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OPTIMAL ARCHITECTURES FOR STORAGE OF "SPATIALLY" CORRELATED DATA IN NEURAL NETWORK MEMORIES*

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Using replica approach we investigate storage capacity for "spatially" correlated patterns in diluted attractor neural networks. We investigate analog *clipped-sign* networks which are generalizations of the standard networks of the Hopfield type. We consider two kinds of dilution: a *band*, and a *random* (periodic) ones. "Spatial" associations of data significantly improve the storage properties of the network. The band-type dilution affects the critical capacity much weaker than the random one, especially when the stored data are strongly correlated.

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1. Introduction

The problem of critical storage capacity in neural network memories has attracted much attention recently. A new method of investigation was proposed by E. Gardner in 1988 [1]. Gardner's idea has been greatly extended since then (see, for example, Refs. [2–9]). Only recently, however, it has been applied to the problem of storage of correlated data (see Refs. [10–13]). In this paper we continue the line of research of Refs. [10, 11] and treat the problem of optimal storage capacity of *"spatially" correlated* patterns in a *diluted* Hopfield neural network. This problem for fully–connected network was studied in Refs. [11, 14, 15]. In those papers the assumption was made that the states of neurons, and consequently patterns stored in the network take binary values. This leads to serious difficulties in performing the proper averages over the memorized data. Only the approximate result for the case of short range correlations of patterns could be derived. Here we overcome these difficulties by considering analog attractor networks that are simple generalizations of the Hopfield model and by assuming that patterns to be stored are Gaussian distributed random variables. In particular, we treat here analog *clipped–sign* (CS) networks that have very similar properties to the standard Hopfield one. The variables describing stored patterns in the CS networks can, however, take any real values. We choose them to be statistically correlated and associated through the *correlation* matrix $\widehat{C}$. There are many reasons why such correlated data are of general interest. Obviously, visual data encountered in our everydays experience are of that sort. Advanced theories of visual information processing stress that very often visual patterns consist of locally correlated blocks (see, for instance, Ref. [16]). Local organization of visual data is discussed in cognitive psychology and cognitive science [17].

The main aim of this paper is to study storage of "spatially" correlated patterns in diluted networks and to look for optimal network architectures at a given level of dilution. Dilution seems to be a very important and general property of neural networks. Biological structures in the brain are very strongly diluted (see, for example, Refs. [18, 19]). From the viewpoint of applied sciences, learning in a fully–connected network with N nodes requires $N(N-1)$ connections, *i.e.*, is very exhaustive. In information processing applications, it might be difficult even to store such large connection matrices in a computer memory. Thus it is obvious that one should study diluted networks, and search for optimal means of dilution that would not degrade the desired properties of the network. An important step in this direction has been accomplished in Refs. [20,21]. On the other hand there are suggestions from cognitive science that the structure of the information processing network reflects the structure of processed data. Therefore, one expects that optimal architectures for networks should depend on correlations in the data set.

In the present paper we show that this is indeed the case for "spatially" (*i.e.*, locally) correlated data. We compare storage capacity of two kinds of networks with *band* and *random* dilution. We show that for the first case the storage capacity α depends simply in a linear way on the dilution ratio B. For the second case such dependence is much more complex but evidently random dilution degrades storage capacity much more drastically. This result illustrates in a quantitative way the intuitive expectation that local correlations of data induce local associations of synaptic connection, and thus select optimal means of dilution.

2. Fractional volume in the interaction space

Let us first review the result of Gardner's method applied to the problem of fully-connected discrete networks. In the standard approach, one considers a network of the Hopfield type [2,22,23] with N discrete nodes $\{\sigma_j = \pm 1\}$ which follow deterministic dynamics described by a simple updating rule

$$\sigma_i(t + \Delta t) = \mathrm{sign}\left[\sum_{j \neq i} J_{ij}\sigma_j(t)\right], \tag{1}$$

with $\{J_{ij}\}$ denoting the synaptic connection matrix. The rows of this matrix are, for each i, normalized according to the spherical constraint

$$\sum_{j \neq i} J_{ij}^2 = N. \tag{2}$$

One then defines, following Gardner [1], the fractional volume in the interaction space

