Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 1–2

A MACROSCOPIC MODEL OF NUCLEAR ROTATION* **

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(Received November 24, 1997)

We have formulated a statistical model of nuclear properties that combines the Thomas-Fermi assumption of two fermions per h^3 of phase space with an effective interaction between the nucleons. The model has been employed in the calculation of nuclear masses and density distributions. The initial calculations assumed spherical symmetry but a later extension to three dimensions permits the calculation of fission saddle-point shapes and the corresponding fission barriers. It is also possible to include angular momentum and we have constructed an extension of the model which describes approximately ground-state, superdeformed and fission-isomeric rotational bands of even-even nuclei. The model is based on a three-term energy expression corresponding to: a) a rigid rotation of part of the nucleus, b) the energy of initially counter-rotating gyroscopes that the overall rotation gradually aligns in the direction of the total angular momentum, and c) a potential energy resisting such alignment. The model can be used for a macroscopic description of the angular momentum dependence of nuclear fission barriers.

PACS numbers: 21.10. Dr, 21.10. Re, 21.60. Ev

1. Introduction

Over the last ten years or so we have developed a detailed model of average nuclear properties [1]. It is based on a statistical treatment of the nuclear energy, analogous to the Thomas–Fermi approximation for the description of smoothed electron densities in atoms and molecules. In place of the

* Presented at the XXV Mazurian Lakes School of Physics, Piaski, Poland, August 27–September 6, 1997.

** This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

electrostatic interactions between atomic electrons, an effective short-range nucleon-nucleon potential has been introduced, representing a generalization of the momentum-dependent Seyler–Blanchard Yukawa interaction. At first only spherically symmetric solutions were considered and the resulting model was used to discuss nuclear binding energies, sizes and charge distributions. Then we went on to discuss the relation of the nuclear compressibility to the surface energy and surface diffuseness and the nuclear optical model potential, including its energy and isospin dependences.

The next step was to generalize the discussion to arbitrary nuclear shapes, which makes possible the crucial confrontation of the model with measurements of nuclear fission barriers.

The most recent improvement we have made to this approach is the addition of angular momentum. One reason for extending the model in this way is the need for a macroscopic theory of fission barriers of rotating nuclei. Existing microscopic calculations of fission barriers are not sufficiently advanced to provide comprehensive, quantitative predictions, whereas macroscopic theories [2,3] have not gone beyond the use of rigid moments of inertia.

While these models are probably fine for describing the highly deformed shapes at the fission saddle-point they are clearly inadequate when it comes to describing the band spectra of rapidly rotating nuclei. Deviations of the observed spectra from those of a rigid rotor are usually drastic, indicating that more than an overall rotation of the matter distribution is involved. The deviations are largest for small nuclear deformations and small angular momenta, decreasing for superdeformed nuclei, especially at high spin. The fact that the empirical data exhibit readily discernible overall trends, in addition to local irregularities, provided the motivation for constructing a simple model of the smoothed, average behaviour of rotational bands and of the associated effective moments of inertia.

The model of nuclear rotation to be presented in what follows provides an approximate description of rotational energies in their dependence on nuclear shape and angular momentum, and represents a step toward a macroscopic theory of the fission barriers of rotating nuclei.

2. The model

In addition to the Thomas-Fermi assumption of two fermions per h^3 of phase space we employ an effective interaction between the nucleons having the following form:

$$\begin{aligned} v_{12} \;=\; & -2T_0\rho_0^{-1} f(r_{12}/a)\Big\{ \tfrac{1}{2}(1\mp\xi)\alpha \\ & -\tfrac{1}{2}(1\mp\zeta)\Big[\beta(p_{12}/P_0)^2-\gamma(p_{12}/P_0)^{-1}+\sigma(2\overline{\rho}\rho_0)^{2/3}\Big]\Big\}\,, \qquad (1) \end{aligned}$$

where $\overline{\rho}^{2/3} = ({\rho_1}^{2/3} + {\rho_2}^{2/3})/2$ and ρ_1 and ρ_2 are the relevant densities of the interacting particles (neutrons or protons) at points 1 and 2. The spatial function $f(r_{12}/a)$ has been chosen to be a normalized Yukawa of range a. We have chosen natural units for energy, density and momentum. The equilibrium particle density of standard nuclear matter is ρ_0, the nuclear matter Fermi momentum is P_0 and the nuclear matter Fermi energy is T_0.

