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ON THE BOSE–EINSTEIN EFFECT AND THE W MASS

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We present an implementation of the Bose–Einstein effect in a Monte Carlo generator for W^-W^+ production in the e^-e^+ annihilation by means of the weight method. We check that the shift of the W mass in four jet events due to this effect is similarly small as for the other prescription used recently by Jadach and Zalewski. Possible generalization of this result is shortly discussed.

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1. Introduction

The problem of an exact determination of the W mass is crucial for any test of the standard model. Thus all the effects resulting in the W mass shift are of great importance. In particular, the possible mass shifts from the Bose–Einstein effect in the four jet events from the e^-e^+ annihilation into W^-W^+ pairs have been recently estimated with widely varying results [1–3].

In this note we use the implementation of the Bose–Einstein effect based on the Białas–Krzywicki prescription for weights to be attached to the Monte Carlo generated events [4]. We avoid the prohibitive increase of computational time with multiplicity by the approximation used already to describe the effect of hadronic collisions [5]. We follow closely the recent investigation by Jadach and Zalewski [3] to check if a different prescription for the weights influences the physical conclusions.

We describe our method shortly in the next section stressing the similarities and differences with other approaches. The results are presented in the third section. The last section contains discussion and conclusions.

2. Implementation of the weight method for the Bose–Einstein effect

The original discussion of the Bose–Einstein effect in multiparticle production assumed the knowledge of amplitudes which have to be symmetrized [6]. This is not the case for Monte Carlo-based models, where only probabilities are calculated. On the other hand, the multitude of available data can be analyzed only within the models using Monte Carlo generators. This sort of analysis seems to be necessary to discriminate among various pictures of the space time development of multiple production. Therefore various methods have been devised to implement the effect into Monte Carlo generators. For heavy ion collisions one used the semi-classical description of the process to provide the space time distribution of sources producing plane waves to be symmetrized [7]. For the Lund model of two-jet processes there is a natural measure for probabilities after symmetrization [8]. However, the most widely used method imitates the effect just by suitable shifts of final state momenta to get the experimental two-particle distributions [9]. Apart from other problems this method includes the momentum rescaling which results in serious mass shifts for W bosons [1,3]. Thus there is a need for a reliable and general method to implement the Bose–Einstein effect into any Monte Carlo generator.

Such a method seems to be suggested by the analysis by means of Wigner functions pioneered by Pratt [10] and recently recalled by Białas and Krzywicki [4]. We have described this method in Ref. [5]. For reader's convenience we repeat here its main ingredients.

With few simplifying assumptions one arrives at formulae where the multiparticle density distribution is expressed by a product of original (non-symmetrized) distribution and the weight factor, representing the effect of symmetrization. In this way one gets a simple prescription for the Monte Carlo generators: one should generate events according to the original generator and then attach to each event its weight calculated from a simple formula

$$W(n) = \sum_{\{P(k)\}} \prod_{i=1}^{n} w_{iP(i)}. \tag{1}$$

Here n is the number of identical particles, $w_{iP(i)}$ is a two particle weight factor calculated for the pair of momenta (of the i-th particle and the particle which occupies the $i-th$ place in the permutation $P(k)$). The sum extends over all the permutations of n elements. Since all factors are positive and $w_{ii} = 1$, the resulting weight is not smaller than one (a contribution from identity permutation). One may rescale the weights to keep, *e.g.*, the average number of particles fixed; we return to this point later.

Since most of the particles detected in experiments are pions, the final weight should be actually given by a product of weights calculated separately for positive, negative, and neutral pions. In fact, the BE interference for neutral particles is not observable (apart from the possible effects for direct photons [11]): neutral pions decay before detection, and for the resulting photons the effective source size is so big that the BE effects must be negligible for momentum differences above a few eV. However, the procedure should not change the observable correlations between the numbers of charged and neutral pions. Therefore weights for all signs of pions must be taken into account.