$$V_T = \frac{\prod_i \left[\int \prod_{j\neq i} dJ_{ij}\, \delta\left(\sum_{j\neq i} J_{ij}^2 - N\right) \prod_\mu \Theta\left(\xi_i^\mu \sum_{j\neq i} \frac{J_{ij}}{\sqrt{N}} \xi_j^\mu - \kappa\right)\right]}{\prod_i \left[\int \prod_{j\neq i} dJ_{ij}\, \delta\left(\sum_{j\neq i} J_{ij}^2 - N\right)\right]}, \tag{3}$$

where the set $\left\{\xi_j^\mu = \pm 1\right\}$ denotes the so-called patterns stored in the memory of the considered network, *i.e.*, the stationary states of the dynamics (1). Note that the index $j = 1, \ldots, N$ enumerates the sites, whereas $\mu = 1, \ldots, \alpha N$ – the patterns (memories), and α is a storage ratio (*i.e.*, a number of patterns stored per neuron). Note also, that for the attractor Hopfield networks with binary nodes, the stored patters must necessarily be also binary. The quantity κ designates a stability parameter which determines the size of the basins of attraction of the stored memory configuration $\left\{\xi_j^\mu\right\}$. One sees that $V_T = \prod_i V_i$, where V_i are volumes referring to the node i.

3. Analog clipped–sign networks

It is easy to generalize the results of discussed above to the case of analog networks which we call *clipped–sign* (CS) ones. To this aim we consider a network with N *analog* nodes $\{\sigma_j\}$, where σ_j, $j = 1, \ldots, N$, are real numbers. We define the deterministic dynamics now by

$$\sigma_i(t + \Delta t) = \sigma_i(t)\,\mathrm{sign}\left[\sigma_i(t)\sum_{j\neq i} J_{ij}\sigma_j(t) - \kappa\right]. \tag{4}$$

Note that this dynamics allows for stationary states, *i.e.*, describes an attractor neural network. These stationary patterns $\left\{\xi_j^\mu\right\}$ are in general defined as sequences of *real* numbers such that

$$\mathrm{sign}\left[\xi_i^\mu \sum_{j\neq i} J_{ij}\xi_j^\mu - \kappa\right] = 1, \tag{5}$$

for $i, j = 1, \ldots, N$, $\mu = 1, \ldots, \alpha N$. One can formulate Gardner's program for the CS networks in the full analogy with Eq. (3). In particular we can ask what is the storage capacity of the CS networks for a set of patterns with given statistical properties. Obviously, a CS network that memorizes a given set of patterns will typically store much more patterns, *i.e.*, have much more stationary states. The standard Hopfield networkhas also spurious memories, but the proliferation of the stationary states for CS networks is much more dramatic. For instance, for $\kappa = 0$ the stationary states of the dynamics (4) are *rays* in the vector space of patterns, *i.e.*, if ξ_j^μ is a stationary configuration then $h\xi_j^\mu$, with arbitrary h, is also. Nevertheless, it is legitimate to ask questions, whether a given statistically sample of patterns may be stored in the CS networks, and what is the critical size of such a sample, or in other words what is the critical capacity of the network in such a case.

In particular, if the memorized patterns are chosen to be statistically independent and binary, the critical storage capacity of the CS network is exactly the same as for the standard Hopfield model. In general, one can consider any kind of real (analog or discrete) patterns $\left\{\xi_j^\mu\right\}$, and use the same Eq. (3) to calculate the fractional volume in the interaction space for CS networks. In this sense, CS networks provide an elegant generalization of the standard Hopfield model with discrete neurons.

4. Storage of "spatially" correlated data

We shall deal with "spatially" correlated patterns, and require that the proper *quenched* averages over the $\left\{\xi_j^\mu\right\}$ fulfil

$$\left\langle\!\left\langle \xi_j^\mu \right\rangle\!\right\rangle = 0, \tag{6}$$

and

$$\left\langle\!\left\langle \xi_j^\mu \xi_{j'}^{\mu'} \right\rangle\!\right\rangle = \delta_{\mu\mu'}\, C_{jj'} \equiv \delta_{\mu\mu'}\, e^{-b|j-j'|}, \tag{7}$$

where $C_{jj'}$ denotes elements of a *correlation matrix.* The specific exponential form of those elements (7) is quite generic for correlated patterns [10, 11, 13]. The parameter b can be viewed as an inverse of the *correlation length*, $b = 1/L_c$. Exponentially correlated patterns may be easily generated in numerical simulations.

