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SUPERSYMMETRY AND EXOTIC NUCLEI* **

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The analysis of binding energies of the *sd* shell nuclei appeared to be a new, interesting application of the supersymmetric model. After fitting the model parameters from masses and excited energy levels of well established nuclei it is possible to describe other exotic nuclei from the edge of a stability line, belonging to the same supermultiplet. We have applied such a procedure to the oxygen isotopes ^{26,28}O. The method can be treated more generally for the construction of a supersymmetric mass formula for all of the *sd* shell nuclei. The results are quite satisfactory in comparison with experimental data and also with other theoretical predictions. It provides an additional argument for the approximate supersymmetry of the *sd* shell nuclei.

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1. Introduction

Following our recent and preliminary reports [1] as well as the method and some of the results of our earlier publications [2, 3], we have given here the full account of the supersymmetry application to the binding energies of the *sd* shell nuclei including the procedure of dealing with exotic nuclei.

The main idea is the following. Let us apply, with positive results, the approximate (dynamical) supersymmetry to the well established nuclei belonging to the same supermultiplet. It means that the supersymmetric model is able to describe as well excited levels as electromagnetic transitions by the means of the constructed Hamiltonian and the transition operators. The model needs several phenomenologically fixed parameters (from three to five) for a given supermultiplet. Suppose then, that to the same supermultiplet

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belongs an exotic nucleus from the edge of a stability line. If our supersymmetry assumption is good enough, then the same Hamiltonian with the same parameters must be able to describe an exotic nucleus including the primary question whether or not that nucleus exists.

The paper is organized in the following way. In Sec. 2 we briefly sketch the supersymmetric model in a description of the excited states of nuclei from a given supermultiplet with the fixed model parameters. In this part we have discussed the $N = 5$ supermultiplet to which belongs the exotic oxygen nucleus $A = 26$. Sec. 3 comprises the extension of the Hamilton operator toward the binding energy calculation. In this part we also consider other supermultiplets with the rest of oxygen isotopes from $A = 18$ to $A = 28$. The problem of smooth variation of the model parameter from one to another supermultiplet has been also discussed. In Sec. 4 we have performed the further generalization of the binding energy formula in such a way as to comprise all of the nuclei of the *sd* shell.

2. Supersymmetry and excited levels of *sd* shell nuclei

Nuclei from the first half and from the second half of the *sd* shell need to be differently treated. This remark is also valid in the shell model calculation. In the supermultiplet model we have described the second half *sd* nuclei in several papers [4] with a quite satisfactory result. Nuclei from the first half of the *sd* shell, which are under our present consideration because of the oxygen isotopes, have been also described under the supersymmetry assumption with some specific changes of the model [2]. We briefly sketch that consideration.

The starting point is the Interacting Boson Model with $l = 0$ and $l = 2$ bosons (s and d bosons) which approximately describe pairs of properly correlated valence nucleons. The non-paired nucleons, one for even-odd and two for odd-odd nuclei together with bosons form a system on which we impose the supersymmetry condition. We also introduce in the model the full isospin formalism, contrary to the so called F-spin, and hence, the number of single particle boson states, is equal to 18. We also assume, for nucleons, the level $j = 5/2$ and hence the number of nucleon single particle states is equal to 12. The unitary-unitary supergroup is then U(18/12) and the chain of subgroups has been then formed with the standard angular and isospin groups $\mathrm{SO}_L(3)$ and $\mathrm{SU}_T(2)$ at the end of the chain.

