CORRELATION SINGS OF INSTANTONS IN MULTIGLUON PRODUCTION PROCESSES AT HIGH ENERGY

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General formula for inclusive gluon distribution on rapidities is obtained for the processes of multigluon production in classical instanton field with the first quantum correction. On the basis of this formula second correlation function is calculated in QCD and analyzed. The features of the correlation function behaviour can be used as a signal of instanton at HERA.

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1. Introduction

There exists now large interest to the high energy processes induced by both SU(2)× U(1) weak instantons [1–2] and SU(3) strong instantons [3–4] because of their important role in HEP.

As it is known violation of baryon and lepton numbers conservation law [5] due to quantum anomaly [6] can be induced by SU(2)× U(1) weak instantons [7] which represent tunneling processes, associated with the highly degenerated vacuum structure. Possibility of the baryon and lepton numbers nonconservation at high energy is connected with the problem of the baryon and antibaryon asymmetry in the observable part of the Universe [8]. At low energies (when energy of the process is less than barrier between different degenerated vacuum stages) cross section $\sigma^I_{\rm tot} \approx 10^{-78}$ [5]. In high energy particle collisions (in multi TeV regime) the cross section can increase exponentially if it is associated with multi W, Z^0 and H-bosons production in the weak instanton field [1].

The needed energies are so high that it is very difficult to see such processes at the nowadays experiments.

On the other hand for strong SU(3) instantons in QCD such phenomenon can exist at hundreds MeV [2] and be important in deep inelastic ep-scattering for decreasing Bjorken variable $X_{\rm Bj}$ and high photon virtuality Q^2 [3]. Search for QCD-instantons has started already in ep-collisions at HERA(H1). The processes have some characteristic features: instanton contribution to structure function $F_2(X_{\rm Bj}, Q^2)$ rises strongly with decreasing $X_{\rm Bj}$; $\sigma^I_{\rm tot}$ strongly peaks with decreasing of $X_{\rm Bj}$; hadronic band emission of semi-hard partons is isotropic in the instanton rest system; current quark jet and characteristic flavour, strangeness-K^0, charm and muon flow take place [4, 9, 10].

Moreover instanton-induced processes manifest new mechanism of multiparticle production and can contribute to intermittency exponent [11].

In this paper we study properties of the second correlation function as signature of SU(3) instanton-induced multi-gluon state for classical instanton with the first quantum correction in QCD.

It should be noted, that for this effect the problem of taking into account the hadronization exists. Here it can be solved by the use of the local parton-hadron duality [12].

2. Exclusive distribution on gluon rapidities for instanton-induced multiparticle production processes with the first quantum correction

The following instanton solution is used [7]:

$$A^{a(cl)}_{\mu}(x) = \frac{2\eta^a_{\mu\nu}(x-z)_\nu}{g((x-z)^2+\rho^2)}\,, \tag{1}$$

where $A^a_\mu(x)$ are gluon fields; $\eta^a_{\mu\nu}$ is a 't Hooft symbol [5]; ρ and z_ν are size and position of the instanton correspondingly; g is a constant of strong interaction: greek indexes $\mu, \nu ... = 1,2,3,4$ are the four-vector indexes, latin indexes $a, b ... = 1,2,...,8$ are SU(3)-group indexes; subscript "cl" denotes quasiclassical approximation.

It is convenient to use reduction formula [13] for the calculation of the amplitude of n gluon production in instanton field

$$T^{a_1 a_2 ... a_n}_{\mu_1\mu_2...\mu_n}(k_1, k_2, ..., k_n) = \int \prod_{j=1}^{n} dy_j\, e^{ik_j y_j}(k_j^2+m^2)$$

$$\times \int [DA]\, e^{-S[A]}\, A^{a_1}_{\mu_1}(x) ... A^{a_n}_{\mu_n}(x)$$

$$= \int \prod_{j=1}^{n} dy_j\, e^{ik_j y_j}(k_j^2+m^2) \int [DA]\, e^{-S[A]}(A^{a_1({\rm cl})}_{\mu_1}(x) + A^{a_1({\rm qu})}_{\mu_1}(x)) ...$$

$$\times (A^{a_n({\rm cl})}_{\mu_n}(x) + A^{a_n({\rm qu})}_{\mu_n}(x)) = \int \prod_{j=1}^{n} dy_j\, e^{ik_j y_j}(k_j^2+m^2)$$

$$\times \int [DA]\, \mathrm{e}^{-S[A]}\, A_{\mu_1}^{a_1(\mathrm{cl})}(x)...A_{\mu_n}^{a_n(\mathrm{cl})}(x) + \int \prod_{j=1}^{n} dy_j\, \mathrm{e}^{ik_j y_j}(k_j^2 + m^2)$$

$$\times \int [DA]\, \mathrm{e}^{-S[A]}\, A_{\mu_1}^{a_1(\mathrm{qu})}(x) A_{\mu_2}^{a_2(\mathrm{qu})}(x) A_{\mu_3}^{a_3(\mathrm{cl})}(x)...A_{\mu_n}^{a_n(\mathrm{cl})}(x) + ... \quad . \quad (2)$$

