TEXTURE DYNAMICS: PART TWO*

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A model of texture dynamics, initially constructed for charged leptons, is now described in some detail for up and down quarks of three families. It nicely correlates and reproduces all six quark masses and four mixing parameters in terms of nine constants which display some simple relations. These, if assumed, may reduce the number of free constants (and so enhance the number of predictions). Possible extensions of the model to neutrinos are briefly discussed.

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Recently, the quantum dynamics was introduced into the "texture" of fundamental–fermion mass matrices by means of annihilation and creation operators acting in the space of three fermion families [1]. A model of such a texture dynamics was constructed for charged leptons, predicting the tauon mass to be $m_\tau = 1776.80$ MeV, when the experimental electron and muon masses m_e and m_μ were used as an input (the recent experimental value is $m_\tau = 1777.00^{+0.30}_{-0.27}$ MeV [2]). In the present note, along similar texture–dynamical lines, we discussed in some detail the masses and mixing parameters for up and down quarks.

Our starting point will be the mass matrices $\widehat{M}^{(u)}$ and $\widehat{M}^{(d)}$ for up and down quarks, both assumed to have the generic form given in Eq. (13) of Ref. [1]:

$$\widehat{M} = \begin{pmatrix} \mu(0)\rho_0^2 & \alpha\rho_0\rho_1\, e^{i\varphi} & 0 \\ \alpha\rho_0\rho_1\, e^{-i\varphi} & \mu(1)\rho_1^2 & (\alpha+\beta)\sqrt{2}\rho_1\rho_2\, e^{i\varphi} \\ 0 & (\alpha+\beta)\sqrt{2}\rho_1\rho_2\, e^{-i\varphi} & \mu(2)\rho_2^2 \end{pmatrix}, \tag{1}$$

where, due to Eqs. (17) or (31) and (16) of Ref. [1],

$$\mu(0) = \mu\varepsilon^2\,,\ \ \mu(1) = \mu\frac{1}{9}(80+\varepsilon^2)\,,\ \ \mu(2) = \mu\frac{1}{25}(624+25C+\varepsilon^2) \tag{2}$$

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and

$$\rho_0 = \frac{1}{\sqrt{29}}, \quad \rho_1 = \sqrt{\frac{4}{29}}, \quad \rho_2 = \sqrt{\frac{24}{29}}. \tag{3}$$

Here, μ , ε^2 , C , α, $\alpha + \beta$ and φ denote constants dependent on the label u or d, while ρ_0, ρ_1 and ρ_2 are the same for both labels u and d. The formula (1) follows from our model of texture dynamics, where

$$\widehat{M} = \widehat{\rho}\widehat{h}\widehat{\rho}\,, \quad \widehat{h} = \mu(\widehat{n}) + (\alpha\widehat{1} + \beta\widehat{n})\widehat{a}e^{i\varphi} + \widehat{a}^{\,\dagger}(\alpha\widehat{1} + \beta\widehat{n})e^{-i\varphi}$$

and

$$\mu(\widehat{n}) = \mu\left[\left(\widehat{1} + 2\widehat{n}\right)^2 - \left(1 - \varepsilon^2\right)\left(\widehat{1} + 2\widehat{n}\right)^{-2} + \widehat{C}\right]$$

with $\widehat{\rho} = (\rho_0\,,\ \rho_1\,,\ \rho_2)$, $\widehat{n} = \mathrm{diag}\,(0, 1, 2) = \widehat{a}^\dagger\widehat{a}$ and $\widehat{C} = \mathrm{diag}\,(0\,,\ 0\,,\ C)$, according to Eqs. (11), (12) and (17) or (31) of Ref. [1]. Here, $\widehat{a}$ and $\widehat{a}^\dagger$ are the (truncated) annihilation and creation operators defined in Eq. (3) of Ref. [1]. The universal form of $\widehat{\rho}$, as given in Eq. (16) of Ref. [1], is based on an additional argument mentioned there.

Our aim is to diagonalize the mass matrix (1) *i.e.*, to solve the eigenvalue equation

$$\widehat{M}\vec{\phi}_i = M_i\vec{\phi}_i \quad (i = 0, 1, 2) \quad (\widehat{M}^{\,\dagger} = \widehat{M}). \tag{4}$$

Thus,

$$\widehat{U}^{-1}\widehat{M}\widehat{U} = \mathrm{diag}\,(M_0, M_1, M_2) \quad (\widehat{U}^{\,\dagger} = \widehat{U}^{-1}) \tag{5}$$

and

$$\widehat{U}^{-1}\vec{\phi}_i = \vec{e}_i \quad (i = 0, 1, 2) \tag{6}$$

with

$$\det(\widehat{M} - \widehat{1}M_i) = 0 \quad (i = 0, 1, 2) \tag{7}$$

and $\vec{e}_i = (\delta_{ij}\,;\ j = 0, 1, 2)$.

Writing $\widehat{M} \equiv (M_{ij}\,;\ i, j = 0, 1, 2)$ and $\vec{\phi}_i \equiv (\phi_{ij}\,;\ j = 0, 1, 2)$, we obtain from Eq. (4) (where $M_{02} = 0 = M_{20}$):

$$\begin{aligned}
&\phi_{00} = N_0\,, && \phi_{10} = -N_1\frac{M_{01}}{M_{00} - M_1}\,, && \phi_{20} = N_2\frac{M_{22} - M_2}{M_{21}}\frac{M_{01}}{M_{00} - M_2}\,,\\
&\phi_{01} = -N_0\frac{M_{00} - M_0}{M_{01}}\,, && \phi_{11} = N_1\,, && \phi_{21} = -N_2\frac{M_{22} - M_2}{M_{21}}\,,\\
&\phi_{02} = N_0\frac{M_{00} - M_0}{M_{01}}\frac{M_{21}}{M_{22} - M_0}\,, && \phi_{12} = -N_1\frac{M_{21}}{M_{22} - M_1}\,, && \phi_{22} = N_2\,.
\end{aligned} \tag{8}$$

Here,

$$N_0 = \left\{1 + \frac{(M_{00} - M_0)^2}{|M_{01}|^2}\left[1 + \frac{|M_{12}|^2}{(M_{22} - M_0)^2}\right]\right\}^{-1/2},$$
$$N_1 = \left[1 + \frac{|M_{01}|^2}{(M_{00} - M_1)^2} + \frac{|M_{12}|^2}{(M_{22} - M_1)^2}\right]^{-1/2},$$
$$N_2 = \left\{1 + \frac{(M_{22} - M_2)^2}{|M_{12}|^2}\left[1 + \frac{|M_{01}|^2}{(M_{00} - M_2)^2}\right]\right\}^{-1/2}. \tag{9}$$