Thus in principle the only arbitrary factor is the function of the difference of two momenta $w_{ij}(p_i - p_j)$. As in Ref. [5] we use here the Gaussian function of four-momentum difference squared

$$w_{ij} = \mathrm{e}^{(p_i - p_j)^2/2\sigma^2} \tag{2}$$

which is motivated by a commonly used experimental parametrization of BE effects.

Of course, different components of momentum difference squared may be multiplied by different coefficients, and the shape may be modified. In this note we do not discuss these possibilities. Therefore the only parameter is a Gaussian half-width of the distribution σ.

Unfortunately, for more than ten pions of a given sign the calculations become prohibitively long. Symmetrizing separately in hemispheres [12] one shifts only the problem to higher energies. There exists a scheme for calculating the sum (1) in a reasonable time [13], but the method is still under investigation. In our calculations we have separated the sum of all the $n!$ permutations into terms where only the permutations which change places of exactly K particles are taken into account:

$$w = \sum_K w^{(K)}. \tag{3}$$

The higher terms in this expansion correspond to configurations where many particles have approximately the same momenta, which is very unlikely. These terms for $K < 6$ are

$$w^{(0)} = 1;\ w^{(1)} = 0;\ w^{(2)} = \sum_{i=1}^{n-1}\sum_{j>i}(w_{ij})^2;\ w^{(3)} = 2\sum_{i=1}^{n-2}\sum_{j>i}\sum_{k>j} w_{ij}w_{jk}w_{ki};$$

$$\begin{aligned} w^{(4)} = & \sum_{i=1}^{n-3}\sum_{j>i}\sum_{k>j}\sum_{l>k}[2w_{ij}w_{ik}w_{jl}w_{kl} + 2w_{ij}w_{il}w_{jk}w_{kl} \\ & +2w_{ik}w_{il}w_{jk}w_{jl} + (w_{il}w_{jk})^2 + (w_{ij}w_{kl})^2 + (w_{ik}w_{jl})^2]; \end{aligned}$$

$$
\begin{aligned}
w^{(5)} = & \; 2\sum_{i=1}^{n-4}\sum_{j>i}\sum_{k>j}\sum_{l>k}\sum_{m>l}[(w_{ij})^2 w_{lk}w_{ml}w_{km} + (w_{ik})^2 w_{jl}w_{ml}w_{jm} \\
& + (w_{il})^2 w_{jk}w_{jm}w_{km} + (w_{im})^2 w_{jk}w_{kl}w_{jl} + (w_{jk})^2 w_{il}w_{lm}w_{im} \\
& + (w_{jl})^2 w_{ik}w_{km}w_{im} + (w_{jm})^2 w_{ik}w_{kl}w_{il} + (w_{kl})^2 w_{ij}w_{jm}w_{im} \\
& + (w_{lm})^2 w_{ij}w_{jk}w_{ik} + (w_{km})^2 w_{ij}w_{jl}w_{il} + w_{ij}w_{jk}w_{kl}w_{lm}w_{im} \\
& + w_{ik}w_{jl}w_{km}w_{jm}w_{il} + w_{il}w_{ij}w_{kl}w_{jm}w_{km} + w_{ij}w_{ik}w_{jl}w_{lm}w_{km} \\
& + w_{ik}w_{im}w_{jk}w_{jl}w_{lm} + w_{il}w_{jl}w_{jk}w_{km}w_{im} + w_{ij}w_{ik}w_{kl}w_{lm}w_{jm} \\
& + w_{ij}w_{il}w_{lm}w_{jk}w_{km} + w_{ij}w_{im}w_{jl}w_{km}w_{kl} + w_{ik}w_{il}w_{jk}w_{lm}w_{jm} \\
& + w_{ik}w_{im}w_{jm}w_{jl}w_{kl} + w_{il}w_{im}w_{kl}w_{jk}w_{jm}]. \qquad (4)
\end{aligned}
$$