The dynamical supersymmetric Hamiltonian is then constructed as a linear combination of the second order Casimir invariants of the relevant subgroups of the chain

$$H = H_0' \; + \; A\, C_2[\mathrm{SO}^B(5)] + B\, C_2[\mathrm{SO}_L^B(3)] + \alpha\, C_2[\mathrm{SU}_{J_f}^F(2)]$$

$$+\ \beta\, C_2[\mathrm{SU}_J^{BF}(2)] + \gamma\, C_2[\mathrm{SU}_{T_f}^{F}(2)] + \delta\, C_2[\mathrm{SU}_T^{B}(2)]$$
$$+\ \varepsilon\, C_2[\mathrm{SU}_T^{BF}(2)]\,. \qquad (1)$$

Diagonalization of the Hamiltonian with a little algebra gives

$$E = E_0' \ +\ A\,\tau(\tau+3) + B\,L(L+1) + (-1)^{T_f}\alpha_0\, J_f(J_f+1)$$
$$+\ \beta\, J(J+1) + \gamma\, T_f(T_f+1) \qquad (2)$$

with the phenomenological parameters A; B; α; β; γ and with the label τ for the completely symmetric irreducible representation of the boson orthogonal group SO(5).

Suppose, we consider the $N = 5$ supermultiplet (N — the number of valence bosons and fermions) to which belong the following nuclei ^{26}Mg; ^{26}Si; ^{25}Mg; ^{25}Al and ^{24}Na. If we adjust the parameters for one of the supermultiplet nucleus, then, the same parameters must be as well good for other nuclei, if the supersymmetry assumption is approximately valid. In Fig. 1 we display the typical agreement (or disagreement) of the calculated [2] and experimental data for the nucleus ^{24}Na with adjusted parameters for the whole supermultiplet $N = 5$: $A = 0.19$; $B = 0.12$; $\beta = 0.08$; $11\alpha_0 - \gamma = 0.82$ (in MeV). In excited energy calculations the parameters α_0

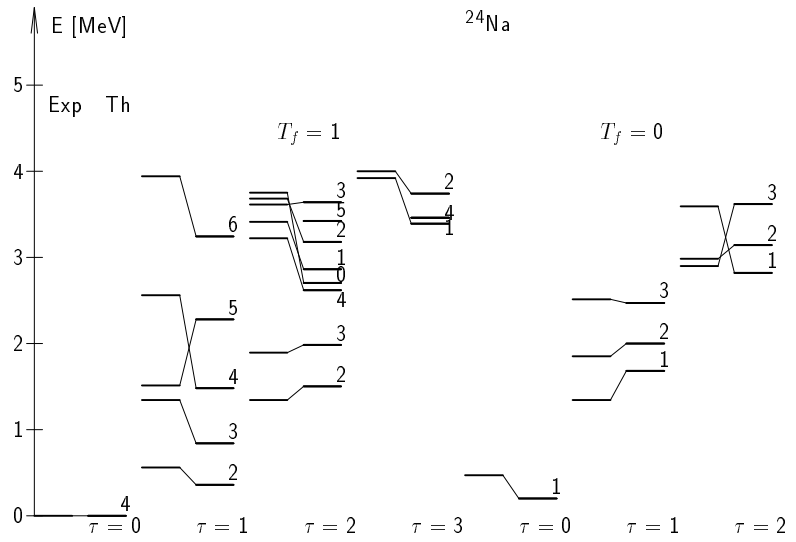

Fig. 1. The example of calculated and experimental data for the odd-odd nucleus ^{24}Na from the supermultiplet $N = 5$. Levels are organized in the SO(5) representations (τ).

and γ enter the calculations in the shown combination. We should stress that all of the experimental (up to a proper energy) and all of the calculated energy levels were taken into account. There is, almost perfect one to one mapping of experimental and theoretical energy and spin levels.

3. Supersymmetry and binding energy

In the formula (1) the H_0' involves the core contribution but also such terms which are the same for a given nucleus. These terms do not enter the relative energy calculation of excited states. However, these terms have to be taken in calculations of a nucleus ground state energy. Hence, we extract from the H_0' the most important three terms: single particle energies for fermions and bosons and the total quadratic isotopic term