Note that $\widehat{U} \equiv (U_{ij}\,;\ i,j = 0,1,2) = (\phi_{ji}\,;\ i,j = 0,1,2)$ since $U_{ij} = \vec{e}_i^T \widehat{U} \vec{e}_j = \vec{e}_i^T \vec{\phi}_j = \phi_{ji}$. Thus, the diagonalizing matrix for $\widehat{M}$ is of the form

$$\widehat{U} = \begin{pmatrix} N_0 & -N_1 \frac{M_{01}}{M_{00}-M_1} & N_2 \frac{M_{22}-M_2}{M_{21}} \frac{M_{01}}{M_{00}-M_2} \\ -N_0 \frac{M_{00}-M_0}{M_{01}} & N_1 & -N_2 \frac{M_{22}-M_2}{M_{21}} \\ N_0 \frac{M_{00}-M_0}{M_{01}} \frac{M_{21}}{M_{22}-M_0} & -N_1 \frac{M_{21}}{M_{22}-M_1} & N_2 \end{pmatrix}. \tag{10}$$

The Cabibbo–Kobayashi–Maskawa matrix is given as $\widehat{V} = \widehat{U}^{(u)\dagger}\widehat{U}^{(d)}$.

From Eq. (7) we can see (due to $M_{02} = 0 = M_{20}$) that

$$M_{11} - M_i = \frac{|M_{01}|^2}{M_{00} - M_i} + \frac{|M_{12}|^2}{M_{22} - M_i} \quad (i = 0, 1, 2). \tag{11}$$

Hence, for $i = 0$ and $i = 2$ we obtain

$$\begin{aligned} M_{11} - M_0 &= \frac{|M_{01}|^2}{M_{00} - M_0} + \frac{|M_{12}|^2}{M_{22} - M_0} \\ &\simeq \frac{|M_{01}|^2}{M_{00} - M_0} \quad \text{or} \quad \frac{M_{00} - M_0}{M_{01}} \simeq \frac{M_{10}}{M_{11} - M_0} \end{aligned} \tag{12}$$

and

$$\begin{aligned} M_{11} - M_2 &= \frac{|M_{01}|^2}{M_{00} - M_2} + \frac{|M_{12}|^2}{M_{22} - M_2} \\ &\simeq \frac{|M_{12}|^2}{M_{22} - M_2} \quad \text{or} \quad \frac{M_{22} - M_2}{M_{12}} \simeq \frac{M_{21}}{M_{11} - M_2}, \end{aligned} \tag{13}$$

because $|M_{00} - M_0| \ll |M_{22} - M_0|$ and $|M_{00} - M_2| \gg |M_{22} - M_2|$ by a factor that will turn out to be of the order $O(10^4)$ or $O(10^3)$ for u or d, respectively, while $|M_{01}|^2 < |M_{12}|^2$ by the factor $48\,[(\alpha+\beta)/\alpha]^2$ which weakens the approximation (12) and strenghtens the approximation (13) [we will assume that $[(\alpha+\beta)/\alpha]^2 = O(1)$].

From the approximative formulae (12) and (13) we get the second order algebraic equations for M_0 and M_2, ·

$$M_0^2 - (M_{00}+M_{11})M_0 + M_{00}M_{11} - |M_{01}|^2 = 0 \tag{14}$$

and

$$M_2^2 - (M_{11}+M_{22})M_2 + M_{11}M_{22} - |M_{12}|^2 = 0\,, \tag{15}$$

that give

$$M_0 \simeq \frac{1}{2}(M_{00}+M_{11}) - \sqrt{\frac{1}{4}(M_{11}-M_{00})^2 + |M_{01}|^2} \simeq M_{00} - \frac{|M_{01}|^2}{M_{11}-M_{00}} \tag{16}$$

and

$$M_2 \simeq \frac{1}{2}(M_{11}+M_{22}) + \sqrt{\frac{1}{4}(M_{22}-M_{11})^2 + |M_{12}|^2} \simeq M_{22} + \frac{|M_{12}|^2}{M_{22}-M_{11}}\,. \tag{17}$$

Hence,

$$\begin{aligned} M_1 &= M_{00}+M_{11}+M_{22}-M_0-M_2 \\ &\simeq \frac{1}{2}(M_{00}+M_{22}) + \sqrt{\frac{1}{4}(M_{11}-M_{00})^2 + |M_{01}|^2} \\ &\quad - \sqrt{\frac{1}{4}(M_{22}-M_{11})^2 + |M_{12}|^2} \\ &\simeq M_{11} + \frac{|M_{01}|^2}{M_{11}-M_{00}} - \frac{|M_{12}|^2}{M_{22}-M_{11}}\,. \end{aligned} \tag{18}$$

Here, due to Eqs. (1), (2) and (3), we have

$$M_{00} = \frac{\mu}{29}\varepsilon^2\,,\quad M_{11} = \frac{\mu}{29}\frac{4}{9}\left(80+\varepsilon^2\right)\,,\quad M_{22} = \frac{\mu}{29}\frac{24}{25}\left(625+25C+\varepsilon^2\right) \tag{19}$$

and

$$M_{01} = \frac{\alpha\sqrt{4}}{29}\,e^{i\varphi}\,,\quad M_{12} = \frac{(\alpha+\beta)\sqrt{192}}{29}\,e^{i\varphi}\,. \tag{20}$$

Thus, the second approximations in Eqs. (16), (17) and (18) hold if (1/4) $(M_{11}-M_{00})^2 \gg |M_{01}|^2$ and $(1/4)(M_{22}-M_{11})^2 \gg |M_{12}|^2$. This is certainly the case, when $(\alpha/2\mu)^2 = O(1)$ and $[(\alpha+\beta)/2\mu]^2 = O(1)$, what we will anticipate.

