HOW TO INCLUDE A THREE-NUCLEON FORCE INTO FADDEEV EQUATIONS FOR THE 3N CONTINUUM: A NEW FORM

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A new, more efficient approach to include a three nucleon force into three-nucleon continuum calculations is presented. Results obtained in the new and our old approach are compared both for elastic nucleon-deuteron scattering as well as for the breakup process. The advantages of the new scheme are discussed.

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Three-nucleon forces (3NF) are getting more and more into the focus of few-nucleon studies, both theoretically [1] and experimentally [2]. Therefore optimal forms of equations should be used to handle those forces most efficiently.

One of the first 3N continuum calculations including a 3N force were performed in [3]. Separable nucleon-nucleon (NN) forces together with a $\pi - \pi$ exchange 3NF [5] averaged over spin and isospin degrees of freedom have been used. Thereby the standard Lovelace equations [6] have been generalized to include a 3NF. Starting from the AGS form [7] of the arrangement operators for identical particles and including a 3N force one gets [8]

$$U = PG_0^{-1} + (1 + P)t_4 + PtG_0U + (1 + P)t_4G_0tG_0U . \qquad (1)$$

Here G_0 is the free 3N propagator, t the NN t-matrix, t_4 the corresponding quantity now driven by the 3NF V_4 alone, and P the sum of a cyclical and

anticyclical permutation of three objects. That operator relation (1) has to be applied on a channel state $|\phi\rangle$ corresponding to Nd scattering, which is a product of the deuteron wave function and the momentum eigenstate of the projectile. Once U has been determined one gets the operator for the breakup process by quadrature. For more details and our notation in general we refer to [8].

Eq. (1) has been solved in [3] assuming NN t-matrices of finite rank. For the sake of a simple notation we just take the schematic form

$$t = |g> \tau < g| \tag{2}$$

which converts (1) into the structure of a single particle Lipmann–Schwinger equation

$$X = (Z^{(2)} + Z^{(3)}) + (Z^{(2)} + Z^{(3)})\tau X , \tag{3}$$

where

$$X \equiv \langle g|G_0 U G_0|g\rangle , \tag{4}$$

$$Z^{(2)} = \langle g|P G_0|\rangle , \tag{5}$$

and

$$Z^{(3)} \equiv \tfrac{1}{3}\langle g|G_0(1 + P)t_4(1 + P)G_0|g\rangle . \tag{6}$$

In addition to the well known particle exchange term $Z^{(2)}$ there is now a potential term $Z^{(3)}$ driven by the 3NF. We used

$$(1 + P)t_4 = t_4(1 + P) = \tfrac{1}{3}(1 + P)t_4(1 + P) . \tag{7}$$

In Ref. [3] Eq.(3) has been solved under the simplifying assumption $t_4 = V_4$. We would like to mention another approach in the context of finite rank NN forces [4]. There a simple ad hoc ansatz for t_4 has been chosen.

For general forces, which are not of finite rank, another set of equations within the Faddeev scheme was formulated [8–10] and has been heavily used in [11–13]. This is a set of two coupled equations in case of identical particles. They have the form

$$T = tP + tG_0T_4 + tPG_0T ,$$
$$T_4 = (1 + P)t_4 + (1 + P)t_4G_0T . \tag{8}$$

Once T and T_4 are found the operator for the 3N breakup process is given as

$$U_0 = (1 + P)T + T_4 \tag{9}$$

and the operator for elastic Nd scattering as

$$U = PG_0^{-1} + PT + T_4. \tag{10}$$

All these operator relations have to be applied onto the initial channel state $|\varphi\rangle$.

We have solved that coupled set [11–13] by expanding t_4, T and T_4 in powers of V_4, which leads to a recursive set of equations [9]. This is feasible and numerically precise but requires extensive resources in storage and computer time.