$$H_0' = H_0 + a\,N_f + b\,N_b + \varepsilon\,\boldsymbol{T^2}\,. \tag{3}$$

In the H_0 there is mostly the core contribution although some small other terms will be discussed in the next section. The formula (3) should be introduced to (1) and then to (2). For simplicity, we have made another assumption: because the parameters α_0 and γ taken separately do not spoil the excited level calculation under the condition $11\alpha_0 - \gamma = 0.82$ MeV, we assume $\alpha_0 = 0$ and then $\gamma = -0.82$ MeV. Hence, the binding energy formula reads

$$\begin{aligned} E = E_0 \;+\; & aN_f + bN_b + A\tau(\tau+3) + B\,L(l+1) \\ +\; & \beta\,J(J+1) + \gamma\,T_f(T_f+1) + \varepsilon\,T(T+1)\,. \end{aligned} \tag{4}$$

In the formula (4) and for the supermultiplet $N = 5$, the E_0 is the core contribution of the oxygen isotope $A = 16$. We should also remember, that the binding energy (4) has no Coulomb contribution. Hence, while comparing with experimental data we should take the Coulomb energy into account.

Let us now adjust the new parameters a; b; ε to the experimental binding energies of $N = 5$ nuclei. From the formula (4) and from Table I

TABLE I

The quantum numbers in the supersymmetric model for the supermultiplet $N = 5$ nuclei

Nucleus	N_f	N_b	τ	L	J	T_f	T	E
^{24}Na	2	3	0	0	4	1	1	E_1
^{25}Mg or ^{25}Al	1	4	0	0	5/2	1/2	1/2	E_3
^{26}Mg or ^{26}Si	0	5	0	0	0	0	1	E_5

we get

$$E_1 = E_0 + 2a + 3b + 2\varepsilon - 0.04\,,$$
$$E_3 = E_0 + a + 4b + 0.75\varepsilon + 0.08\,,$$
$$E_5 = E_0 - 5b + 2\varepsilon\,. \tag{5}$$

Now we make use of the experimental binding energies [5] together with the Coulomb contribution [6] which we display in Table II.

TABLE II

Experimental binding energy [5] and Coulomb contribution E_C [6] for the core and $N = 5$ supermultiplet nuclei.

	$^{16}_{8}O_8$	$^{24}_{11}Na_{13}$	$^{25}_{12}Mg_{13}$	$^{25}_{13}Al_{12}$	$^{26}_{12}Mg_{14}$	$^{26}_{14}Si_{12}$
	E_0	E_1	E_2	E_3	E_4	E_5
$(E_B)_{\rm exp}$	–127.62	–193.52	–205.59	–200.53	–216.68	–206.05
E_C	18.29	30.26	35.10	40.18	35.10	45.77
$E_i \equiv E_B - E_C$	–145.91	–223.78	–240.69	–240.71	–251.78	–251.82

Solving the equations (5) we get $a = -0.12$; $b = -22.14$ and $\varepsilon = 2.41$ (MeV).

Now, there is a crucial question of our consideration, namely, whether or not the oxygen isotope ^{26}O, which belongs to the same supermultiplet is a stable one and may, or not be found experimentally. For this nucleus we get in our model $N_f = 0$; $N_b = 5$; $\tau = L = J = T_f = 0$; $T = 5$ and

$$E_6 = E_0 + 5b + 30\varepsilon = -184.31 \text{ MeV}\,.$$

Let us compare the calculated binding energy of ^{26}O with — from one side — the experimental value of ^{24}O and — from the other side — with one of the last theoretical prediction for ^{26}O [7] obtained in the relativistic mean field approach (in MeV):

$(E_B - E_C)_{\rm exp}$ for ^{24}O	$(E_B - E_C)_{\rm mean\ field}$ for ^{26}O	$(E_B - E_C)_{\rm supersymmetric}$ for ^{26}O
–187.17	–191.03	–184.31