The shape of the weight factor (2) should be chosen to fit the "BE ratio", defined for the pair of identical pions as a function of $Q = \sqrt{-(p_1 - p_2)^2}$

$$
c_2(Q) = \frac{\int d^3p_1 d^3p_2 \rho_2(p_1, p_2)\delta[Q - \sqrt{-(p_1 - p_2)^2}]}{\int d^3p_1 d^3p_2 \rho_1(p_1)\rho_1(p_2)\delta[Q - \sqrt{-(p_1 - p_2)^2}]} \frac{\langle n\rangle^2}{\langle n(n-1)\rangle}. \qquad (5)
$$

Without weights it is rather flat and close to one for typical Monte Carlo generators, if we normalize separately the numerator and the denominator of Eq. (5) to the same number of entries (which is achieved by the second factor in (5)). Including weights produces a maximum at smallest Q^2 with the height about 2 (*i.e.* one unit above the value at large Q^2) and a width σ' close to σ of formula (2). Thus we reproduce satisfactorily the shape assumed for the two-particle weight factor.

We have checked in Ref. [5] for PYTHIA/JETSET generated $p\bar{p}$ events at 630 GeV that cutting the series (3) at $K = 3$ and at $K = 4$ we get quite similar shapes of the Q^2 spectra, although the normalization is significantly different. Including the term with $K = 5$ we change even less all the distributions. Thus we feel that cutting the series (3) at $K = 5$ we get a reliable estimate of the results for Q^2 spectra from the weight method (up to the possible change of normalization).

This may seem surprising if we remember that our approximation does not take into account, *e.g.*, the contribution from such a simple configuration as three pairs of very close (pair wise) momenta. Indeed, in this case there is a contribution from a permutation of 6 elements. However, the full contribution of such a configuration to the sum (1) is equal $1+3+3+1 = 8$ (from permutations moving 0, 2, 4 and 6 elements, respectively) and our approximation counts all but the last term in this sum. We have checked that for all reasonably probable configurations our approximation seems similarly satisfactory.

Since the JETSET/PYTHIA parameters were fitted to reproduce inclusive experimental data without weights, the change, *e.g.*, of the average multiplicity induced by weights should be compensated by the proper refitting procedures. Instead we have applied (as in Ref. [3]) a simple method of multiplying weights by an extra cV^n factor, where n is the number of all "direct" pions, and c and V are constants fixed by the requirements to restore the original number of events and the original average multiplicity. This is done by assuming that the original multiplicity distribution of "direct" pions may be well approximated by the negative binomial formula, *i.e.* that the NBD parameters $\overline{n}$ and $1/k$ are given by the experimental values of $\langle n \rangle$ and $\langle n(n-1)\rangle/\langle n\rangle^2 - 1$. If with the weights we get a new average multiplicity $\langle n' \rangle$, the original value may then be restored by rescaling the weights with

$$V = \frac{\langle n\rangle(\langle n'\rangle + k)}{\langle n'\rangle(\langle n\rangle + k)} \tag{6}$$

and

$$c = \frac{[1 + (1 - V)\langle n'\rangle/k]^k}{\langle w\rangle}, \tag{7}$$

where $\langle w \rangle$ is the average value of weights before rescaling. We have checked that this procedure restores indeed the original average multiplicity with accuracy of a few percent. If this accuracy is not satisfactory, the quantities c and V can be estimated by direct minimization of differences between the multiplicity distributions without weights and with the rescaled weights, respectively. On the other hand, the BE ratios are little affected by rescaling (only the normalization, which is anyway mainly a matter of convention, changes by a few percent). As will be shown later this is also true for e^-e^+ collisions. The BE ratio still reflects mainly the assumed shape of the two-particle weight (plus 1): for larger σ it is wider and starts to increase above 2 for smallest Q^2.