In this paper we assume that the patterns $\left\{\xi_j^\mu\right\}$ are not binary, but rather are distributed according to a correlated Gaussian distribution

$$\Pr\left(\left\{\xi_j^\mu\right\}\right) = \frac{\left(\prod_{\mu,j} d\xi_j^\mu\right)}{\left[(2\pi)^N \det \widehat{C}\right]^{\frac{\alpha N}{2}}} \exp\left\{-\frac{1}{2}\sum_{\mu,j,j'} \xi_j^\mu \left(C^{-1}\right)_{jj'} \xi_{j'}^\mu\right\}, \tag{8}$$

where $C_{jj'}$ are the elements of the positively defined association matrix $\widehat{C}$. This assumption allows one to perform all the *quenched* averages over the patterns exactly.

As usual in the theory of disordered systems we need to calculate the quenched average of $(\ln V_T)$. For $N \to \infty$ due to the self-averaging: $\ln V_T = \sum_i \ln V_i \cong N\langle\!\langle \ln V_i \rangle\!\rangle$, and the calculation reduces to the evaluation of the fractional volume at an arbitrarily chosen node i. This is achieved with the help of the replica method $\langle\!\langle \ln V_i \rangle\!\rangle = \lim_{n\to 0}(\langle\!\langle V_i^n \rangle\!\rangle - 1)/n$. We first calculate $\langle\!\langle V_i^n \rangle\!\rangle$ for natural n and then perform an analytic continuation to $n = 0$.

To this aim, we introduce, as usual, the order parameters

$$q^{\alpha\beta} = \frac{1}{N} \sum_{j\neq i, j'\neq i} C_{jj'} J_{ij}^\alpha J_{ij'}^\beta, \tag{9}$$

$$Q^\alpha = \frac{1}{N} \sum_{j\neq i, j'\neq i} C_{jj'} J_{ij}^\alpha J_{ij'}^\alpha, \tag{10}$$

$$m^\alpha = \frac{1}{\sqrt{N}} \sum_{j\neq i} C_{ij} J_{ij}^\alpha \tag{11}$$

and their conjugated counterparts $\left\{f^{\alpha\beta}\right\}$, $\{F^{\alpha}\}$, and $\{M^{\alpha}\}$. Note that $\alpha,\beta = 1,\ldots,n$ designate here the replica indices [1]. The average of V^n then reads

$$\langle\langle V^n\rangle\rangle = \mathcal{C}\int \mathcal{D}E^{\alpha}\mathcal{D}q^{\alpha\beta}\mathcal{D}Q^{\alpha}\mathcal{D}f^{\alpha\beta}\mathcal{D}F^{\alpha}\mathcal{D}m^{\alpha}\mathcal{D}M^{\alpha}e^{-Nn\mathcal{F}(E,q,Q,f,F,m,M)}, \quad (12)$$

where $\{E^{\alpha}\}$ assure the spherical normalization constraint (2), and $\mathcal{C}$ is a constant.

In order to calculate the free-energy density $\mathcal{F}$, that is to perform the integrals over $\left\{J_{ij}^{\alpha}\right\}$ analytically, we assume that neurons are located on a torus (see Ref. [11]). Note that in such a case the correlation matrix can be diagonalized using a Fourier transform.

To proceed further one makes a replica-symmetric Ansatz $q^{\alpha\beta} = q$, $Q^{\alpha} = Q$, $r^{\alpha} = r$, $R^{\alpha} = R$, $f^{\alpha\beta} = f$, $F^{\alpha} = F$, $m^{\alpha} = m$, $M^{\alpha} = M$, and $E^{\alpha} = E$, for each α, β. The quantity $\mathcal{F}$ thus takes the following form

$$\mathcal{F} = -\alpha G_1 - \frac{1}{N}G_2 - \frac{1}{2}fq + \frac{1}{2}FQ + Mm - \frac{1}{2}E, \quad (13)$$

where the functions G_1, G_2 are given by

$$G_1 = \int Dt\ln H\left[\frac{\kappa - m + t\sqrt{q}}{\sqrt{Q-q}}\right] \quad (14)$$

