OPEN PARABOSONIC STRING BRST TRANSFORMATIONS AND THE CRITICAL DIMENSION*

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Open parabosonic string BRST transformations are constructed and the critical space-time dimension is derived.

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1. Introduction

In the usual procedure of canonical quantization, to transit from the classical to quantum theory, one has to consider the canonical coordinates q_i and p_i $(i = 1 - 3)$ as Hilbert space operators satisfying the so called canonical commutation relations

$$[q_i, q_j] = [p_i, p_j] = 0 \quad [q_i, p_j] = i\delta_{ij}\,. \tag{1.1}$$

It turns out that Eqs (1.1) are sufficient but by no means necessary to guarantee the Heisenberg equations of motion [1]

$$-i\frac{\partial A}{\partial q_\mu} = [A, P_\mu] \qquad (\mu = 0 - 3)\,, \tag{1.2}$$

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where A is an arbitrary linear operator (here $P_0 = E$ (energy) and $q = t$ (time)). Note that Eqs (1.2) are necessary to provide a unified description of the so called wave-particle duality. Thus, one has to take them as the most fundamental and therefore the canonical variables q_i and p_j satisfy bilinear (like Eq. 1.1) or trilinear commutation relations, depending on the generalization of the quantization which is characterized by a given parameter Q called the order of the quantization [2–9].This procedure is called paraquantization.

One may wonder why there is such necessity for this generalized procedure of quantization and unnecessary mathematical complication . It is clear with regards to the present day unsolved problems that beset quantum field theories like the non-renormalizability of quantum gravity and the compactification problems in the superstring theories, that one has to improve or completely reformulate the foundations of the existing theories and must know their structures in a more general context in order to have a better understanding [6].

In fact, as it was shown in Refs [10–13], the paraquantization of bosonic string and superstring leads to a relationship between the order of paraquantization Q and the critical dimension D ($D = 2 + 24/Q$ and $D = 2 + 8/Q$). Moreover, for the closed parabosonic string, the spectrum was shown to contain a massless spin two para-bosonic particle which may be identified as the graviton.

The aim of this paper is to derive the paraquantum BRST transformations and the corresponding charge for an open bosonic string and show that the nilpotency property applied to the Green components of the BRST charge leads to $D = 2 + 24/Q$. In Section 2, we present the formalism and in Section 3 we draw our conclusions.

2. Formalism

The classical Polyakov Lagrangian for an open bosonic string is given by [10, 11]:

$$L_{\mathrm{Cl}} = -\frac{1}{2\alpha'}\tilde{g}^{ab}\partial_a x^\mu \partial_b x_\mu + \theta\left(\tilde{g} + 1\right) , \tag{2.1}$$

where

$$\tilde{g}^{ab} = \sqrt{-g}g^{ab} , \qquad a, b = 0, 1 ,$$

and

$$\tilde{g} = \det \tilde{g}^{a,b} .$$

Here ∂_0 and ∂_1 mean ∂_τ and ∂_σ, where τ and σ are the string world-sheet coordinates. In what follows we take $(\tau, \sigma) = (\zeta^0, \zeta^1)$.

It is important to mention that the Lagrangian (2.1) is invariant under the following infinitesimal world-sheet coordinates transformations [14, 15]:

$$
\begin{aligned}
\delta x^\mu &= -\varepsilon^a \partial_a x^\mu \,, \\
\delta \widetilde{g}^{ab} &= \partial_c \varepsilon^a \widetilde{g}^{cb} + \partial_c \varepsilon^b \widetilde{g}^{ac} - \partial_c \left(\varepsilon^c \widetilde{g}^{ab}\right) , \\
\delta \left(\widetilde{g}\right) &= -\varepsilon^a \partial_a \left(\widetilde{g}\right) , \\
\delta \theta &= -\varepsilon^a \partial_a \theta \,.
\end{aligned}
\tag{2.2}
$$