The numbers show that in the relativistic mean field approach the nucleus ^{26}O seems to be stable against the neutron pair emission while in the

supersymmetric model it is not bound. However, this statement should be falsified by the calculation, in our supersymmetric model, the binding energies of other oxygen isotopes. Hence, let us consider the supermultiplets $N = 2$; 3; 4; 5; and 6 to which belong the oxygen isotopes ^{20}O; ^{22}O; ^{24}O; ^{26}O and ^{28}O respectively. In this part of discussion we take only even-even nuclei for which the supersymmetric binding energy formula takes on the simpler form

$$E = E_0 + b\,N_b + \varepsilon\,T(T+1)\,, \tag{6}$$

where E_0, as before, is the binding energy of the core ^{16}O. After inspection of more than twenty nuclei, we have found the following variation of the parameters b and ε:

1°. The parameter ε is, *in the first approximation*, the same for considered supermultiplets and is equal $\varepsilon = 2.41$ MeV, as for $N = 5$.

2°. The parameter b smoothly changes from one to another supermultiplet according to the rule

$$b_{N-1} - b_N = 0.60 \text{ MeV}\,.$$

Taking the values of the parameters, we present in Tables III and IV the comparison of experimental and theoretical binding energies for even-even nuclei in the supermultiplets $N = 2$, 3, 4, 5, 6.

TABLE III

Comparison of experimental and theoretical binding energies for even-even nuclei belonging to the supermultiplets $N = 2, 3, 4$. We omit here the mirror nuclei from those supermultiplets.

	$N=2$		$N=3$		$N=4$		
	^{20}O	^{20}Ne	^{22}O	^{22}Ne	^{24}O	^{24}Ne	^{24}Mg
$(E_B - E_C)_{\text{exp}}$	–169.66	–186.58	–180.16	–208.68	–187.14	–217.75	–233.36
$(E_B - E_C)_{\text{th}}$	–172	–186	–180	–204	–184	–218	–232

TABLE IV

The same as Table III but for supermultiplets $N = 5, 6$.

	$N=5$		$N=6$			
	^{26}O	^{26}Mg	^{28}O	^{28}Ne	^{28}Mg	^{28}Si
$(E_B - E_C)_{\text{exp}}$	—	–251.78	—	–233.09	–266.73	–282.31
$(E_B - E_C)_{\text{th}}$	−184	−252	−181	−234	−268	−282

The most interesting are the calculated binding energies of the nuclei ^{24}O; ^{26}O and ^{28}O which are equal to –184; –184; and –181 (MeV) respectively. Hence, in our model, in the first approximation, there is no conclusion about stability of the exotic nucleus ^{26}O. Beyond any theoretical doubts, the isotope ^{28}O is not a stable one. Hence, we perform more detailed treatment in which we allow for a small variation of the parameter ε in such a way as to get almost perfect reproduction of binding energies of known nuclei. The small variation of the parameter ε is given on Fig. 2. Let us notice the small values of the parameter in the middle of the *sd* neutron shell.

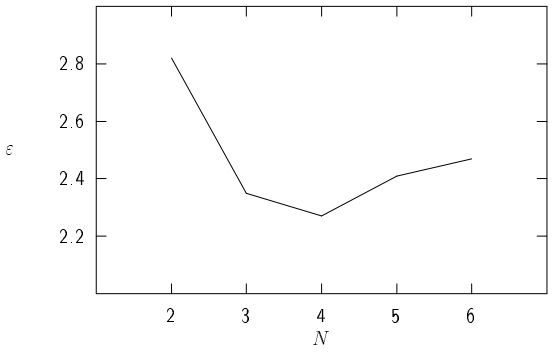

Fig. 2. The small variation of the parameter ε versus the supermultiplet label N.

Table V presents the experimental data and detailed calculation of the binding energies of oxygen nuclei together with other theoretical predictions, known from literature.

TABLE V

Final results for oxygen isotopes, including the Coulomb contribution E_{C}=+18.29 MeV, and comparison with experimental data and other theoretical predictions.