$$\begin{aligned}
G_2 &= -\frac{1}{2}\sum_k \ln(E + fC_k - FC_k) + \frac{1}{2}\sum_k \frac{fC_k}{E + fC_k - FC_k} \\
&+ \frac{1}{2}M^2\sum_k \frac{d_k}{E + fC_k - FC_k} \\
&\equiv -\frac{N}{2}\,\langle\langle \ln[E + fC(\phi) - FC(\phi)]\rangle\rangle_{\phi} \\
&+ \frac{N}{2}\left\langle\!\!\left\langle\frac{fC(\phi)}{E + fC(\phi) - FC(\phi)}\right\rangle\!\!\right\rangle_{\phi} \\
&+ \frac{N}{2}M^2\left\langle\!\!\left\langle\frac{d(\phi)}{E + fC(\phi) - FC(\phi)}\right\rangle\!\!\right\rangle_{\phi}. \quad (15)
\end{aligned}$$

Here $k = 1,\ldots,N$ denotes the Fourier index, whereas $\{C_k\}$ are eigenvalues of the correlation matrix. In the thermodynamic limit $N \to \infty$ (see Ref. [11] for more details) we replace sums over the Fourier components by integrals $\sum_k \to \frac{1}{2\pi}\int_0^{2\pi} d\phi$. Thus, $\langle\langle(\cdot)\rangle\rangle_{\phi} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi\,(\cdot)$ defines the average with respect to the matrix $\widehat{C}$ eigenvalue spectrum [11,12,24]. The eigenvalues of $\widehat{C}$ reads

$$C(\phi) = \frac{1 - e^{-2b}}{1 - 2e^{-b}\cos\phi + e^{-2b}} \quad (16)$$

for $\phi \in [0, 2\pi]$. On the other hand $d(\phi)$ is given by

$$d(\phi) = \left[\frac{2e^{-b}\cos\phi - 2e^{-2b}}{1 - 2e^{-b}\cos\phi + e^{-2b}} \right]^2 . \tag{17}$$

Note that this result holds provided the correlation length in not extensive, *i.e.*, $L_c/N = 1/(bN) \to 0$ as $N \to \infty$. The Gaussian measure

$$Dt = \frac{dt}{\sqrt{2\pi}} e^{-t^2/2}, \tag{18}$$

and the function H is defined as usual

$$H(x) = \int_x^\infty Dt. \tag{19}$$

The integral (12) can be evaluated using a saddle–point technique. After some calculations (see Refs. [11, 15]) we obtain seven saddle–point equations which we then solve numerically. From the condition $q = Q$ we determine the critical capacity of the network. It is worth stressing, however, that the saddle–point equations can also be used in the perturbative approach for the case of a short correlation length (*i.e.*, when $C_{jj'} \simeq \delta_{jj'}$). The approximate value of the storage ratio α_c for the minimal stability parameter $\kappa = 0$ (see Refs. [11, 14]) is

$$\alpha_c \cong 2 + 2 \frac{\left\langle\!\left\langle \frac{d(\phi)}{C(\phi)} \right\rangle\!\right\rangle_\phi}{\left\langle\!\left\langle \frac{1}{C(\phi)} \right\rangle\!\right\rangle_\phi} . \tag{20}$$

This formula holds for the arbitrary positively defined matrix $\widehat{C}$.

The results of numerical analysis for the fully–connected network are plotted in Fig. 1. Gardner's original curve is recovered in the limit $b \to \infty$, *i.e.*, when correlations vanish. It is worth stressing that for small values of the parameter b (*i.e.*, for the large correlation length L_c) the critical storage ratio greatly exceeds Gardner's result. In the limit $b \to 0$ the quantity α_c (for $\kappa = 0$) tends to infinity. Note that this result (which is caused by approximations that cannot be applied in the limit $b \to 0$) does not have sense for binary networks, but is legitimate for the analog CS systems considered here. One should also notice that a similar situation ($\alpha_c \to \infty$ for $\kappa \to 0$) occurs in the case of strongly biased (discrete) patterns [1], and for "semantically" correlated binary data in a perceptron [10, 11]. In the Hopfield network with "semantically" associated patterns, however, the capacity remains finite (see Ref. [11]).