For the 3N bound state in a momentum space treatment an algorithm has been proposed [14], which has been taken up again in [15]. There the property of 3NF's applied up to now have been used, that it can be split naturally into 3 parts, each one of which is symmetrical under exchange of 2 particles, like the pair forces. For the $\pi-\pi$ exchange 3NF for instance that splitting occurs automatically, since there are three possibilities for choosing a nucleon which undergoes the (off-shell) $\pi-N$ scattering. Using that decomposition one can combine each of the three NN forces with a corresponding part of the 3NF having the same symmetry under particle exchanges. Then the derivation of the Faddeev equation from the Schrödinger equation follows exactly the same line as for NN forces alone and one arrives at one Faddeev equation using the identity of the particles. For the 3N bound state it has the form

$$\psi = G_0 t P \psi + G_0(1 + tG_0)V_4^{(1)}(1 + P)\psi. \tag{11}$$

This equation has been used in [15] and later work, for instance in [16].

Based on that formal insight it is obvious that a corresponding form should exist for 3N scattering. The elaboration of that expectation is the content of the present work.

Let us split the 3NF into 3 parts

$$V_4 = \sum_{i=1}^{3} V_4^{(i)} \tag{12}$$

as described above and let us arrange the 3N Schrödinger equation in integral form as

$$\Psi = G_0 \sum_{i=1}^{3} (V_i + V_4^{(i)})\Psi. \tag{13}$$

We used the usual "odd man out" notation for the NN forces. This is a correct homogeneous integral equation for a scattering state, which is initiated in

the nucleon-deuteron (Nd) channel [8]. Then splitting Ψ into three Faddeev components ψ_i one gets for the first component, for instance,

$$\psi_1 = G_0(V_1 + V_4^{(1)})(\psi_1 + \psi_2 + \psi_3) . \tag{14}$$

The NN t-matrix t_1 related to V_1 is then introduced in the normal manner and one ends up easily with

$$\psi_1 = \phi_1 + G_0 t_1(\psi_2 + \psi_3) + (1 + G_0 t_1)G_0 V_4^{(1)}(\psi_1 + \psi_2 + \psi_3) . \tag{15}$$

Here ϕ_1 is the initial channel state, as described above.

For identical particles

$$\psi_2 + \psi_3 \equiv P\psi_1 \tag{16}$$

and dropping the index 1 we get

$$\psi = \phi + G_0 t P\psi + (1 + G_0 t)G_0 V_4^{(1)}(1 + P)\psi . \tag{17}$$

This is the Faddeev equation for 3N scattering initiated in a Nd channel.

Using $G_0 t = GV$, where $G \equiv (E - H_0 - V)^{-1}$, one can extract the asymptotic behaviour in the elastic and breakup channels [17]. Thus the amplitude accompanying the familiar outgoing wave in the hyperradius is

$$\begin{aligned}\tilde{T} &= (1 + tG_0)VP\psi + (1 + tG_0)V_4^{(1)}(1 + P)\psi \\ &= tP\psi + (1 + tG_0)V_4^{(1)}(1 + P)\psi . \end{aligned} \tag{18}$$

The full breakup amplitude constructed from all three Faddeev amplitudes is then

$$U_0 = (1 + P)\tilde{T} . \tag{19}$$

Now we see directly comparing (17) and (18) that

$$\psi = \phi + G_0\tilde{T} \tag{20}$$

and, therefore,

$$\begin{aligned}\tilde{T} &= tP\phi + (1 + tG_0)V_4^{(1)}(1 + P)\phi \\ &\quad + tPG_0\tilde{T} + (1 + tG_0)V_4^{(1)}(1 + P)G_0\tilde{T} . \end{aligned} \tag{21}$$

This is the searched for single equation, which replaces the coupled set (8). For $V_4^{(1)} = 0$ it reduces to the form, which we always use in solving the 3N continuum for NN forces only [12].

The asymptotic behaviour of (17) in the elastic channel yields the operator for the elastic process

$$U = VP\psi + V_4^{(1)}(1 + P)\psi. \tag{22}$$

The elastic amplitude arises by projecting U from the left with the channel state $< \phi|$, which carries the correct energy. Then V in the driving term can be replaced by G_0^{-1}. Using that and the relation (20) we find

$$U = PG_0^{-1} + P\tilde{T} + V_4^{(1)}(1 + P)\phi + V_4^{(1)}(1 + P)G_0\tilde{T}. \tag{23}$$

This is the equation which provides U once $\tilde{T}$ has been found.

It remains to exhibit the formal connection to the previously used forms (8)-(10).