It can be easily shown that the invariance of the action

$$S = \int d\zeta^2 L_{\mathrm{Cl}}$$

under the transformations (2.2) implies that

$$\varepsilon^1 = 0 \quad \text{for} \quad \sigma = 0, \pi \,.$$

Now, to get the δ_{BRST} transformations related to the transformations (2.2) one has to replace the infinitesimal parameter $\varepsilon^a(\zeta)$ by $\lambda\kappa^a(\zeta)$, where λ is an anticommuting parameter and $\kappa^a(\zeta)$ the ghost coordinates. The nilpotency property of the δ_{BRST} implies that

$$\delta_{\mathrm{BRST}} \kappa^a = -\lambda \kappa^b \partial_b \kappa^a \,. \tag{2.3}$$

The world-sheet symmetric tensor $\widetilde{g}^{ab}$ has the following form

$$\widetilde{g}^{ab} = \begin{bmatrix} \omega_1 + \omega_2 & \omega_0 \\ \omega_0 & \omega_1 - \omega_2 \end{bmatrix} . \tag{2.4}$$

With the gauge condition

$$\omega_0 = \omega_1 = 0 \tag{2.5}$$

and

$$\widetilde{g} = -1 \tag{2.6}$$

one has

$$\widetilde{g}^{ab} = \eta^{ab} \,. \tag{2.7}$$

Now, the gauge fixing term takes the form

$$L_{\mathrm{GF}} = \beta_0 \omega_0 + \beta_1 \omega_1 \tag{2.8}$$

such that

$$\delta_{\mathrm{BRST}} \widetilde{\kappa}^a = -i\lambda \beta^a \,, \tag{2.9}$$

where $\widetilde{\kappa}^a$ and β^a are the antighosts and auxiliary coordinates respectively. It is straightforward to show that

$$\delta_{\mathrm{BRST}}\beta^a = 0\,. \tag{2.10}$$

To get the total quantum Lagrangian, one has to construct the Fadeev–Popov term L_{FP}. Direct calculations lead to

$$\begin{aligned} L_{\mathrm{FP}} = -i\,[&\widetilde{\kappa}_0\left(-\kappa^a\partial_a\omega_0 + \left(\partial_1\kappa^0 + \partial_0\kappa^1\right)\omega_1 + \left(\partial_0\kappa^1 - \partial_1\kappa^0\right)\omega_2\right) + \\ &\widetilde{\kappa}_1\left(-\kappa^a\partial_a\omega_1 + \left(\partial_1\kappa^0 + \partial_0\kappa^1\right)\omega_0 + \left(\partial_1\kappa^0 - \partial_0\kappa^1\right)\omega_2\right)]\,, \end{aligned} \tag{2.11}$$

where $\omega_1, \omega_2, \omega_3$, transform as:

$$\begin{aligned} \delta_{\mathrm{BRST}}\omega_0 &= \lambda\left(-\kappa^a\partial_a\omega_0 + \left(\partial_1\kappa^0 + \partial_0\kappa^1\right)\omega_1 + \left(\partial_0\kappa^1 - \partial_1\kappa^0\right)\omega_2\right)\,, \\ \delta_{\mathrm{BRST}}\omega_1 &= \lambda\left(-\kappa^a\partial_a\omega_1 + \left(\partial_1\kappa^0 + \partial_0\kappa^1\right)\omega_0 + \left(\partial_1\kappa^0 - \partial_0\kappa^1\right)\omega_2\right)\,, \\ &\text{and} \\ \delta_{\mathrm{BRST}}\omega_2 &= \lambda\left(\partial_1\kappa^0 - \partial_0\kappa^1\right)\omega_{0-}\left(\kappa^0\partial_0\omega_2 - \kappa^1\partial_1\omega_2\right) - \partial_1\kappa^1\omega_2 + \partial_0\kappa^0\omega_1]\,. \end{aligned} \tag{2.12}$$