The explicit forms of Eqs (16), (18) and (17) are

$$M_0 \simeq \frac{\mu}{29}\left[\varepsilon^2 - \frac{36(\alpha/\mu)^2}{320-5\varepsilon^2}\right] \equiv M_{00} - \frac{\mu}{29}\frac{36(\alpha/\mu)^2}{320-5\varepsilon^2}\,, \tag{21}$$

$$\begin{aligned} M_1 &\simeq \frac{\mu}{29}\left\{\frac{4}{9}\left(80+\varepsilon^2\right) + \frac{36(\alpha/\mu)^2}{320-5\varepsilon^2} - \frac{10800\left[(\alpha+\beta)/\mu\right]^2}{31696+1350C+29\varepsilon^2}\right\} \\ &\simeq \frac{\mu}{29}\frac{4}{9}\left(80+\varepsilon^2\right) \equiv M_{11} \end{aligned} \tag{22}$$

and

$$\begin{aligned} M_2 &\simeq \frac{\mu}{29}\left\{\frac{24}{25}\left(624+25C+\varepsilon^2\right) + \frac{10800\left[(\alpha+\beta)/\mu\right]^2}{31696+1350C+29\varepsilon^2}\right\} \\ &\simeq \frac{\mu}{29}\frac{24}{25}\left(624+25C+\varepsilon^2\right) \equiv M_{22}\,, \end{aligned} \tag{23}$$

where it was anticipated that $(\alpha/2\mu)^2 = O(1)$ and $[(\alpha+\beta)/2\mu]^2 = O(1)$.

From Eqs (22) and (23) we can find μ and C (neglecting ε^2 in comparison with 80):

$$\mu \simeq \frac{261}{320}M_1 \tag{24}$$

and

$$C \simeq \frac{40}{27}\frac{M_2}{M_1} - \frac{624}{25}\,. \tag{25}$$

Then, from Eqs (21) and (24) we calculate ε^2 (neglecting ε^2 vs 320/5=64):

$$\varepsilon^2 \simeq 29\frac{M_0}{\mu} + \frac{9}{80}\left(\frac{\alpha}{\mu}\right)^2 \simeq \frac{9280}{261}\frac{M_0}{M_1} + \frac{1280}{7569}\left(\frac{\alpha}{M_1}\right)^2\,. \tag{26}$$

The constants $\alpha^{(u)}$, $\alpha^{(d)}$ and $\alpha^{(u)}+\beta^{(u)}$, $\alpha^{(d)}+\beta^{(d)}$ as well as $\varphi^{(u)}-\varphi^{(d)}$ will be determined from the experimental data for magnitudes of the elements $V_{us} \equiv V_{01}$, $V_{cb} \equiv V_{12}$ and $V_{ub} \equiv V_{02}$ of Cabibbo–Kobayashi–Maskawa matrix $\widehat{V}$.

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Starting with the formula (10) for the unitary matrices $\widehat{U}^{(u)}$ and $\widehat{U}^{(d)}$ diagonalizing the mass matrices $\widehat{M}^{(u)}$ and $\widehat{M}^{(d)}$ of up and down quarks, we calculate the elements V_{ij} of the Cabibbo–Kobayashi–Maskawa matrix $\widehat{V} \equiv (V_{ij}\,;\ i,j=0,1,2) = \widehat{U}^{(u)\dagger}\widehat{U}^{(d)}$: $V_{ij} = \sum_k U^{(u)*}_{ki}U^{(u)}_{kj}$. Then, making use of the approximations (12) and (13), we get

$$\begin{aligned} V_{us} \equiv V_{01} &= -N_0^{(u)} N_1^{(d)} \left(\frac{M_{01}^{(d)}}{M_{00}^{(d)} - M_1^{(d)}} + \frac{M_{00}^{(u)} - M_0^{(u)}}{M_{01}^{(u)*}} \right. \\ &\left. + \frac{M_{00}^{(u)} - M_0^{(u)}}{M_{01}^{(u)*}} \frac{M_{21}^{(u)*}}{M_{22}^{(u)} - M_0^{(u)}} \frac{M_{21}^{(d)}}{M_{22}^{(d)} - M_1^{(d)}} \right) \\ &\simeq -N_0^{(u)} N_1^{(d)} \left(\frac{M_{01}^{(d)}}{M_{00}^{(d)} - M_1^{(d)}} + \frac{M_{01}^{(u)}}{M_{11}^{(u)} - M_0^{(u)}} \right) \\ &\simeq N_0^{(u)} N_1^{(d)} \left(\frac{M_{01}^{(d)}}{M_1^{(d)}} - \frac{M_{01}^{(u)}}{M_1^{(u)}} \right) \\ &= N_0^{(u)} N_1^{(d)} \frac{\sqrt{4}}{29} \left(\frac{\alpha^{(d)}}{m_s} e^{i\varphi^{(d)}} - \frac{\alpha^{(u)}}{m_c} e^{i\varphi^{(u)}} \right) , \end{aligned} \tag{27}$$

and similarly [with the use of Eqs. (12) and (13)]

$$\begin{aligned} V_{cb} \equiv V_{12} &\simeq N_1^{(u)} N_2^{(d)} \left(\frac{M_{12}^{(d)}}{M_2^{(d)}} - \frac{M_{12}^{(u)}}{M_2^{(u)}} \right) \\ &= N_1^{(u)} N_2^{(d)} \frac{\sqrt{192}}{29} \left(\frac{\alpha^{(d)} + \beta^{(d)}}{m_b} e^{i\varphi^{(d)}} - \frac{\alpha^{(u)} + \beta^{(u)}}{m_t} e^{i\varphi^{(u)}} \right) \\ &\simeq N_1^{(u)} N_2^{(d)} \frac{\sqrt{192}}{29} \frac{\alpha^{(d)} + \beta^{(d)}}{m_b} e^{i\varphi^{(d)}} \end{aligned} \tag{28}$$

and

$$\begin{aligned} V_{ub} \equiv V_{02} &\simeq -N_0^{(u)} N_2^{(d)} \frac{M_{01}^{(u)}}{M_1^{(u)}} \left(\frac{M_{12}^{(d)}}{M_2^{(d)}} - \frac{M_{12}^{(u)}}{M_2^{(u)}} \right) \\ &= -N_0^{(u)} N_2^{(d)} \frac{\sqrt{4 \cdot 192}}{29^2} \frac{\alpha^{(u)}}{m_c} e^{i\varphi^{(u)}} \left(\frac{\alpha^{(d)} + \beta^{(d)}}{m_b} e^{i\varphi^{(d)}} - \frac{\alpha^{(u)} + \beta^{(u)}}{m_t} e^{i\varphi^{(u)}} \right) \\ &\simeq N_0^{(u)} N_2^{(d)} \frac{\sqrt{4 \cdot 192}}{29^2} \frac{\alpha^{(u)} (\alpha^{(d)} + \beta^{(d)})}{m_c\, m_b} e^{i(\varphi^{(u)} + \varphi^{(d)} - 180^\circ)} . \end{aligned} \tag{29}$$