By making the total energy of a nucleus stationary with respect to particle-preserving variations in the density of the neutrons and protons, one obtains Euler–Langrange equations for the ground-state neutron and proton density distributions. With considerably more effort one can determine the unstable saddle-point configurations for nuclear fission and the associated heights of the fission barriers. The (optical model) potential felt by a neutron or proton traveling through a nucleus or through nuclear matter, including its energy and isospin dependence, can also be calculated.

The overall optimization and fine tuning of the seven parameters by comparisons with a full range of diverse data has been performed, with the following results:

$$\begin{array}{llllllll} a & = & 0.59294\text{ fm}, & & & & & \\ \alpha & = & 1.94684, & \beta & = & 0.15311, & \gamma & = 1.13672, \\ \sigma & = & 1.05, & \xi & = & 0.27976, & \zeta & = 0.55665\,. \end{array}$$

These values correspond to the following nuclear properties:

radius constant of nuclear matter,	r_0	=	1.14 fm,
volume energy coefficient,	a_1	=	16.24 MeV,
symmetry energy coefficient,	J	=	32.65 MeV,
surface energy coefficient,	a_2	=	18.63 MeV,
curvature correction coefficient,	a_3	=	12.1 MeV,
compressibility coefficient,	K	=	234 MeV.

3. Nuclear masses

In [1] we discuss in more detail the determination of the force coefficients and the fit to the 1654 measured nuclear binding energies. Figure 1, from Ref. [1] shows the remaining deviations between measurement and theory and compares the model being described here with the predictions of earlier work based on the Finite Range Droplet Model [7].

4. Fission barriers

When we extended our calculation capabilities to include deformed nuclei in three dimensions it became possible to calculate fission barriers. A preliminary comparison between the calculated and measured values is contained

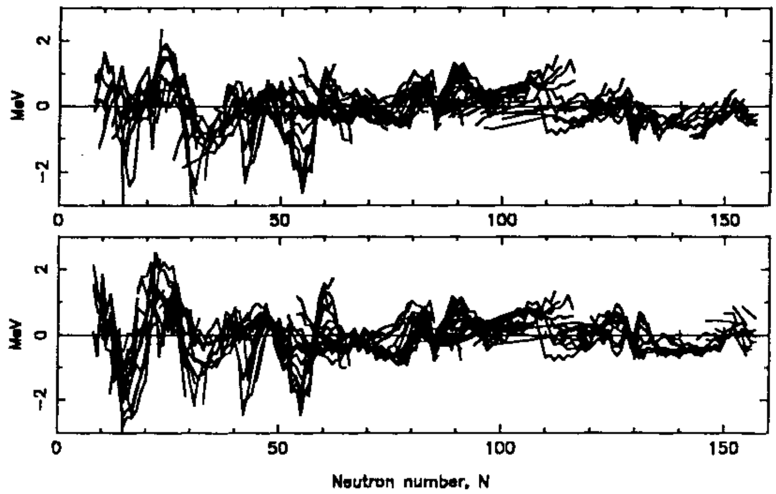

Fig. 1. The difference (measured mass) minus (theoretical mass) for 1654 nuclei. Lines connect isotopes. Upper panel is based on [7], lower panel on the present model.

in Ref. [1] and the results of a more recent comparison [8] are displayed in Fig. 2.