	^{18}O	^{20}O	^{22}O	^{24}O	^{26}O	^{28}O
E_{exp}	–139.81	–151.37	–161.87	–168.85	—	—
E_{th} (this work)	–139.79	–151.37	–161.85	–168.82	–166.02	–160.38
E_{th} [5]					–168.43	—
E_{th} [7]				–170.46	–172.94	–177.40
E_{th} [8]				–165.39	–167.85	–166.01
E_{th} [9, 10]				–168.48	–169.66	–168.88

In our more exact calculations the conclusion is clear: the last bound oxygen isotope is the ^{24}O. This conclusion is contrary to the other theoretical prediction, but it is in accord with the recent experimental search [11].

4. Search for the mass formula

In this section we present briefly somewhat more general approach to the calculations of a nuclear binding energy. Details of the method have been given in [3].

As in the previous sections the main assumption is a dynamical symmetry of the Hamiltonian. We try to construct the most general Hamilton operator taking into account the first and second order Casimir invariants ($\hat{C}_i$) of all groups (algebras) of the appropriate chain:

$$H = \sum_i e_{1i}\hat{C}_1[g_i] + \sum_i e_{2i}\hat{C}_2[g_i]\,, \tag{7}$$

where e_{1i}, e_{2i} are the coefficients constant, in principle, for a given supermultiplet. The next step is to determine the irreducible representations (IRs) of considered groups which describe the lowest energy states. The values of quantum numbers such as the total spin and isospin, and respective groups, have direct physical meaning, but the proper choice of IRs of some other groups needs more detailed studies of excited levels and also electromagnetic transitions.

In [3] we discuss the IBM4 bosons, *i.e.* bosons with isospin and spin quantum numbers, the same as those of a pair of nucleons in LS coupling. Fermions, *i.e.* unpaired nucleons can occupy $j = 1/2$, $3/2$ and $5/2$ levels. Obviously, the fermion spin and isospin are equal to $1/2$. The largest supersymmetry group is then the unitary supergroup U(36/24). Not entering the details, it should be added that in the orbital part of the group chain we adopted the SO(6) symmetry while in the spin-isospin part we constructed boson-fermion SU(4) group. The lowest representations of this group are determined, in analogy to the shell model, by the lowest value of the Casimir invariant.

The lowest eigenvalue of the Hamiltonian (7) corresponds to the nuclear part of the binding energy and can be written in the form

$$E_\eta = \alpha_\eta N + \beta N^2 + \gamma_\eta T + \delta T^2 + \kappa_\eta\,, \tag{8}$$

where N is a total number of supersymmetric particles (*i.e.* sum of a number of bosons and fermions). T is the isospin of the ground state, and the coefficients α_η, β, γ_η, δ and κ_η are the linear combinations of the original ones e_i. The subscript η distinguishes even-even, odd-odd and odd nuclei and also both halves of the *sd* shell, what is explained below.

The values of parameters α, β, γ, δ and κ are, in principle, given separately for each supermultiplet. However, we made the following observation. The parameters e_i smoothly change, in principle, from one to the other supermultiplet either increasing or decreasing. The final parameters (8) being linear combinations of the parameters e_i, mostly diminish the differences of their values between the *sd* shell supermultiplets. It is the ground for introducing the approximation by taking the constant parameters for each half of the *sd* shell and separately for even-even, odd-odd and odd nuclei.

We are aware that in comparison with experimental binding energies expected differences will point out not only on the approximation of the supersymmetry, but also on the approximated constant values of the final parameters which then are fitted to experimental energies.