The procedure seems to produce too high a value of the BE ratio for smallest Q^2. As already noted, it is about twice the value for large Q^2, whereas in most of the data it is only by some 50% higher. To explain why the BE ratio does not increase up to the value of 2, one may invoke some coherent component [6], but a more obvious effect (which also lowers the BE ratio) is the existence of longer living resonances. Pions coming from their decay are effectively "born" more than 10 fm from the collision point. Thus the Gaussian width parameter in a two-particle weight for these pions should be smaller by an order of magnitude, which allows practically to neglect their contribution to the BE effect in the experimentally accessible Q^2 range. Therefore the Białas–Krzywicki weights are calculated taking into account only the permutations of momenta of pions produced directly, or resulting from the decay of the widest resonances.

3. Results and comparison with data

We have generated 150 000 events of e^+e^- annihilation into W^+W^- at 172 GeV CM energy by the default version of the PYTHIA/JETSET generator [9]. For each event the weight factor was calculated by taking the 4-momenta of "direct" pions of each sign, calculating for them a matrix of two-particle weights w_{ij} according to (2) with $\sigma = 0.14$ GeV (the same value as used to describe the $p\bar{p}$ collisions), and then the weight w as a series (3) cut at $K = 5$ and rescaled as described in the previous section [1]. As already noted, the event weight is a product of weight factors for all three kinds of pions. In Fig. 1 we present the ratio of "BE ratios" (5) for pairs of positive pions as a function of Q^2 for the events from our prescription with series (3) cut at $K = 5$ and from the standard PYTHIA/JETSET generator (without weights). Results are shown for the rescaled and unrescaled weights.

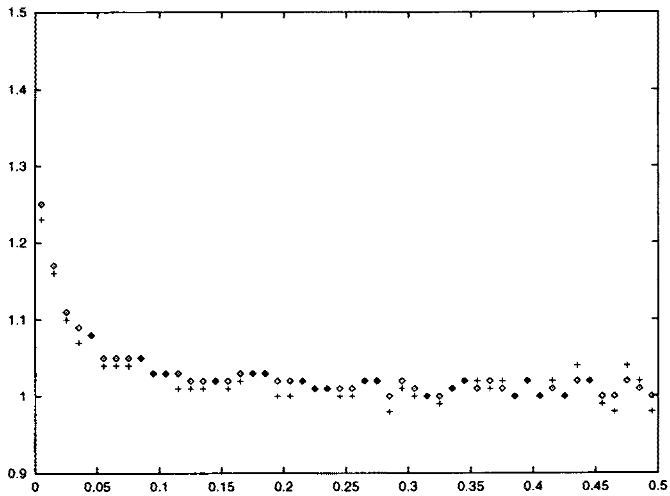

Fig. 1. The ratio of "BE ratios" (5) for positive pions with and without weights as a function of Q^2 [GeV^2]. Diamonds and crosses correspond to rescaled and unrescaled weights, respectively.

We see that without any fitting one can reproduce the main features of inclusive hadroproduction data at similar energy [14] (there are no data yet for W^+W^- events) and that the two curves are hardly distinguishable.

Next we analyze the events using the LUCLUS procedure [15] to select 4 jet events and to reconstruct W as the 2-jet system. To reduce the com-

[1] More precisely, we have used the values of c and V fitted to restore the original multiplicity distribution for the hadronic decay of a single W.

binatorial background we select as a partner for the first jet this jet which maximizes the two-jet invariant mass. Then, as in Ref. [3], we plot the average of this mass and the mass of the system of two remaining jets. The resulting W mass distribution with and without weights is shown in Fig. 2. We see that the introduction of weights hardly affects the distribution. The average mass for two curves of Fig. 2 differs by less than 20 MeV. If we fit these curves by the Breit–Wigner formula (plus background), the fitted W mass would differ even less.