Thus, the total BRST invariant Lagrangian L_{TOT} is

$$L_{\mathrm{TOT}} = L_{\mathrm{Cl}} + L_{\mathrm{FP}} + L_{\mathrm{GF}}$$

with the boundary conditions:

$$\kappa^1 = 0 \quad , \quad \partial_0\kappa^1 \quad \text{at} \quad \sigma = 0\ ,\ \pi. \tag{2.13}$$

Notice that after simple redefinitions of the auxiliary fields coordinates β^a and θ the total Lagrangian takes the simple form

$$L_{\mathrm{TOT}} = -\frac{1}{2\alpha'}\eta^{ab}\partial_a x^\mu\partial_b x_\mu - i\widetilde{\kappa_a}D^a{}_b\kappa^b + \theta\left(1-\omega_2^2\right) + \beta_0\omega_0 + \beta_1\omega_1 \tag{2.14}$$

with

$$D^a{}_b = \begin{bmatrix} \partial_1 & -\partial_0 \\ -\partial_0 & \partial_1 \end{bmatrix}.$$

Now, it is easy to show that the application of the variational principle to the action gives the following equations of motion:

$$\begin{aligned} \left(\partial_0^2 - \partial_1^2\right)x^\mu &= 0\,, \\ D^a{}_b\kappa^b &= 0\,, \\ D^a{}_b\widetilde{\kappa}^b &= 0\,, \\ \omega_0 &= \omega_1 = \omega_2^2 - 1 = \beta_0 = \beta_1 = \theta = 0\,, \end{aligned} \tag{2.15}$$

with the boundary conditions

$$\partial_1 x^\mu = \widetilde{\kappa}_0 = \widetilde{\kappa}_1 = 0 \qquad \text{at} \qquad \sigma = 0\,,\ \pi\,. \tag{2.16}$$

Straightforward calculation shows that the solutions of Eq. (2.15) with the boundary conditions (2.13) and (2.16) are

$$\begin{aligned}
x^\mu(\sigma,\tau) &= q^\mu + \frac{\alpha'}{\pi} p^\mu \tau - \left(\frac{\alpha'}{\pi}\right)^{1/2} \sum_{n=1}^{\infty} \left(a_n^+\, e^{in\tau} - a_n\, e^{-in\tau}\right) \frac{\cos(n\sigma)}{\sqrt{n}}\,,\\
\kappa^0(\sigma,\tau) &= \frac{1}{\sqrt{\pi}}\Omega_0 + \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \left(\Omega_n^+\, e^{in\tau} + \Omega_n\, e^{-in\tau}\right) \cos(n\sigma)\,,\\
\kappa^1(\sigma,\tau) &= -\frac{i}{\sqrt{\pi}} \sum_{n=1}^{\infty} \left(\Omega_n\, e^{-in\tau} - \Omega_n^+\, e^{in\tau}\right) \sin(n\sigma)\,,\\
\widetilde{\kappa}^0(\sigma,\tau) &= -\frac{i}{\sqrt{\pi}} \sum_{n=1}^{\infty} \left(\widetilde{\Omega}_n\, e^{-in\tau} - \widetilde{\Omega}_n^+\, e^{in\tau}\right) \sin(n\sigma)\,,\\
\widetilde{\kappa}^1(\sigma,\tau) &= \frac{1}{\sqrt{\pi}}\widetilde{\Omega}_0 + \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \left(\widetilde{\Omega}_n^+\, e^{in\tau} + \widetilde{\Omega}_n\, e^{-in\tau}\right) \cos(n\sigma)
\end{aligned} \tag{2.17}$$

and the momentum conjugates takes the form:

$$\begin{aligned}
\Pi_{x_\mu} &= -\frac{1}{\kappa}\partial_0 x_\mu\,,\\
\Pi_{\kappa_0} &= -i\widetilde{\kappa}_1\,, \qquad \Pi_{\kappa_1} = -i\widetilde{\kappa}_0
\end{aligned} \tag{2.18}$$

and

$$\Pi_{\widetilde{\kappa}_1} = \Pi_{\widetilde{\kappa}_2} = \Pi_{\omega_0} = \Pi_{\omega_1} = \Pi_{\omega_2} = \Pi_{\beta_0} = \Pi_{\beta_1} = \Pi_\theta = 0\,. \tag{2.19}$$