In Eqs. (28) and (29) we anticipated that $(\alpha^{(d)} + \beta^{(d)})/m_b \gg (\alpha^{(u)} + \beta^{(u)})/m_t$. Here, from Eqs. (9) [with the use of Eqs. (12) and (13)]

$$N_0 \simeq \left(1 + \frac{|M_{01}|^2}{M_1^2} \right)^{-1/2} = \left(1 + \frac{4}{29^2} \frac{\alpha^2}{M_1^2} \right)^{-1/2} \simeq 1 ,$$

$$N_1 \simeq \left(1+\frac{|M_{01}|^2}{M_1^2}\right)^{-1/2} = \left(1+\frac{4}{29^2}\,\frac{\alpha^2}{M_1^2}\right)^{-1/2} \simeq 1\,,$$

$$N_2 \simeq \left(1+\frac{|M_{12}|^2}{M_2^2}\right)^{-1/2} = \left[1+\frac{192}{29^2}\,\frac{(\alpha+\beta)^2}{M_2^2}\right]^{-1/2} \simeq 1\,, \qquad (30)$$

when $(\alpha/2\mu)^2 = O(1)$ and $[(\alpha+\beta)/2\mu]^2 = O(1)$. With the figures for $\alpha^{(u)}$ and $\alpha^{(d)}$, as they will be determined in Eqs. (32) and (33), we evaluate more precisely from Eqs. (30) the normalization constants $N_0^{(u)} \simeq N_1^{(u)} \simeq 0.997$, $N_0^{(d)} \simeq N_1^{(d)} \simeq 0.978$ and $N_2^{(u)} \simeq 1.000$, $N_2^{(d)} \simeq 0.999$.

Taking the experimental values $|V_{cb}| = 0.041 \pm 0.003$ and $|V_{ub}/V_{cb}| = 0.08 \pm 0.002$ [2], we obtain from Eqs. (28) and (29) with $m_c = 1.5$ GeV and $m_b = 4.7$ GeV:

$$\alpha^{(d)} + \beta^{(d)} \simeq \frac{1}{N_1^{(u)}N_2^{(d)}}\,\frac{29}{\sqrt{192}}\,m_b|V_{cb}| \simeq 405\ \text{MeV} \qquad (31)$$

and

$$\alpha^{(u)} \simeq \frac{N_1^{(u)}}{N_0^{(u)}}\,\frac{29}{\sqrt{4}}\,m_c\ \left|\frac{V_{ub}}{V_{cb}}\right| \simeq 1740\ \text{MeV}\,. \qquad (32)$$

Here, $N_1^{(u)}N_2^{(d)} \simeq 0.996$ and $N_1^{(u)}/N_0^{(u)} \simeq 1.000$, so the latter coefficient can be omitted in Eq. (32).

Let us conjecture for coupling constants α and β in our model of texture dynamics (see Eq. (12) of Ref. [1]) that

$$\alpha^{(u)} : \alpha^{(d)} = \beta^{(u)} : \beta^{(d)} = |Q^{(u)}| : |Q^{(d)}| = 2\,, \qquad (33)$$

where $Q^{(u)} = 2/3$ and $Q^{(d)} = -1/3$ are up and down quark charges[1]. Then, from Eqs. (32) and (31), we infer that

$$\alpha^{(u)} \simeq 1740\,\text{MeV}\,,\ \alpha^{(d)} \simeq 870\,\text{MeV} \qquad (34)$$

and

$$\beta^{(u)} \simeq -930\,\text{MeV}\,, \beta^{(d)} \simeq -465\,\text{MeV} \qquad (35)$$

(and hence $\beta^{(u)}/\alpha^{(u)} = \beta^{(d)}/\alpha^{(d)} \simeq -0.534$). In this case, with $m_u = 4$ MeV, $m_d = 7$ MeV and $m_s = 279$ MeV (and the above m_c), Eq. (26) gives

$$\varepsilon^{(u)\,2} \simeq 0.322\,,\ \ \varepsilon^{(d)\,2} \simeq 2.54\,. \qquad (36)$$

[1] Here, the minus sign at $Q^{(d)} = -|Q^{(d)}|$ may be compensated by the rescaling $\varphi^{(d)} \to \varphi^{(d)} + 180°$ in Eq. (12) of Ref. [1], as then $\exp(\pm i\varphi^{(d)}) \to -\exp(\pm i\varphi^{(d)})$.

On the other hand, with $m_t = 175$ GeV (and the above m_c, m_b and m_s), we get from Eq. (25)

$$C^{(u)} \simeq 148\,,\; C^{(d)} \simeq 0\,. \tag{37}$$

Here, the particular value $m_s = 279$ MeV is chosen to imply $C^{(d)} \simeq 0.00$ with $m_b = 4.7$ GeV, what makes top quark exceptional among other fundamental fermions, as then only for t the constant C is (very) different from zero[2]. Finally, Eq. (24) (with the above m_c, and m_s) leads to

$$\mu^{(u)} \simeq 1220\,\mathrm{MeV}\,,\;\; \mu^{(d)} \simeq 228\,\mathrm{MeV}\,. \tag{38}$$

From Eqs (34) and (38) it can be seen that really $(\alpha^{(u)}/2\mu^{(u)})^2 = O(1)$ and $(\alpha^{(d)}/2\mu^{(d)})^2 = O(1)$.