We use now the following property of V_4 underlying its decomposition used throughout:

$$(1 + P)V_4 = (1 + P)V_4^{(1)}(1 + P). \tag{24}$$

Let us then define

$$\hat{V}_4 \equiv V_4^{(1)}(1 + P), \tag{25}$$

and

$$\hat{t}_4 = \hat{V}_4 + \hat{V}_4 G_0 \hat{t}_4. \tag{26}$$

Then it follows

$$\begin{aligned}
(1 + P)t_4 &\equiv (1 + P)[V_4 + V_4 G_0 V_4 + V_4 G_0 V_4 G_0 V_4 + \cdots] \\
&= (1 + P)V_4^{(1)}(1 + P) + (1 + P)V_4^{(1)}(1 + P)G_0 V_4 + \cdots \\
&= (1 + P)[V_4^{(1)}(1 + P) + V_4^{(1)}(1 + P)G_0 V_4^{(1)}(1 + P) + \cdots \\
&= (1 + P)[\hat{V}_4 + \hat{V}_4 G_0 \hat{V}_4 + \cdots] \\
&\equiv (1 + P)\hat{t}_4. \tag{27}
\end{aligned}$$

Further we introduce

$$\begin{aligned}
\tau_4 &\equiv \hat{t}_4 + \hat{t}_4 G_0 T \\
&= \hat{V}_4 + \hat{V}_4 G_0 \tau_4 + \hat{V}_4 G_0 T. \tag{28}
\end{aligned}$$

Applying $(1 + P)$ from the left we get

$$\begin{aligned}
(1 + P)\tau_4 &= (1 + P)t_4 + (1 + P)t_4 G_0 T \\
&= T_4 \tag{29}
\end{aligned}$$

according to (8).

Let us now add the information from the first equation of the set (8) to get

$$T + \tau_4 = tP + tG_0(1 + P)\tau_4 + tPG_0T + \hat{V}_4 + \hat{V}_4G_0\tau_4 + \hat{V}_4G_0T . \quad (30)$$

Using again (28) yields

$$\begin{aligned}
\tilde{T} &= tP + \hat{V}_4 + tG_0P\tilde{T} + tG_0\hat{V}_4 + tG_0\hat{V}_4G_0\tilde{T} + \hat{V}_4G_0\tilde{T} \\
&= tP + (1 + tG_0)\hat{V}_4 + tG_0P\tilde{T} + (1 + tG_0)\hat{V}_4G_0\tilde{T}
\end{aligned} \quad (31)$$

with

$$\tilde{T} \equiv \tau_4 + T . \quad (32)$$

This Eq. (31) is identical to (21).

Now the full breakup operator as given in (9) is

$$\begin{aligned}
U_0 &= (1 + P)T + T_4 \\
&= (1 + P)T + (1 + P)\tau_4 \\
&= (1 + P)\tilde{T}
\end{aligned} \quad (33)$$

as required by (19).

Finally the operator for elastic scattering (10) can be rewritten, using (29) and (28) as

$$\begin{aligned}
U &= PG_0^{-1} + PT + T_4 \\
&= PG_0^{-1} + P\tilde{T} + \tau_4 \\
&= PG_0^{-1} + P\tilde{T} + \hat{V}_4 + \hat{V}_4G_0\tilde{T}
\end{aligned} \quad (34)$$

which is identical to (23). This concludes the verification of the equivalence of the old and new formulations. In practice we use the second form of Eq. (34) with the quantity τ_4 to evaluate U, since it occurs naturally in Eq. (21) as an intermediate amplitude.

We developed a computer code for the new forms (21), (19) and (23). This has significant practical advantages over the previous formulation. In the new approach one is simply iterating (21), typically 18 times for total angular momentum and parity $1/2^+$ followed by a Padé summation. The number of iterations for $1/2^-$ is smaller and decreases further with increasing angular momentum. In the old scheme the set (8) expanded in power of V_4 (see [11]) had to be iterated. Since typically for $1/2^+$ 10 powers of V_4 had to be kept and for each order of V_4 one has to iterate the corresponding Faddeev equations typically 13 times, the number of iterations in the new approach is drastically reduced. However, in the new approach the kernel in

Eq. (21) includes V_4 dependent parts and requires therefore more CPU time for its evaluation. Altogether we found in a realistic calculation a reduction of about a factor 4 in the CPU time for the new approach.