It is obvious from Eq. (2.19) that all the constraints are of the second class. Therefore the variables ω_0, ω_1, ω_2, $\widetilde{\kappa}_a$, β_a can be eliminated.

Now, to construct the paraquantum Lagrangian of order Q, one has to use first the following Green decomposition of the coordinates x^μ, κ_0, κ_1, q^μ and p^μ [1–8]:

$$\begin{aligned}
x^\mu &= \sum_{\alpha=1}^{Q} x^{\mu(\alpha)},\\
\kappa_0 &= \sum_{\alpha=1}^{Q} \kappa_0^{(\alpha)}, \quad \widetilde{\kappa}_0 = \sum_{\alpha=1}^{Q} \widetilde{\kappa}_0^{(\alpha)},
\end{aligned}$$

$$\begin{aligned}
\kappa_1 &= \sum_{\alpha=1}^{Q} \kappa_1^{(\alpha)}, \quad \tilde{\kappa}_1 = \sum_{\alpha=1}^{Q} \tilde{\kappa}_1^{(\alpha)} \\
\beta_i &= \sum_{\alpha=1}^{Q} \beta_i^{(\alpha)}\,(i=1,2)\,, \quad \omega_j = \sum_{\alpha=1}^{Q} \omega_j^{(\alpha)}\,(j=1,2,3)\,, \quad \theta = \sum_{\alpha=1}^{Q} \theta^{(\alpha)}\,, \\
q^{\mu} &= \sum_{\alpha=1}^{Q} q^{\mu(\alpha)} \qquad \text{with} \qquad q^{\mu(\alpha)} = q^{\mu}\delta_{\alpha,1}\,, \\
p^{\mu} &= \sum_{\alpha=1}^{Q} p^{\mu(\alpha)} \qquad \text{with} \qquad p^{\mu(\alpha)} = p^{\mu}\delta_{\alpha,1}\,.
\end{aligned} \tag{2.20}$$

(Similar decomposition can be written for the momentum conjugates of the variables x^{μ}, κ_0, κ_1), where the Green components $x^{\mu(\alpha)}$, $\kappa_0^{(\alpha)}$, $\tilde{\kappa}_0^{(\alpha)}$, $\kappa_1^{(\alpha)}$, $\tilde{\kappa}_1^{(\alpha)}$, $\beta_i^{(\alpha)}$, $\omega_j^{(\alpha)}$, $\theta^{(\alpha)}$, $q^{\mu(\alpha)}$, $p^{\mu(\alpha)}$ satisfy anomalous bilinear commutation relations [1–9] and have similar mode decompositions as in Eqs (2.17). For the canonical paraquantum non vanishing commutation relations, one has to postulate the following [1–9]:

$$\begin{aligned}
\left[\Pi_{x_\mu}^{(\alpha)}(\tau,\sigma), x^{\nu(\alpha)}(\tau,\sigma')\right] &= -i\delta_{\mu}^{\nu}\delta(\sigma-\sigma')\,, \\
\left\{\Pi_{x_\mu}^{(\alpha)}(\tau,\sigma), x^{\nu(\beta)}(\tau,\sigma')\right\} &= 0 \quad \text{if} \quad \alpha \neq \beta\,, \\
\left\{\Pi_{\kappa_a}^{(\alpha)}(\tau,\sigma), \kappa^{b(\alpha)}(\tau,\sigma')\right\} &= -i\delta_a{}^{b}\delta(\sigma-\sigma')\,, \\
\left[\Pi_{\kappa_a}^{(\alpha)}(\tau,\sigma), \kappa^{b(\beta)}(\tau,\sigma')\right] &= 0 \quad \text{if} \quad \alpha \neq \beta\,.
\end{aligned} \tag{2.21}$$