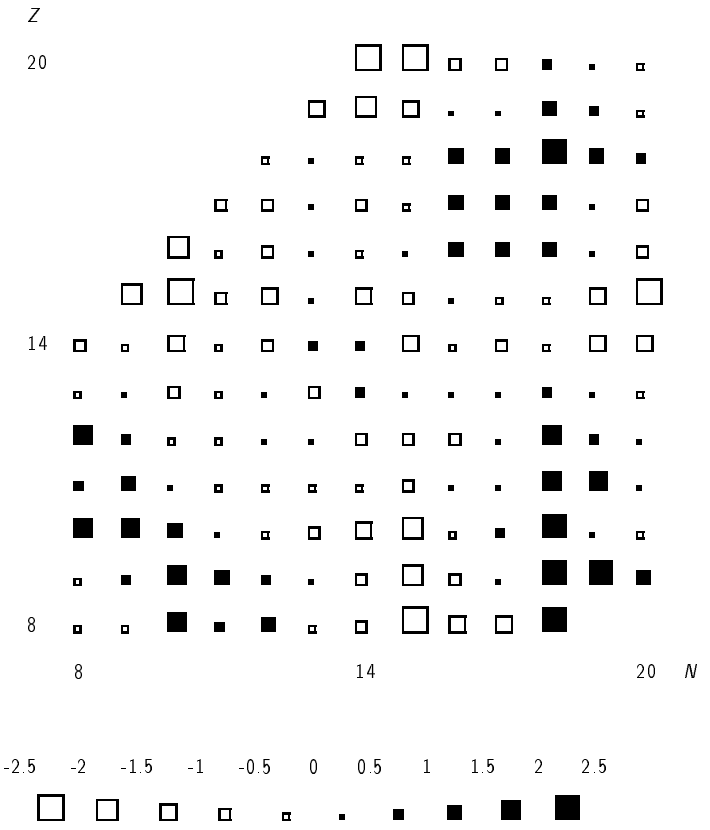

Fig. 3. Difference (in MeV) between calculated and experimental [5] binding energy of nuclei in the *sd* shell. The size of the squares corresponds to the absolute value of the difference.

After adding the Coulomb energy, which is calculated following the shell model approach [6], one can compare obtained results with experimental data [5]. The difference between theoretical and experimental binding energy for the *sd* shell nuclei is presented in Figure 3. For each half of the shell there are about 70 calculated values with ten free parameters of the model.

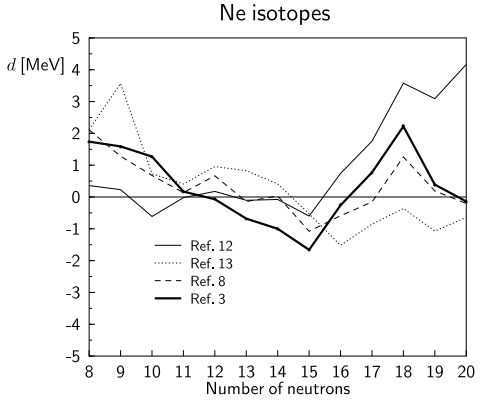

Fig. 4. Difference d (in MeV) between calculated and experimental [5] binding energy of Ne isotopes for the supersymmetric model and other theoretical approaches: the Liran–Zeldes model [12], the Möller–Nix macroscopic-microscopic model [13], the finite range droplet model [8].

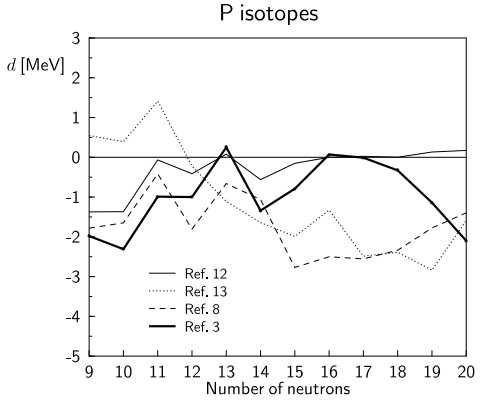

Fig. 5. The same as in Fig. 4 but for P isotopes.

In Figs 4 and 5 we also present the comparison of our results with other theoretical predictions of the binding energies for the Ne and P isotopes. Figure 4 shows that our supersymmetry results are, at least, at the same level of accuracy as other theoretical predictions [8, 12, 13].

5. Conclusions

The results of binding energy calculations in the supersymmetric model provide one more argument for existing of the approximate supersymmetry in the *sd* shell nuclei. Besides that, we have also shown the way of dealing with the exotic nuclei in the supersymmetric model.

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