5. The rotation model

The model nucleus rotating with angular momentum $\vec{L}$ is assumed to consist of two components, and its energy to be composed of: a) the energy of part of the nuclear matter assumed to rotate rigidly about the direction of $\vec{L}$; b) the energy of any number of pairs of identical gyroscopes initially counter-rotating around axes at right angles to $\vec{L}$, which the overall nuclear rotation gradually aligns along $\vec{L}$; and c) a potential energy which resists such alignment. Thus:

$$E(L,\theta) = \frac{(L-\ell\sin\theta)^2}{2(1-\nu)J} + \frac{\ell^2}{2\nu J} + \frac{1}{2}k\sin^2\theta \ . \tag{2}$$

In this expression ℓ (assumed to be a function of L) is the sum of the magnitudes of all the gyroscopes' angular momenta, νJ is the sum of their moments of inertia (about their respective rotation axes), $(1-\nu)J$ is the moment of inertia of the rigidly rotating part, whose angular momentum is L reduced by the angular momentum contributed by the gyroscopes. This contribution is equal to $\ell\sin\theta$, where θ is the common angle between the axis of a gyroscope and the plane normal to $\vec{L}$. The parameter k characterises

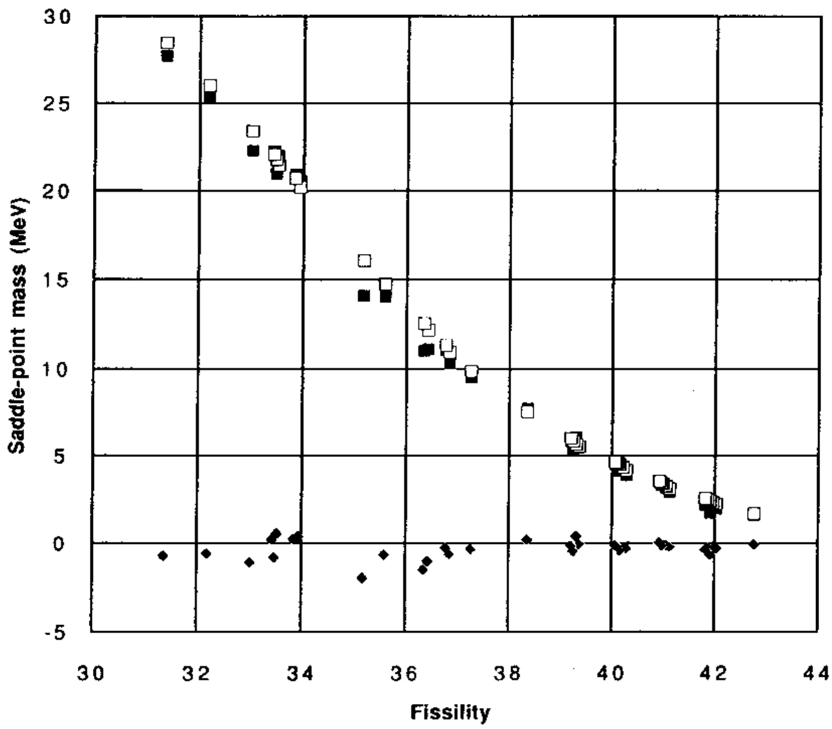

Fig. 2. A comparison of measured (solid squares) and calculated fission barriers, with the shape dependence of the congruence energy [8] included. The black diamonds show the remaining differences. "Fissility" is defined as $Z/A(1 - 2.2I^2)$.

the strength of the potential resisting the alignment of the gyroscopes along $\vec{L}$. This potential, designed to simulate nuclear pairing, is taken to depend in a simple cyclic way on the angle θ. The symbol J represents the moment of inertia of the nuclear density distribution.