Notice that from Eqs (2.17), (2.18) and (2.21) we obtain:

$$\begin{aligned}
[P^{\mu}, q^{\nu}] &= ig^{\mu\nu}\,, \\
\left[a_n^{\mu(\alpha)}, a_m^{+\nu(\alpha)}\right] &= -g^{\mu\nu}\delta_{nm}\,, \\
\left\{a_n^{\mu(\alpha)}, a_m^{+\nu(\beta)}\right\} &= 0 \qquad \text{if} \qquad \alpha\neq\beta\,, \\
\left\{\tilde{\Omega}_0^{(\alpha)}, \Omega_0^{(\alpha)}\right\} &= 1\,, \\
\left\{\tilde{\Omega}_0^{(\alpha)}, \Omega_0^{(\beta)}\right\} &= 0 \qquad \text{if} \qquad \alpha\neq\beta\,, \\
\left\{\tilde{\Omega}_n^{+(\alpha)}, \Omega_m^{(\alpha)}\right\} &= \left\{\tilde{\Omega}_n^{(\alpha)}, \Omega_m^{+(\alpha)}\right\} = \delta_{nm}\,, \\
\left\{\tilde{\Omega}_n^{+(\alpha)}, \Omega_m^{(\alpha)}\right\} &= \left\{\tilde{\Omega}_n^{(\alpha)}, \Omega_m^{+(\alpha)}\right\} = 0 \qquad \text{if} \qquad \alpha\neq\beta\,.
\end{aligned} \tag{2.22}$$

It is important to mention that one has to require that the space-time properties should not be affected by the paraquantization. In particular, since q^μ and p^μ are observables (dynamical variables) characterizing the string center of mass, they have to satisfy to ordinary commutation relations. To respect this condition, one has to specify a direction in the Green para-space. For example, one can choose the direction corresponding to $\alpha = 1$ and require that the center of mass coordinates have one component (along this direction).

Now, it can be shown easily that the total paraquantum Lagrangian $L^{\mathrm{PQ}}_{\mathrm{TOT}}$ of open string can be written as

$$\begin{aligned} L^{\mathrm{PQ}}_{\mathrm{TOT}} &= -\frac{1}{2\alpha'}\sum_{\alpha=1}^{Q}\Big[\eta^{ab}\partial_a x^{\mu(\alpha)}\partial_b x^{(\alpha)}_{\mu} - i\widetilde{\kappa_a}^{(\alpha)}D^a{}_b\kappa^{b(\alpha)} \\ &+\theta^{(\alpha)}\left(1-\omega_2^{2(\alpha)}\right) \\ &+\beta_0^{(\alpha)}\omega_0^{(\alpha)} + \beta_1^{(\alpha)}\omega_1^{(\alpha)}\Big]. \end{aligned} \tag{2.23}$$

Thus, the paraquantum BRST transformations get the form:

$$\begin{aligned} \delta_{\mathrm{BRST}} x^{\mu(\alpha)} &= -\lambda\sum_{\beta=1}^{Q}\kappa^{a(\beta)}\partial_a x^{\mu(\alpha)}\,, \\ \delta_{\mathrm{BRST}}\kappa^{a(\alpha)} &= -\lambda\sum_{\beta=1}^{Q}\kappa^{b(\beta)}\partial_b\kappa^{a(\alpha)}\,, \\ \delta_{\mathrm{BRST}}\omega_0^{(\alpha)} &= \lambda\sum_{\beta=1}^{Q}\Big[-\kappa^{a(\beta)}\partial_a\omega_0^{(\alpha)} + (\partial_1\kappa^{0(\beta)}+\partial_0\kappa^{1(\beta)})\omega_1^{(\alpha)} \\ &+(\partial_0\kappa^{1(\beta)}-\partial_1\kappa^{0(\beta)})\omega_0^{(\alpha)}\Big]\,, \\ \delta_{\mathrm{BRST}}\omega_1^{(\alpha)} &= \lambda\sum_{\beta=1}^{Q}\Big[-\kappa^{a(\beta)}\partial_a\omega_1^{(\alpha)} + (\partial_1\kappa^{0(\beta)}+\partial_0\kappa^{1(\beta)})\omega_0^{(\alpha)} \\ &+(\partial_0\kappa^{0(\beta)}-\partial_1\kappa^{1(\beta)})\omega_2^{(\alpha)}\Big]\,, \\ \delta_{\mathrm{BRST}}\omega_2^{(\alpha)} &= \lambda\sum_{\beta=1}^{Q}\Big[(\partial_1\kappa^{0(\beta)}-\partial_0\kappa^{1(\beta)})\omega_0^{(\alpha)} \\ &-(\kappa^{0(\beta)}\partial_0\omega_2^{(\alpha)}-\kappa^{1(\beta)}\partial_1\omega_2^{(\alpha)}) \\ &-\partial_1\kappa^{1(\beta)}\omega_1^{(\alpha)}+\partial_0\kappa^{0(\beta)})\omega_1^{(\alpha)}\Big]\,, \\ \delta_{\mathrm{BRST}}\theta^{(\alpha)} &= -\lambda\partial_a\sum_{\beta=1}^{Q}(\kappa^{a(\beta)}\theta^{(\alpha)})\,. \end{aligned} \tag{2.24}$$

Similarly, one can show that the Green component of the BRST charge $Q^{(\beta)}_{\mathrm{BRST}}$ has the following expression :

$$Q^{(\beta)}_{\mathrm{BRST}} = \sum_{\alpha=1}^{Q} \left\{ \kappa_0^{(\beta)} \Lambda_1^{(\alpha)} + \tilde{\kappa}_0^{(\alpha)} \Lambda_2^{(\alpha)} + \left(\frac{\alpha'}{\pi}\right)^{\frac{1}{2}} \Lambda_3^{(\alpha)} \right\}, \tag{2.25}$$