With $\alpha^{(u)}$ and $\alpha^{(d)}$ determined as in Eqs. (34), we find from Eq. (27) that

$$V_{us} \simeq N_0^{(u)} N_1^{(d)} \frac{\sqrt{4}}{27}\left[\frac{870}{279} - \frac{1740}{1500}\, e^{i(\varphi^{(u)}-\varphi^{(d)})}\right] e^{i\,\varphi^{(d)}} \tag{39}$$

and thus

$$|V_{us}| \simeq N_0^{(u)} N_1^{(d)} \frac{\sqrt{4}}{27}\sqrt{11.07 - 7.23\cos(\varphi^{(u)} - \varphi^{(d)})}\,. \tag{40}$$

Here, $N_0^{(u)} N_1^{(d)} \simeq 0.975$. Hence, taking the experimental value $|V_{us}| = 0.2205 \pm 0.0018$ [2], we calculate $\cos(\varphi^{(u)} - \varphi^{(d)}) \simeq 0.0422$ and

$$\varphi^{(u)} - \varphi^{(d)} \simeq 87.6^\circ\,. \tag{41}$$

Then, from Eq. (35) we evaluate

$$\arg V_{us} \simeq -20.7^\circ + \varphi^{(d)}\,. \tag{42}$$

By a quark rephasing *i.e.*, rescaling of quark phases one may achieve that

$$\begin{gathered} \arg V_{ud} \to 0\,,\; \arg V_{us} \to 0\;\;,\; \arg V_{ub} \to -72.0^\circ\,, \\ \arg V_{cd} \to -180^\circ\,,\; \arg V_{cs} \to 0\;\;,\; \arg V_{cb} \to 0\,. \\ \arg V_{td} \to -21.0^\circ\,,\; \arg V_{ts} \to -180^\circ\,,\; \arg V_{tb} \to 0\,. \end{gathered} \tag{43}$$

[2] For a little smaller $C^{(u)} \simeq 144$ one has $m_c = 1.53$ GeV with $m_t = 175$ GeV. Note that the values $C^{(u)} = 144$ and $C^{(d)} = 0$ for up and down quarks as well as $C^{(\nu)} = 0$ and $C^{(e)} = 0$ for neutrinos and charged leptons may be provided by the formula

$$\widehat{C}^{(f)} = \mathrm{diag}\,(0\,,\;0\,,\;C^{(f)}) = 2\,\widehat{n}\,2\,(\widehat{n} - \widehat{1})N_C^{(f)}(N_C^{(f)} - 1)\left[N_C^{(f)}\,(Q^{(f)} + B^{(f)})\right]\,,$$

where $\widehat{n} = \mathrm{diag}$ (0, 1, 2), while $N_C^{(f)}$, $Q^{(f)}$ and $B^{(f)}$ are the number of colors, the charge and the baryon number, respectively, all for $f = u\,,\;d\,,\;\nu\,,\;e$ (in particular, $N_C^{(u)} = N_C^{(d)} = 3$ and $N_C^{(\nu)} = N_C^{(e)} = 1$).

In fact, the CP violating phases $\arg(V^*_{us} V^*_{cb} V_{ub} V_{cs}) \simeq 87.6° - 180° + 20.7° - 0.3° = -72.0°$ and $\arg(V^*_{cd} V^*_{ts} V_{td} V_{cs}) \simeq -20.7° - 0.3° = -21.0°$ are invariant under any quark rephasing and reduce to $\arg V_{ub} \simeq -72.0°$ and $\arg V_{td} \simeq -21.0°$ in our special quark phasing. Here, the contribution -0.3 comes from $\arg V_{cs} \simeq -0.3°$ as it will be given in Eq. (48).

As to the second half of nondiagonal elements of Cabibbo–Kobayashi–Maskawa matrix, V_{cd}, V_{ts} and V_{td}, we obtain the following formulae when we proceed in a similar way as in the case of Eqs. (27), (28) and (29) for V_{us}, V_{cb} and V_{ub}:

$$V_{cd} \equiv V_{10} \simeq -N_1^{(u)} N_0^{(d)} \frac{\sqrt{4}}{29} \left(\frac{\alpha^{(d)}}{m_s} e^{-i\varphi^{(d)}} - \frac{\alpha^{(u)}}{m_c} e^{-i\varphi^{(u)}} \right), \tag{44}$$

$$V_{ts} \equiv V_{21} \simeq -N_2^{(u)} N_1^{(d)} \frac{\sqrt{192}}{29} \frac{\alpha^{(d)} + \beta^{(d)}}{m_b} e^{-i\varphi^{(d)}} \tag{45}$$

and

$$V_{td} \equiv V_{20} \simeq -N_2^{(u)} N_0^{(d)} \frac{\sqrt{4 \cdot 192}}{29^2} \frac{\alpha^{(d)}(\alpha^{(d)} + \beta^{(d)})}{m_s\, m_b} e^{-i(2\varphi^{(d)} - 180°)}. \tag{46}$$

They were used in Eq. (43). Thus, we can infer that, both in our old and new quark phasings, the relations

$$V_{cd} \simeq -V^*_{us}, \quad V_{ts} \simeq -\frac{N_2^{(u)}}{N_2^{(d)}} V^*_{cb},$$

$$V_{td} \simeq -\frac{\alpha^{(d)}}{\alpha^{(u)}} \frac{m_c}{m_s} \frac{N_2^{(u)}}{N_2^{(d)}} V^*_{ub} \exp\left[i(\varphi^{(u)} - \varphi^{(d)} + 2\arg V_{cs} - 0.6°)\right] \tag{47}$$

hold. Here, $N_2^{(u)}/N_2^{(d)} \simeq 1.001$ and $(\alpha^{(d)}/\alpha^{(u)})(m_c N_2^{(u)}/m_s N_2^{(d)}) \simeq 2.69$.

Finally, for diagonal elements, when making use of Eqs. (30), we get

$$V_{ud} \equiv V_{00} \simeq N_0^{(u)} N_0^{(d)} e^{i0.3°} \simeq 0.975 e^{i0.3°},$$

$$V_{cs} \equiv V_{11} \simeq N_1^{(u)} N_1^{(d)} e^{-i0.3°} \simeq 0.975 e^{-i0.3°},$$

$$V_{tb} \equiv V_{22} \simeq N_2^{(u)} N_2^{(d)} \simeq 0.999. \tag{48}$$

These formulae also were used in Eq. (43).

Of course, due to the unitarity of matrix $\widehat{V}$, the elements V_{ud}, V_{cb} and V_{ub} determine the rest of its elements [in our approximation, three of them are given as in Eqs. (47) and three — as in Eqs. (48)].