The form of Eq. (2) was inspired in the first place by the paper of Stephens and Simon [4], but also by the theories of pair breaking associated with the names of Mottelson, Valatin, Belyaev and Migdal, as described in Ref. [5]. The common thread in these approaches is that pairing of nucleons in time-reversed orbits inhibits the nucleons from contributing their share to the total angular momentum L. With increasing L the pairing is more or less gradually (or more or less abruptly) broken. The Mottelson–Valatin–Belyaev–Migdal approach draws on the macroscopic analogy with the destruction of superconductivity by a magnetic field. Stephens and Simon pointed out that microscopic effects can be very important, so that the alignment of one or two high-j pairs can produce large oscillations (even back-bending) on top of any gradual quenching of the pairing correlations. The underlying physics is, however, the same: The destruction of pairing, caused by rotation, in (a portion of) the nuclear matter. Our gyroscopes are meant to represent, in some average sense, this originally paired matter, giving reduced moments of inertia for low spins and rigid rotation at high spins (when pairing has been destroyed).

Assuming that $\ell = \nu L$ (see below) and minimizing the energy with respect to θ, we find:

$$E(L) = \frac{L^2}{2J}\left[\nu + \frac{1}{1-\nu+\nu(L/L_1)^2}\right] \quad \text{for } L \le L_1 \tag{3}$$

$$E(L) = \frac{L^2}{2J} + \frac{1}{2}k \quad \text{for } L \ge L_1 \ , \tag{4}$$

where

$${L_1}^2 = kJ/\nu. \tag{5}$$

There are two limiting cases which have guided our choice of the dependence of ℓ on L. As regards large L, the choice of $\ell = \nu L$ follows from the requirement that after pairing has been destroyed, the moment of inertia should be rigid. For small L it would be reasonable to have ℓ tend to a constant $\ell_\circ$ related to some average intrinsic angular momentum of the paired nucleons. In Ref. [6] we have tested the sensitivity of our model to the finiteness of $\ell_\circ$. We find that neglecting $\ell_\circ$ and having $\ell = \nu L$ for all L might be an acceptable approximation.

If the rigid-body moment of inertia J is assumed to be known, eqs. (3,4) become formulae with two adjustable parameters, ν and k (or ν and L_1), which can be used to fit rotational spectra of individual nuclei. In this sense they are similar to the two-parameter scheme of the Variable Moment of Inertia Model (VMI) [9]. For many ground-state bands the quality of the fits using either the VMI or our current model turned out to be quite similar. But, unlike the VMI formulae, the present model is able to describe the approach (and transition) to rigid rotation, which is observed to take place for large angular momenta and/or large deformations.

As regards the pairing strength k we shall assume that in momentum space pairing effects are confined to a fixed neighborhood of the Fermi sphere, which implies that k should be proportional to a fixed fraction of the particle number A. Thus

$$k = \kappa A \ , \tag{6}$$

where κ is a constant, which we adjusted to have the value 0.036 MeV.

In searching for a global prescription for the shape dependence of the parameter ν we became aware of a curious systematics of low-spin effective moments of inertia, illustrated in Fig. 3. The various symbols in this figure show the experimental values of the effective moment of inertia at angular momenta close to zero, divided by the rigid moment of inertia (taken about a minor axis) of a spheroid with semi-axes $a, a, c > a$ and volume $(4\pi/3)R^3$, where $R = 1.2A^{1/3}$ fm. The values of c/a at which the points are plotted were deduced from the measured intrinsic quadrupole moments Q of the nuclei in question.

The lower curve in Fig. 3 corresponds to irrotational hydrodynamical flow inside the spheroid. The middle curve corresponds to the moment of inertia of what remains of a spheroid (its 'tips') after removal of an inscribed sphere whose radius is equal to the minor semi-axis a. Thus, at low spin, we have a curious 'as if' feature of nuclear rotations: nuclei behave approximately as if their tips were rotating rigidly, and the inscribed sphere were standing still. Without implying that paired nuclear matter simulated by the gyroscopes is actually confined inside such a sphere and that the rigidly rotating matter is located in the tips, we experimented with the prescription that νJ in Eq. (2) should be the moment of inertia of a suitably defined inscribed sphere. The moment of inertia $(1-\nu)J$ in the first term in Eq. (2) would then be the moment of inertia of the complementary tips. This assumption turned out to work fairly well in reproducing observed rotational bands, but its theoretical justification remains an open question.