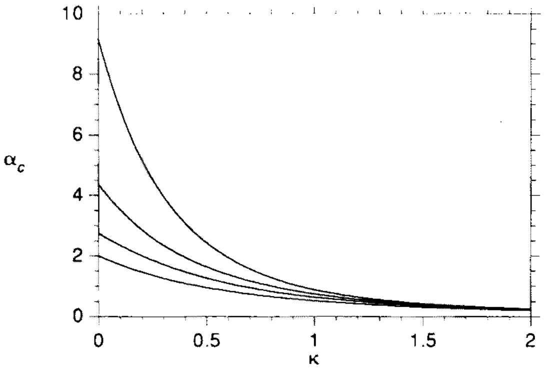

Fig. 1. Critical storage ratio α_c as a function of the stability parameter κ for $b = 0.5, 0.6, 0.8$, and $b \to \infty$ (from uppermost to lowest curve, respectively).

5. Band dilution

Let us turn now to a discussion of the main subject of this paper – diluted networks. First we consider a *band* dilution. In that case we set $J_{ij} = 0$ for all j's which fulfill $|i - j| \leq N(1 - B)$, with $0 \leq B \leq 1$. We also modify the spherical normalization (2) to the form

$$\sum_{j \neq i} J_{ij}^2 = AN \tag{21}$$

for arbitrary positive constant A. It is easy to observe that the free–energy density $\mathcal{F}$ remains invariant with respect to the transformation $E \to E/A$, $F \to F/A$, $f \to f/A$, $M \to M/A$ and $\alpha \to \alpha/B$ $(B \neq 0)$. This fact implies simply that the storage ratio

$$\alpha_c(\kappa) = B\alpha_c'(\kappa)\,, \tag{22}$$

independently of A, where α_c' is the critical capacity for $B = 1$.

6. Random dilution

One can also dilute the network in another way. A *random* dilution can be realized in a following manner: For a given dilution ratio B, if $B \in \left[\frac{1}{2}, 1\right]$, one cuts $(1/[1 - B])_I$–*th* successive synapse J_{ij}, where the function $(\cdot)_I$ takes the integer value closest to its argument. In another words, $J_{ij} \neq 0$ for $j = 1, \ldots, \ell - 1, \ell + 1, \ldots, \left(\frac{1}{1-B}\right)_I + \ell - 1, \left(\frac{1}{1-B}\right)_I + \ell + 1, \ldots, \left(\frac{2}{1-B}\right)_I + \ell -$

$1, \left(\frac{2}{1-B}\right)_I + \ell + 1, \ldots,$ *etc.*, where $\ell \in \left\{1, \ldots, \left(\frac{1}{1-B}\right)_I\right\}$. This procedure is then averaged over all possible choices of the starting point of the dilution, ℓ. A very similar procedure is used for $B \in \left[0, \frac{1}{2}\right]$, but here one drops every $(1/B)_I$*-th* successive coupling and sets all the others to zero. In this case $J_{ij} \neq 0$ for $j = \ell, \left(\frac{1}{B}\right)_I + \ell, \left(\frac{2}{B}\right)_I + \ell, \ldots,$ *etc.*, where $\ell \in \left\{1, \ldots, \left(\frac{1}{B}\right)_I\right\}$.

Random dilution leads to the following modification of the eigenvalue spectrum of the correlation matrix $\widehat{C}$ (see Eq. (16)) and the quantity d (Eq. (17)). For $B \geq 1/2$

$$
\begin{aligned}
C(\phi) &= \frac{1 - e^{-2b}}{1 - 2e^{-b}\cos\phi + e^{-2b}} \\
&- (1-B)\left\{\frac{1 - e^{-\frac{2b}{1-B}}}{1 - 2e^{-\frac{b}{1-B}}\cos\left[\frac{\phi}{1-B}\right] + e^{-\frac{2b}{1-B}}}\right. \\
&\left. + \sum_{r=1}^{\left(\frac{B}{1-B}\right)_I} \frac{e^{-br}\left(2\cos[r\phi] - 2e^{-\frac{b}{1-B}}\cos\left[\left(\frac{1}{1-B} - r\right)\phi\right]\right)}{1 - 2e^{-\frac{b}{1-B}}\cos\left[\frac{\phi}{1-B}\right] + e^{-\frac{2b}{1-B}}}\right\}, \qquad (23) \\
d(\phi) &= \left\{B\frac{2e^{-b}\cos\phi - 2e^{-2b}}{1 - 2e^{-b}\cos\phi + e^{-2b}}\right. \\
&\left. - (1-B)\sum_{r=1}^{\left(\frac{1}{1-B}\right)_I} \frac{e^{-br}\left(2\cos[r\phi] - 2e^{-\frac{b}{1-B}}\cos\left[\left(\frac{1}{1-B} - r\right)\phi\right]\right)}{1 - 2e^{-\frac{b}{1-B}}\cos\left[\frac{\phi}{1-B}\right] + e^{-\frac{2b}{1-B}}}\right\}^2, \qquad (24)
\end{aligned}
$$