* * *

To summarize, the predicted Cabibbo–Kobayashi–Maskawa matrix has approximately the form:

$$\hat{V} \simeq \begin{pmatrix} 0.975 & 0.221 & 0.003\, e^{-i\,72^\circ} \\ -0.221 & 0.975 & 0.041 \\ 0.009\, e^{-i\,21^\circ} & -0.041 & 0.999 \end{pmatrix} \tag{49}$$

(where our convenient quark rephasing, leading to all $\arg V_{ij} = 0$ except for $ij = ub$ and td, was carried out). Here, the input $|V_{us}| = 0.2205$, $|V_{cb}| = 0.041$ and $|V_{ub}/V_{cb}| = 0.08$ as well as $m_c/m_s = 1500/279$ was actually used. In fact, we derived from the proposed texture–dynamical model the approximate formulae (27), (28), (29), which can be rewritten as

$$\begin{aligned} V_{us} &\simeq N_1^{(u)} N_1^{(d)} \left|\frac{V_{ub}}{V_{cb}}\right| \left[\frac{1}{2}\frac{m_c}{m_s} - e^{i(\varphi^{(u)}-\varphi^{(d)})}\right] e^{i\varphi^{(d)}} , \\ V_{cb} &\simeq |V_{cb}|\, e^{i\varphi^{(d)}} ,\ V_{ub} \simeq |V_{ub}|\, e^{i(\varphi^{(u)}+\varphi^{(d)}-180^\circ)} \end{aligned} \tag{50}$$

(all valid before quark rephasing), and (47)

$$\begin{aligned} V_{cd} &\simeq -V_{us}^* , \qquad V_{ts} \simeq -\frac{N_2^{(u)}}{N_2^{(d)}} V_{cb}^* , \\ V_{td} &\simeq -\frac{1}{2}\frac{m_c}{m_s}\frac{N_2^{(u)}}{N_2^{(d)}} V_{ub}^* \exp\left[i(\varphi^{(u)} - \varphi^{(d)} + 2\arg V_{cs} - 0.6^\circ)\right] . \end{aligned}$$

Here, $N_1^{(u)} N_1^{(d)} \simeq 0.975$ and $N_2^{(u)}/N_2^{(d)} \simeq 1.001$. Hence, we determined the model constant $\varphi^{(u)} - \varphi^{(d)} \simeq 87.6^\circ$ and then predicted the CP violating phases $\arg V_{ub} \simeq -72.0^\circ$ and $\arg V_{td} \simeq -21.0^\circ$. Beside four quark mixing parameters, all six experimentally suggested values of quark masses are the physical quantities we were also able to fit to our model by determining all of its nine free constants (the CP violating phase $\arg V_{ub}$, implying also $\arg V_{td}$, was a prediction). Note that, for the approximate matrix $\hat{V}$ given in Eq. (49), some deviations from the unitarity condition $\hat{V}^\dagger \hat{V} = \hat{1} = \hat{V}\hat{V}^\dagger$ appear in the order $O(10^{-3})$ and so, are irrelevant in our approximation.

It is interesting to compare our predicted Cabibbo–Kobayashi–Maskawa matrix (49) (in particular, the CP violating phases $\arg V_{ub} \simeq -72.0^\circ$ and $\arg V_{td} \simeq -21.0^\circ$ there) with its Wolfenstein approximate parametrization [3]. To this end, let us take as the input our prediction for the phase $\arg V_{ub} \simeq -72.0^\circ$ and determine the parameters ρ and η as well as λ and A from the Wolfenstein formulae:

$$V_{us} \simeq \lambda \simeq -V_{cd} ,\ \ V_{cb} \simeq A\,\lambda^2 \simeq -V_{ts} , \tag{51}$$

$$V_{ub} \simeq A\,\lambda^3(\rho - i\eta) ,\ V_{td} \simeq A\,\lambda^3(1 - \rho - i\eta) \tag{52}$$

and

$$V_{ud} \simeq 1 - \frac{1}{2}\lambda^2 \simeq -V_{cs}\,, \quad V_{tb} \simeq 1\,. \tag{53}$$

Also the experimental values $|V_{us}| = 0.2205$, $|V_{cb}| = 0.041$ and $|V_{ub}/Vcb| = 0.08$ will be included in the input. Then, since Eqs. (52) imply

$$|V_{ub}| \simeq A\,\lambda^3\sqrt{\rho^2+\eta^2}\,,\ |V_{td}| \simeq A\,\lambda^3\sqrt{(1-\rho)^2+\eta^2} \tag{54}$$

and

$$\arg V_{ub} \simeq -\arctan\frac{\eta}{\rho}\,,\ \arg V_{td} \simeq -\arctan\frac{\eta}{1-\rho}\,, \tag{55}$$

we obtain

$$\mid \frac{V_{ub}}{V_{cb}} \mid \simeq \lambda\sqrt{\rho^2+\eta^2}\,,\ \mid \frac{V_{td}}{V_{ub}} \mid \simeq \frac{\sqrt{(1-\rho)^2+\eta^2}}{\sqrt{\rho^2+\eta^2}}\,. \tag{56}$$

Combining the first formulae (55) and (56), we calculate

$$\rho \simeq \frac{1}{\lambda} \mid \frac{V_{ub}}{V_{cb}} \mid \left[\tan^2(\arg V_{ub}) + 1\right]^{-1/2} \simeq 0.112 \tag{57}$$

and

$$\eta \simeq \left(\frac{1}{\lambda^2} \mid \frac{V_{ub}}{V_{cb}} \mid^2 - \rho^2\right)^{1/2} \simeq 0.345\,, \tag{58}$$

as well as $\sqrt{\rho^2+\eta^2} \simeq 0.363$ and $\sqrt{(1-\rho)^2+\eta^2} \simeq 0.953$ (since $\arg V_{ub} < 0$, both ρ and η are of the same sign that we choose positive). The present experimental data, when analyzed jointly with theoretical errors of lattice QCD results, give the limits [3]

$$-0.35 \leq \rho \leq 0.35\,,\ 0.20 \leq \eta \leq 0.52 \tag{59}$$

which confine the values (57) and (58) (it is interesting to mention that the data prefer $|\arg V_{ub}|$ smaller but not much smaller than 90°). Finally, Eqs. (51) imply $\lambda \simeq 0.2205$ and $A \simeq 0.843$.

Now, with the figures (57) and (58), we evaluate from the second equations (55) and (56) the quantities $\arg V_{td} \simeq -21.2$ and $\mid V_{td}/V_{ub} \mid \simeq 2.63$, the latter value giving $\mid V_{td} \mid \simeq 0.00086$ when $|V_{ub}| = 0.08 \mid V_{cb} \mid = 0.0033$. These numbers are to be compared with our previous, a little larger estimates $\arg V_{td} \simeq -21.0$ and $\mid V_{td}/V_{ub} \mid \simeq 2.69$, the second figure implying $\mid V_{td} \mid \simeq 0.00088$ when $\mid V_{ub} \mid = 0.0033$. Thus, we can see that our approximate form (49) of the Cabibbo–Kobayashi–Maskawa matrix is still valid in the Wolfenstein parametrization. This consistency of our work with the structure of Wolfenstein parametrization verifies nicely the approximation used here to solve the proposed model of texture dynamics. The model itself may be confirmed or refuted (perhaps, in a near future) by a precise measurement of, at least, two of four quantities $\arg V_{ub}$, $\arg V_{td}$, $|V_{ub}|$ and $|V_{td}|$.