where

$$\begin{aligned}
\Lambda_1^{(\alpha)} &= \frac{1}{\sqrt{\pi}} \left[\frac{1}{2} \frac{\alpha'}{\pi} p^\mu p_\mu \delta_{\alpha,1} + \frac{1}{2} \sum_{n=1}^{\infty} n \left(a_n^{\mu(\alpha)} a_{n_\mu}^{+(\alpha)} + a_n^{+\mu(\alpha)} a_{n_\mu}^{(\alpha)} \right) \right. \\
&\quad \left. - \sum_{n=1}^{\infty} n \left(\Omega_n^{+(\alpha)} \tilde{\Omega}_n^{(\alpha)} + \Omega_n^{(\alpha)} \tilde{\Omega}_n^{+(\alpha)} \right) \right] + \frac{\alpha(0)}{\sqrt{\pi}} \delta_{\alpha,1}, \\
\Lambda_2^{(\alpha)} &= \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} 2n \Omega_n^{+(\alpha)} \Omega_n^{(\alpha)}, \\
\Lambda_3^{(\alpha)} &= \sum_{\beta=1}^{Q} \left[\frac{i}{\sqrt{\pi}} \sum_{n=1}^{\infty} \sqrt{n} p^\mu \delta_{\alpha,1} \left(a_{n_\mu}^{+(\alpha)} \Omega_n^{(\beta)} + \Omega_n^{+(\beta)} a_{n_\mu}^{(\alpha)} \right) \right. \\
&\quad + \frac{1}{2\sqrt{\alpha'}} \sum_{n=1}^{\infty} \sum_{m=1}^{n-1} \sqrt{n(n-m)} \left(a_{n-m}^{+\mu(\alpha)} a_{m_\mu}^{+(\alpha)} \Omega_n^{(\beta)} + \Omega_n^{+(\beta)} a_{n-m}^{\mu(\alpha)} a_{n_\mu}^{(\alpha)} \right) \\
&\quad + \frac{1}{\sqrt{\alpha'}} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sqrt{n(n+m)} \left(a_{n+m}^{+\mu(\alpha)} a_{m_\mu}^{(\alpha)} \Omega_n^{(\beta)} + \Omega_n^{+(\beta)} a_m^{+\mu(\alpha)} a_{m+n_\mu}^{(\alpha)} \right) \\
&\quad + \frac{1}{\sqrt{\alpha'}} \sum_{n=1} \sum_{m=1} m \left(\tilde{\Omega}_{n+m}^{+(\beta)} \Omega_m^{(\alpha)} \Omega_n^{(\alpha)} - \Omega_n^{+(\alpha)} \Omega_m^{+(\alpha)} \tilde{\Omega}_{n+m}^{(\beta)} \right) \\
&\quad \left. + (n+2m) \left(\tilde{\Omega}_n^{+(\beta)} \Omega_m^{+(\alpha)} \Omega_{n+m}^{(\alpha)} + \Omega_{n+m}^{+(\alpha)} \Omega_m^{(\beta)} \tilde{\Omega}_n^{(\beta)} \right) \right].
\end{aligned} \tag{2.26}$$

Now, a straightforward calculation and use of the Riemann zeta function regularization gives:

$$\begin{aligned}
{Q^{(\alpha)}_{\mathrm{BRST}}}^2 &= \frac{2}{\pi} \left[\left(\frac{D}{12} - \frac{2Q}{12} - 2 \right) \sum_{n=1}^{\infty} n^3 \Omega_n^{+(\alpha)} \Omega_n^{(\alpha)} \right. \\
&\quad \left. \times \left(\frac{D}{12} - \frac{2Q}{12} - 1 - \alpha(0) \right) \sum_{n=1}^{\infty} n \Omega_n^{+(\alpha)} \Omega_n^{(\alpha)} \right].
\end{aligned} \tag{2.27}$$

The nilpotency property applied to $Q^{(\alpha)}_{\mathrm{BRST}}$ gives

$$D = 2 + \frac{24}{Q}.$$

Notice the Q dependence of the critical space-time dimension D.

3. Conclusion

One concludes that the nilpotency property applied to the Green component of the parastring BRST charge $Q^{(\alpha)}_{\rm BRST}$ leads to the relation

$$D = 2 + \frac{24}{Q}$$

between the critical dimension D and the order of paraquatization Q already derived in [12]. This suggests that the nilpotency property of the $Q^{(\alpha)}_{\rm BRST}$ plays the same role as the Virasoro constraints applied to physical states [12]. In fact, one can define a physical state $|\Psi\rangle_{\rm phys}$ as

$$|\Psi\rangle_{\rm phys} = \sum_{\alpha=1}^{Q} |\Phi\rangle^{(\alpha)}$$

such that

$$Q^{(\alpha)}_{\rm BRST}|\Phi\rangle^{(\alpha)} = 0\,.$$

(The index (α) means the $\alpha^{\rm th}$Green component.)

Thus one can have a paraquantum consistent open string theory at $D = 26$, 14, 10, and 4 (which is not the case of the ordinary quantum open string). This is a very interesting result, in the sense that one can first study the topological and geometrical properties of the resulting manifolds with the extra new space-time dimensions as well as the compactification procedure (one may have interesting phenomenological solutions) and, second, one may solve the compactification problem by choosing a generalized quantum theory of order $Q = 12$ $(D = 4)$. (More details are under investigations.)

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