The formulation just described can be applied directly to nuclei idealized as sharp-surfaced or diffuse spheroids. Obvious improvements of such an idealization include, first, replacing the spheroids by density distributions that follow, for a given Q, from solving the Thomas-Fermi equations of Ref. [1]. (This is essential for the discussion of fission barriers). Second, the radius r of the inscribed sphere used to estimate ν may be considered as a (somewhat) adjustable parameter. We adopted the following procedure for calculating r. An effective neck radius a was deduced from the Thomas-Fermi calculations and then slightly reduced by the amount δ. The radius r of the inscribed sphere is then calculated as

$$r = a - \delta \ , \tag{7}$$

where δ is an adjustable parameter, for which we adopted the value 0.5 fm. The relative moment of inertia ν was then calculated as the ratio of the rigid moment of inertia of a sphere of density $\rho_\circ$ and radius r to the rigid moment of inertia J of the Thomas-Fermi density distribution, taken about the rotation axis.

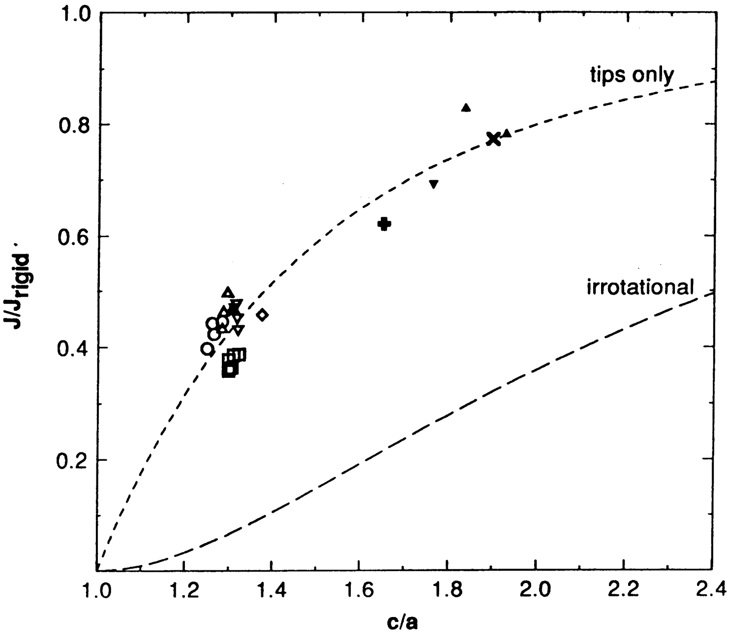

Fig. 3. Systematics of experimental effective moments of inertia divided by the rigid moment of inertia of a spheroid (with semi-axes $a, a, c > a$) rotating about a minor axis. The cluster of points on the left refers to rare-earth and actinide nuclei. The vertical cross refers to superdeformed ^{194}Hg, the next point up to superdeformed ^{194}Pb and the three remaining points to fission-isomeric states in ^{236}U, ^{238}U and ^{240}Pu. The values of c/a were deduced from the measured quadrupole moments, using the standard radius constant $r_0 = 1.14$ fm. The moments of inertia are based on $r_0 = 1.2$ fm, allowing approximately for surface diffuseness. The long-dashed curve refers to the assumption of irrotational flow inside the spheroid and the short-dashed curve to the moment of inertia of what remains of a rigidly rotating spheroid after the removal of an inscribed core of radius a.