In formula (2) we consider $A_\mu^a(x) = A_\mu^{a(\mathrm{cl})}(x) + A_\mu^{a(\mathrm{qu})}(x)$, where $A_\mu^{a(\mathrm{qu})}(x)$ is small fluctuations near classical field $A_\mu^{a(\mathrm{cl})}(x)$; subscript "qu" denotes quantum correction [14]. The first term in formula (2) corresponds to the main (quasiclassical) approximation and is given by the following expression [15]:

$$T_{\mu_1\mu_2...\mu_n}^{(\mathrm{cl})a_1a_2...a_n}(k_1, k_2, ..., k_n) = (C_1)^n \prod_{j=1}^{n} \eta_{\mu_j\nu_j}^{a_j} k_j^{\nu_j}, \quad (3)$$

where $k_1, k_2, ..., k_n$ are the 4-momenta of the produced gluons, $k_j = (\vec{k}_j, iE_j)$, $C_1 = 4\pi^2 i\rho^2/g$, m is effective gluon mass.

In the formula (3) we as usual do not write δ-function, which is connected with law of conservation of 4-momentum [1]. For the correct normalization we must rewrite formula (3) in the following way:

$$T_{\mu_1\mu_2...\mu_n}^{(\mathrm{cl})a_1a_2...a_n}(k_1, k_2, ..., k_n) = (C_1)^n \prod_{j=1}^{n} \eta_{\mu_j\nu_j}^{a_j} k_j^{\nu_j}\, \Theta(n_{\max} - n), \quad (3')$$

where $n_{\max} = \sqrt{s}/m$.

Then the second term in (2) is the first quantum correction to the classical amplitude. It is calculated on the basis of gluon propogation function in instanton field [16] and is given by the following formula [2] (for two produced gluons):

$$T_{\mu\nu}^{(\mathrm{qu})ab}(k_1, k_2) = (C_2)^2 \eta_{\mu\alpha}^{d} \eta_{\nu\beta}^{d} k_1^\alpha k_2^\beta \varepsilon^{abc} \eta_{\kappa\lambda}^{c} \frac{k_1^\kappa k_2^\lambda}{(k_1, k_2)^2}, \quad (4)$$

where $C_2 = 2\pi i\rho$; $(k_1, k_2) = (\vec{k}_1 \vec{k}_2) - E_1 E_2$.

We make the following natural assumption (in laboratory subsystem in *ep*-collision) [17]:

$$(k_i^L)^2 \gg (\vec{k}_i^T)^2 \gg m^2, \qquad k_i^T \equiv |\vec{k}_i^T| = k^T,$$
$$E_i \approx k^T \cosh y_i, \qquad k_i^L \approx k^T \sinh y_i. \quad (5)$$

Let us write the expressions for the probabilities of the gluon production processes going through instanton mechanism in dependence on rapidity variables:

$$
\begin{aligned}
P_n(y_1, ..., y_n) &= | T_n^{(\mathrm{cl})}(y_1, ..., y_n) + T_n^{(\mathrm{qu})}(y_1, ..., y_n) |^2 \\
&\approx T_n^{(\mathrm{cl})}(y_1, ..., y_n)\left[T_n^{(\mathrm{cl})}(y_1, ..., y_n)\right]^* + T_n^{(\mathrm{qu})}(y_1, ..., y_n)\left[T_n^{(\mathrm{cl})}(y_1, ..., y_n)\right]^* \\
&+ T_n^{(\mathrm{cl})}(y_1, ..., y_n)\left[T_n^{(\mathrm{qu})}(y_1, ..., y_n)\right]^* = A^n \prod_{j=1}^{n} \cosh 2y_j \Theta(n_{\max} - n) \\
&- \alpha A^{n-2}\Big[\Pi(y_1, y_2) \cosh 2y_3 ... \cosh 2y_n + ... \\
&+ \Pi(y_{n-1}, y_n) \cosh 2y_1 ... \cosh 2y_{n-2}\Big]\Theta(n_{\max} - n) , \qquad (6)
\end{aligned}
$$

where $A = 3(k^T)^2(C_1)^2$, $\alpha = 8(C_1)^2(C_2)^2(k^T)^2$, $\Pi(y_1, y_2) = \sinh(y_1 - y_2) \times \tanh(y_1 - y_2)$. In the formula (6) the first term corresponds to the classical approximation and the others are the first quantum correction contribution.