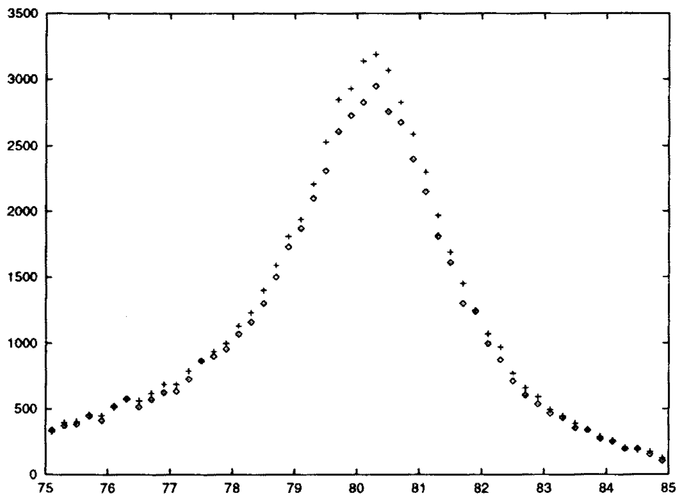

Fig. 2. The two-jet invariant mass distribution (in GeV/c^2) as described in the text. Diamonds and crosses correspond to distributions with and without weights.

It is important to underline that our results are very similar to those of Ref. [3] both for the BE ratio and for the m_W distribution, although our distribution of weights has a much more extended tail, even after rescaling. As shown in Fig. 3, this tail decreases quite slowly and for 175 events the values of weights are outside the plot (the maximal value is about hundred).

There are good reasons for this difference: *e.g.*, we do not have a "coherence factor" $1 - p$ decreasing the maximal possible value of weights even for big clusters of particles with similar momenta values. We do not damp doubly the momentum differences (by the definition of a cluster and by the prescription of weights). We also have only one free parameter instead of three as in Ref. [3]. This, altogether, makes the similarity of the results for the Q^2 and m_W distributions quite intriguing.

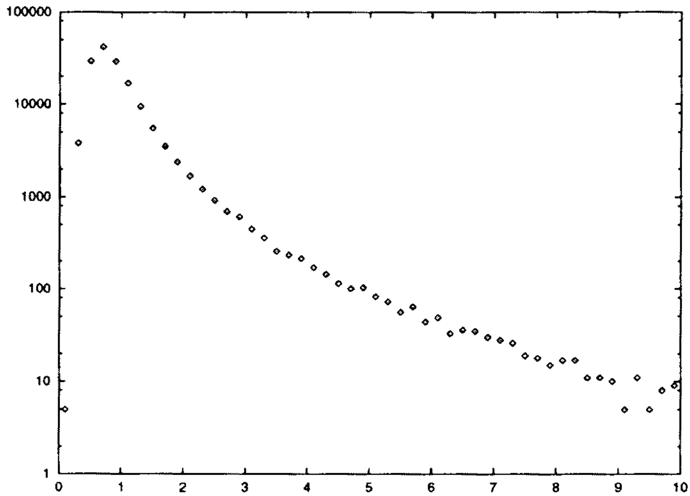

Fig. 3. The rescaled weight distribution in a semi-log scale.

4. Summary and conclusions

We have applied the weight method to implement the Bose–Einstein interference effect into the Monte Carlo generator for the $e^+e^- \to W^+W^-$ process with a four jet final state. With the same value of the only free parameter σ as used for the $p\bar{p}$ collision we get a reasonable qualitative description of the Bose–Einstein ratio. The reconstructed value of the W mass is practically the same with and without weights.

One may argue that this last result is almost trivial: since the weights do not change the momentum structure of events, there is no reason to expect mass shifts for the reconstructed unstable particles. This is, however, a too simplistic argument. Jet algorithms assign the decay products to the two W-s only statistically. One may easily imagine nonzero correlations between the degree of misassignment and the weight values, which would introduce a significant mass shift. The absence of such shifts for two different weight methods supports the suggestion that such correlations do not exist.

Summarizing, we confirm the claim of Ref. [3] that there seems to be no significant W mass shift in four jet events due to the Bose–Einstein interference effect. Since our algorithm is based directly on the original formula for weights [4] (apart from cutting the full series (3)) and contains no *ad hoc* extra assumptions and free parameters, we regard it as a reliable, general method of implementing the Bose–Einstein effect. Therefore we believe that our results are relevant for the coming precise W mass measurements.

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