We demonstrate now the feasibility of the new formalism and the numerical accuracy achieved with that form by showing a few 3N scattering observables evaluated with the old and new code. These are fullfledged calculations based on the NN potential CD Bonn (np) [18] kept up to $j_{max} = 2$ (j is the total 2-body angular momentum) and the Tucson–Melbourne $\pi - \pi$ exchange 3NF [5], also kept up to $j_{max} = 2$. The cut-off parameter in the 3NF has been chosen as $\Lambda = 5.8\ m_\pi$.

Fig. 1 shows two spin observables in elastic nd scattering at 10.3 MeV. We have chosen cases where the effect of the 3NF is large. (The elastic differential cross section shows no effect of the 3NF at all.) The two curves with the 3NF included and evaluated in the old and new scheme overlap perfectly. This is generally true for all elastic observables.
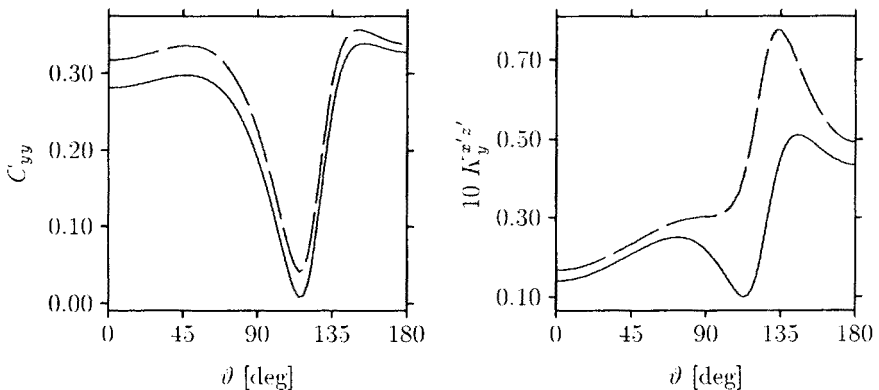

Fig. 1. Spin correlation coefficient C_{yy} and neutron to deuteron spin transfer coefficient $K_y^{x'z'}$ at $E_N^{lab} = 10.3$ MeV. The solid curve is without 3NF and the long and short dashed curves are with 3NF evaluated with the old and new approach, respectively.

For the breakup process we show again two examples in Fig. 2 with large 3NF effects. One is a breakup cross section under np QFS conditions and the other a deuteron analyzing power under the same conditions. For that very small observable A_y one can see tiny differences between the predictions of the old and new scheme. This is the worst case we found by checking in the standard breakup configurations cross sections and all analyzing powers, spin correlation coefficients and vector spin transfer coefficients.
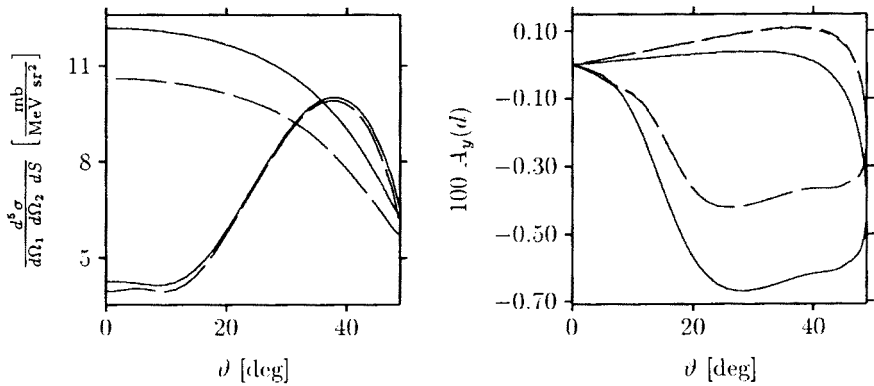
Fig. 2. The differential cross section and the deuteron analyzing power at $E_N^{\text{lab}} = 10.3$ MeV under np QFS conditions as a function of the scattering angle of one nucleon. Curves as in Fig. 1

Summarizing, we have introduced a new single equation for 3N scattering including a 3NF. It is a direct generalisation of what has already been used for the 3N bound state. It is significantly faster to solve than our previous form (8), used up to now [12]. We demonstrated the feasibility and accuracy of the new form.

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