3. Inclusive distribution and second correlation function for instanton-induced multigluon production processes with the first quantum correction

In order to calculate inclusive distribution $\rho_n(y_1, ..., y_n)$ we must by standard method take into account all channels of the process. In our case we obtain:

$$
\begin{aligned}
&\rho_n(y_1, ..., y_n) = A^n \cosh 2y_1 ... \cosh 2y_n[\Theta(n_{\max} - n) + R_0(n)] \\
&- \alpha A^{n-2}[\Pi(y_1, y_2) \cosh 2y_3 ... \cosh 2y_n + ... \\
&+ \Pi(y_{n-1}, y_n) \cosh 2y_1 ... \cosh 2y_{n-2}] \\
&\times (\Theta(n_{\max} - n) + R_0(n)) - \frac{\alpha}{2} A^n \cosh 2y_1 ... \cosh 2y_n \Pi(Y) R_2(n) \\
&- \alpha A^{n-1}[Q(Y, y_1) \cosh 2y_2 ... \cosh 2y_n + ... \\
&+ Q(Y, y_n) \cosh 2y_1 ... \cosh 2y_{n-1}] R_1(n) , \qquad (7)
\end{aligned}
$$

where we denoted

$$
\begin{aligned}
&\Pi(Y) = \int_{-Y}^{Y} \Pi(y_1, y_2) dy_1 dy_2 = \int_{-Y}^{Y} \sinh(y_1 - y_2) \tanh(y_1 - y_2) dy_1 dy_2 \\
&= 4\left(\sinh\left(\tfrac{Y}{2}\right)\right)^2 - \pi Y + 8 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} exp\left(-\tfrac{Y}{2}[2k+1]\right) \sinh\left(\tfrac{Y}{2}[2k+1]\right); \\
&Q(Y, y) = 2 \sinh Y \cosh y + \arctan[\sinh(y - Y)] - \arctan[\sinh(y + Y)]; \\
&R_i(n) = \sum_{m=1}^{\infty} \frac{[A \sinh 2Y]^{m-i}}{(m-i)!} \Theta(n_{\max} - n - m), \\
&i = 0, 1, 2, \qquad (0 \le R_2(n) \le R_1(n) \le R_0(n) \le 1) . \qquad (8)
\end{aligned}
$$

Two-particle correlation function in dependence on rapidities

$$C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$$

if $n_{\max} \gg 1$ and $\Theta(n_{\max} - 1) = \Theta(n_{\max} - 2) = 1; R_i(1) \approx R_i(2) \approx 1$; $i = 0, 1, 2$, has the following form:

$$\begin{aligned} C_2(y_1, y_2) = & \left[-2A^2 + \tfrac{3}{2}\alpha A^2 \Pi(Y)\right] \cosh 2y_1 \cosh 2y_2 \\ - & \ 4\alpha A\pi \cosh(y_1 + y_2) \tanh(y_1 - y_2) - 2\alpha \sinh(y_1 - y_2) \tanh(y_1 - y_2) \\ + & \ 2\alpha A \sinh Y \left[\cosh y_1 \cosh 2y_2 + \cosh y_2 \cosh 2y_1\right] \\ + & \ 2\alpha A \left[arctg(\frac{\cosh y_1}{\sinh Y}) \cosh 2y_2 + arctg(\frac{\cosh y_2}{\sinh Y}) \cosh 2y_1\right]. \end{aligned} \tag{9}$$

Corresponding curve lies in negative region of the plot $C_2(y_1, 0)$, has maximum at $y_1 = 0$ and minima at $y_1 = +3,5; -3,5$ (see Fig.1).Central maximum corresponds to the quasiclassical part of (9); two minima are contribution of the first quantum correction. For the estimation the parameters are taken to have the following values: $\sqrt{s} = 50$ GeV, $m = 100$ MeV, $\rho = 1$ GeV^{-1}, $Y \approx 4$, $k^T = 0,1$ GeV.

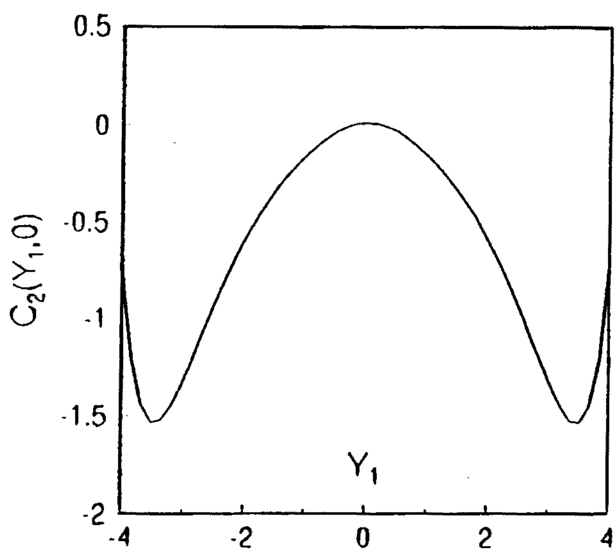

Fig. 1. Correlation function *vs.* rapidity y_1 of one of the particle, when $y_2 = 0$.

4. Conclusion

Thus, in addition to the known footprints of instanton induced events we have obtained two gluon correlation function which is negative and has specific structure at the rest system of one of the particle. With the help of local parton-hadron duality the result can be used for hadrons and is of interest for HERA experiments [4].

The estimations of the higher order quantum corrections with adequate kinematical restrictions on separation on hadrons from semi-hard quarks and from accompanying gluons to be used in Monte-Carlo analysis are now in progress.

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