Concluding, we can say that our texture–dynamical model for mass matrices of up and down quarks, described in Eqs. (1), (2) and (3), correlates neatly and reproduces reasonably all six quark masses and four quark mixing parameters (and so all elements of Cabibbo–Kobayashi–Maskawa matrix) in terms of the model constants $\mu^{(u)}$, $\mu^{(d)}$, $C^{(u)}$, $C^{(d)}$, $\varepsilon^{(u)\,2}$, $\varepsilon^{(d)\,2}$, $\alpha^{(u)}$, $\beta^{(d)}$ and $\varphi^{(u)} - \varphi^{(d)}$. These take the values as determined in Eqs. (38), (37), (36), (34), (35) and (41). They display some relations (or correlations) such as $C^{(d)} \simeq 0$ and $\alpha^{(u)}/\mu^{(u)}\sqrt{2} \simeq 1$. If all quark (current) masses were better known, in particular the mass m_s, the fixing of free constants within our model would be more certain and, perhaps, would uncover new illuminating relationships for these specific constants.

It is worthwhile to stress that the mass matrix described in Ref. [1], working successfully for charged leptons, has the same generic form as those discussed in the present paper for up and down quarks.

* * *

How to extend our generic form of mass matrix also to neutrinos is evidently an open question. A proposal, how to do it, can be found in Ref. [4]. If $m_{\nu_e} \neq 0$, that leads to an "inverted" order of neutrino masses: $m_{\nu_e} \simeq 2m_{\nu_\mu} \simeq m_{\nu_\tau}$ or more precisely $m_{\nu_e} : 2m_{\nu_\mu} : m_{\nu_\tau} \simeq 1 : (4/9) : (24/25)$ (possibly, with "=" instead of "$\simeq$").

However, this proposal is not in the spirit of the extension of charged-lepton mass matrix to up and down quarks, considered in the present paper. A natural extension to neutrinos, following this spirit, may be based on our generic mass matrix of the form (1), where now $\varepsilon^{(\nu)\,2} = 0$, $C^{(\nu)} = 0$ and $\alpha^{(\nu)} \simeq 0 \simeq \beta^{(\nu)\,2}$ or, more naturally, $\alpha^{(\nu)} = 0 = \beta^{(\nu)}$, where the latter more restrictive condition would exclude neutrino oscillation (note that for charged leptons $\varepsilon^{(e)\,2} \neq 0$, $C^{(e)} = 0$ and $\alpha^{(e)} \simeq 0 \simeq \beta^{(e)}$ or even $\alpha^{(e)} = 0 = \beta^{(e)}$). Then, in place of charged-lepton spectrum

$$
\begin{aligned}
m_e &\equiv M_0^{(e)} \simeq \frac{\mu(e)}{29}\,\varepsilon^{(e)\,2}\,, \\
m_\mu &\equiv M_1^{(e)} \simeq \frac{4}{9}\,\frac{\mu(e)}{29}\,\left(80 + \varepsilon^{(e)\,2}\right)\,, \\
m_\tau &\equiv M_2^{(e)} \simeq \frac{24}{25}\,\frac{\mu(e)}{29}\,\left(624 + \varepsilon^{(e)\,2}\right)\,,
\end{aligned}
\tag{60}
$$

predicting

$$m_\tau \simeq 1766.80\,\text{MeV}\,,\ \varepsilon^{(e)\,2} \simeq 0.172329\,,\ \mu^{(e)} \simeq 85.9924\,\text{MeV} \qquad (61)$$

from the experimental values of m_e and m_μ, one obtains the neutrino spectrum

$$\begin{aligned} m_{\nu_e} \simeq m_{\nu_0} &\equiv M_0^{(\nu)} \simeq 0\,, \\ m_{\nu_\mu} \simeq m_{\nu_1} &\equiv M_1^{(\nu)} \simeq \frac{320}{261}\,\mu^{(\nu)}\,, \\ m_{\nu_\tau} \simeq m_{\nu_2} &\equiv M_2^{(\nu)} \simeq \frac{14376}{725}\,\mu^{(\nu)} \end{aligned} \qquad (62)$$

or, more naturally, with "=" instead of "$\simeq$". If $m_{\nu_\mu} \neq 0$,

$$m_{\nu_\mu} : m_{\nu_\tau} \simeq 1 : \frac{2106}{125} \qquad (63)$$

or with "=" in place of "$\simeq$". Here, the mass scale $\mu^{(\nu)}$ must be tiny enough to provide small mass differences between neutrinos of three kinds, what seems to be consistent with cosmic neutrino experiments (*cf. e.g.*, some citations in Ref. [4]).

Another, perhaps the most natural, extension may consist in putting $\mu^{(\nu)} = 0$ and $\alpha^{(\nu)} = 0 = \beta^{(\nu)}$ in our generic mass matrix of the form (1). Then,

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0\,, \qquad (64)$$

what, at the present moment, seems not to be a popular option as excluding neutrino oscillations, though it is still possible. A less natural version of this option may be $\mu^{(\nu)} = 0$ and $\alpha^{(\nu)} \simeq 0 \simeq \beta^{(\nu)}$, where the second relaxed condition would allow for neutrino oscillations. In this case, the neutrino mass matrix of the generic form (1) gets the shape

$$\widehat{M}^{(\nu)} = \begin{pmatrix} 0 & \alpha^{(\nu)}\rho_0\rho_1 e^{i\varphi^{(\nu)}} & 0 \\ \alpha^{(\nu)}\rho_0\rho_1 e^{-i\varphi^{(\nu)}} & 0 & (\alpha^{(\nu)}+\beta^{(\nu)})\sqrt{2}\rho_1\rho_2 e^{i\varphi^{(\nu)}} \\ 0 & (\alpha^{(\nu)}+\beta^{(\nu)})\sqrt{2}\rho_1\rho_2 e^{-i\varphi^{(\nu)}} & 0 \end{pmatrix}, \qquad (65)$$

leading to three eigenvalues

$$M_{0,1,2}^{(\nu)} = 0\,,\ \pm\frac{\sqrt{4}}{29}\sqrt{\alpha^{(\nu)\,2} + 48(\alpha^{(\nu)} + \beta^{(\nu)})^2}\,. \qquad (66)$$

