.* **D54**, 914 (1996).
[13] C. Sorensen, *Phys. Rev.* **D24**, 2976 (1981).
[14] O. Babelon *et al.*, *Nucl. Phys.* **B113**, 445 (1976).
[15] N.A. Törnqvist, Z.Phys. C68, 647 (1995) and papers cited therein.
[16] J.L. Basdevant, E.L. Berger, *Phys. Rev.* **D16**, 657 (1977).
[17] K.N. Watson, *Phys. Rev.* **95**, 228 (1954).
[18] L.-L. Chau, H.-Y. Cheng, *Phys. Rev.* **D39**, 2788 (1989).
[19] J.F.Donoghue, B.R.Holstein, *Phys. Rev.* **D21**, 1334 (1980).
[20] N.A.Törnqvist, *Phys. Rev. Lett.* **49**, 624 (1982); *Acta Phys. Pol.* **B16**, 503 (1985); in *The Hadron Mass Spectrum*, Proceedings of the Rheinfels Workshop, St. Goar, Germany, 1990, edited by E. Klempt, K. Peters, *Nucl. Phys.* B (Proc. Suppl.) **21** (1991).
[21] D.Morgan, M.R.Pennington, *Phys. Rev.* **D48**, 1185(1993); **D48**, 5422 (1993); N.N.Achasov, G.N.Shestakov, *Phys. Rev.* **D49**, 5779 (1994); R.Kamiński, L.Leśniak, J.-P.Maillet, *Phys. Rev.* **D50**, 3145 (1994).
[22] Particle Data Group, *Review of Particle Properties*, *Phys. Rev.* **D50**, 1173 (1994).