and for $B \leq 1/2$

$$
\begin{aligned}
C(\phi) &= B\left\{\frac{1 - e^{-\frac{2b}{B}}}{1 - 2e^{-\frac{b}{B}}\cos\left[\frac{\phi}{B}\right] + e^{-\frac{2b}{B}}}\right. \\
&\left. + \sum_{r=1}^{\left(\frac{1-B}{B}\right)_I} \frac{e^{-br}\left(2\cos[r\phi] - 2e^{-\frac{b}{B}}\cos\left[\left(\frac{1}{B} - r\right)\phi\right]\right)}{1 - 2e^{-\frac{b}{B}}\cos\left[\frac{\phi}{B}\right] + e^{-\frac{2b}{B}}}\right\}, \qquad (25) \\
d(\phi) &= \left\{B\sum_{r=1}^{\left(\frac{1}{B}\right)_I} \frac{e^{-br}\left(2\cos[r\phi] - 2e^{-\frac{b}{B}}\cos\left[\left(\frac{1}{B} - r\right)\phi\right]\right)}{1 - 2e^{-\frac{b}{B}}\cos\left[\frac{\phi}{B}\right] + e^{-\frac{2b}{B}}}\right\}^2. \qquad (26)
\end{aligned}
$$

Numerical solutions of the corresponding saddle-point equations are presented in Fig. 2. Here we have plotted the relative values of the critical storage capacity $\alpha_c^r(B) = \alpha_c(B)/\alpha_c(B=1)$ as a function of the dilution ratio B for several values of the association strength b. It is easy to see that an increase of the correlation length L_c decreases significantly the relative storage properties of randomly diluted networks. This is caused by the fact that random dilution destroys strong local associations between the synaptic couplings. Note that J_{ij} might be of the order of $\sqrt{N}$ for j close to i. On the other hand, the straight "diagonal" line in Fig. 2 corresponds to the case of the band dilution. Here the relative storage capacity $\alpha_c^r(\kappa)$ does not depend on the correlation parameter b at all. All the curves in Fig. 2 are plotted for the stability parameter $\kappa = 1$. For the other values of κ the situation is, of course, qualitatively similar.

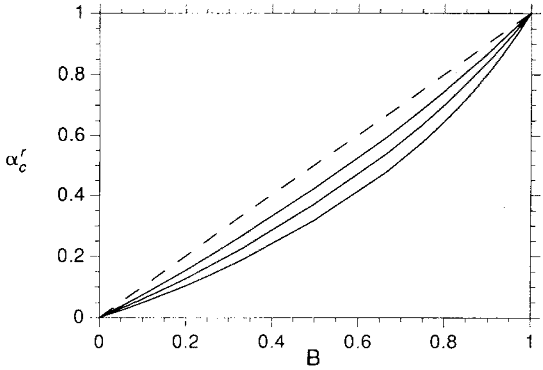

Fig. 2. Relative critical storage ratio α_c^r as a function of the parameter B in the case of random dilution for $b = 0.5$, 0.6, and 0.8 (lower to upper solid curve, respectively). All curves are plotted for $\kappa = 1$. The dashed line corresponds to band dilution for arbitrary (non-negative) b and to the random clipping for $b \to \infty$.

Summarizing, we have analyzed storage capacity of diluted neural networks that store "spatially" correlated patterns. We considered a new type of analog systems, the so-called *clipped-sign* networks which provide a generalization of the standard Hopfield model. The main result of the paper concerns optimal architectures for CS networks with "spatially" correlated stored patterns. We have shown that *band* dilution in CS networks changes the capacity proportionally to the dilution ratio whereas *random* dilution affects it in a more complex manner, but much more significantly. These results clearly indicates quantitative advantage of the band dilution for the storage of "spatially" correlated data.

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