Thus, for neutrino masses squared, one obtains $m^2_{\nu_0} = 0$ and $m^2_{\nu_1} = m^2_{\nu_2}$, where $m^2_{\nu_i} \equiv M_i^{(\nu)2}$ $(i = 0, 1, 2)$. Here, the smallness of neutrino masses is measured by the smallness of $\alpha^{(\nu)}$ and $\beta^{(\nu)}$ compared to $\alpha^{(e)}$ and $\beta^{(e)}$ that also are tiny or even zero (note that, however, for charged leptons $\mu^{(e)}$ as given in Eq. (61) is large). The eigenvectors $\vec{\phi}^{(\nu)} \equiv (\phi^{(\nu)}_{ij}\,;\; j = 0,1,2)$ corresponding to the eigenvalues $M^{(\nu)}_i$ evaluated in Eq. (66) can be presented in the form:

$$
\begin{array}{lll}
\phi^{(\nu)}_{00} = N^{(\nu)}_0 \;, & \phi^{(\nu)}_{10} = N^{(\nu)}_1 \dfrac{M^{(\nu)}_{01}}{M^{(\nu)}_1} \;, & \phi^{(\nu)}_{20} = N^{(\nu)}_2 \dfrac{M^{(\nu)}_{01}}{M^{(\nu)}_{21}} \;, \\[2ex]
\phi^{(\nu)}_{01} = 0 \;, & \phi^{(\nu)}_{11} = N^{(\nu)}_1 \;, & \phi^{(\nu)}_{21} = N^{(\nu)}_2 \dfrac{M^{(\nu)}_2}{M^{(\nu)}_{21}} \;, \\[2ex]
\phi^{(\nu)}_{02} = -N^{(\nu)}_0 \dfrac{M^{(\nu)}_{10}}{M^{(\nu)}_{12}} \;, & \phi^{(\nu)}_{12} = N^{(\nu)}_1 \dfrac{M^{(\nu)}_{21}}{M^{(\nu)}_1} \;, & \phi^{(\nu)}_{22} = N^{(\nu)}_2 \;,
\end{array}
\tag{67}
$$

where

$$
M^{(\nu)}_{01} = \frac{\sqrt{4}}{29}\alpha^{(\nu)}\, e^{i\varphi^{(\nu)}} = M^{(\nu)\,*}_{10} \;,\; M^{(\nu)}_{12} = \frac{\sqrt{192}}{29}(\alpha^{(\nu)} + \beta^{(\nu)})\, e^{i\varphi^{(\nu)}} = M^{(\nu)\,*}_{21}
\tag{68}
$$

with $\epsilon \equiv |M^{(\nu)}_{01}/M^{(\nu)}_{12}| = (1/\sqrt{48})\alpha^{(\nu)}/(\alpha^{(\nu)} + \beta^{(\nu)})$. Evidently, the diagonalizing matrix $\widehat{U}^{(\nu)} \equiv \left(U^{(\nu)}_{ij}\right)$ for neutrino mass matrix $\widehat{M}^{(\nu)} \equiv \left(M^{(\nu)}_{ij}\right)$ [compare Eq. (5)] is given as $\widehat{U}^{(\nu)} \equiv \left(\phi^{(\nu)}_{ji}\,;\; i,\,j = 0,1,2\right)$. If the ratio ϵ happens to be small enough, then from Eq. (67)

$$
\vec{\phi}^{(\nu)}_0 \sim \begin{pmatrix} 1 \\ 0 \\ O(\epsilon) \end{pmatrix} ,\; \vec{\phi}^{(\nu)}_1 \sim \begin{pmatrix} O(\epsilon) \\ 1 \\ O(1-\epsilon^2/2) \end{pmatrix} ,\; \vec{\phi}^{(\nu)}_2 \sim \begin{pmatrix} O(\epsilon) \\ O(1+\epsilon^2/2) \\ 1 \end{pmatrix} .
\tag{69}
$$

This shows that, in such a case, among the neutrino mass eigenstates ν_0, ν_1, ν_2, the first is dominated by the electron–neutrino weak–interaction state ν_e, while the second and third are nearly equal–weighted superpositions of muon–neutrino and tauon–neutrino weak–interaction states ν_μ and ν_τ. In fact, the relations

$$
\begin{pmatrix} \nu_0 \\ \nu_1 \\ \nu_2 \end{pmatrix} = \widehat{U}^{(\nu)\,\dagger} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}
\tag{70}
$$

and $\widehat{U}^{(\nu)\,\dagger} \equiv \left(U^{(\nu)\,*}_{ji}\right) = \left(\phi^{(\nu)\,*}_{ij}\right)$ imply the superpositions

$$
\nu_i = \phi^{(\nu)\,*}_{i0}\nu_e + \phi^{(\nu)\,*}_{i1}\nu_\mu + \phi^{(\nu)\,*}_{i2}\nu_\tau \;\; (i = 0,1,2)\,,
\tag{71}
$$

where coefficients form the vectors $\vec{\phi}_i^\dagger \equiv \left(\phi_{ij}^{(\nu)*}\,;\; j=0,1,2\right)^T$ with $\phi_{ij}^{(\nu)}$ as given in Eq. (67).

Concluding the last part concerning neutrinos, we can see that in the framework of our generic mass matrix (1) there are two, more or less natural, options for neutrino masses: when $\varepsilon^{(\nu)2} = 0$ or $\mu^{(\nu)} = 0$. If neutrino oscillations can take place (when $\alpha^{(\nu)}$ and $\beta^{(\nu)}$ are small, but at least one of them differs from zero), these options are:

$$m_{\nu_0} \simeq 0,\; m_{\nu_1} \simeq m_{\nu_2}/16.85 > 0 \tag{72}$$

(Eqs. (62) with $\nu_0 \simeq \nu_e$, $\nu_1 \simeq \nu_\mu$ and $\nu_2 \simeq \nu_\tau$) or

$$m^2_{\nu_0} = 0,\; m^2_{\nu_1} = m^2_{\nu_2} > 0 \tag{73}$$

(Eqs. 66) with $\nu_0 \simeq \nu_e$, but not $\nu_1 \simeq \nu_\mu$ nor $\nu_2 \simeq \nu_\tau$).

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