6. Comparison with measurements

In Ref. [6] we compare measured and calculated gamma ray energies γ_I and the associated level energies E_I using the global parameter set $\kappa = 0.036$ MeV, δ =0.5 fm. We display eight ground-state bands (^{154,156}Dy, ^{160}Er, ^{172}Hf, ^{178}Os, ^{232}Th, ^{238}U and ^{244}Pu), four superdeformed bands (^{194}Pb, ^{194}Hg, ^{152}Dy and ^{132}Ce) and two fission-isomeric bands (^{236}U and ^{240}Pu). Here we only have space to consider a few of these cases. In Fig. 4 the data correspond to the ground-state bands of ^{160}Er and ^{172}Hf followed by a band (or bands) estimated to be the lowest-energy (yrast) band at the given spin. (All experimental data are from Ref. [10].)

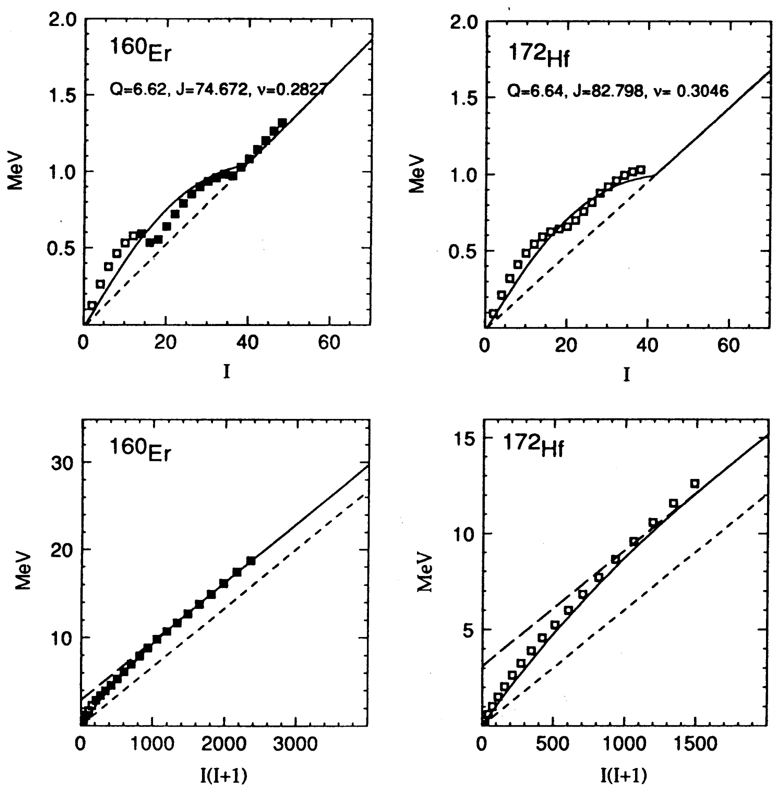

Fig. 4. The upper panels show the gamma-ray energies as functions of the spin I, the lower panels the corresponding level energies as functions of $I(I+1)$. The experimental points for ^{160}Er and ^{172}Hf are from Ref. [10]. The solid curves are the model's predictions, based on the measured quadrupole moment Q indicated, and on the rigid moment of inertia J and core moment νJ that follow from a Thomas-Fermi calculation constrained to have that quadrupole moment. (We used Eqs. (3), (4), with $L^2 = I(I+1)\hbar^2$). The short-dashed lines refer to the rigid rotor predictions. The long-dashed line is the energy of a rigid rotor augmented by the alignment energy $k/2$.

In the case of the superdeformed bands in ^{194}Pb and ^{194}Hg in Fig. 5 the agreement between theory and experiment is quite close. We should point out that in the case of ^{194}Hg we took the liberty of assuming the quadrupole moment to be $Q = 19.2$ barn, when the quoted measurement is 17.2 ± 2 barn. In other cases we took the nominal measured value of Q as the constraint in our Thomas-Fermi calculations (from which followed the rigid moment of inertia J and the parameter ν).

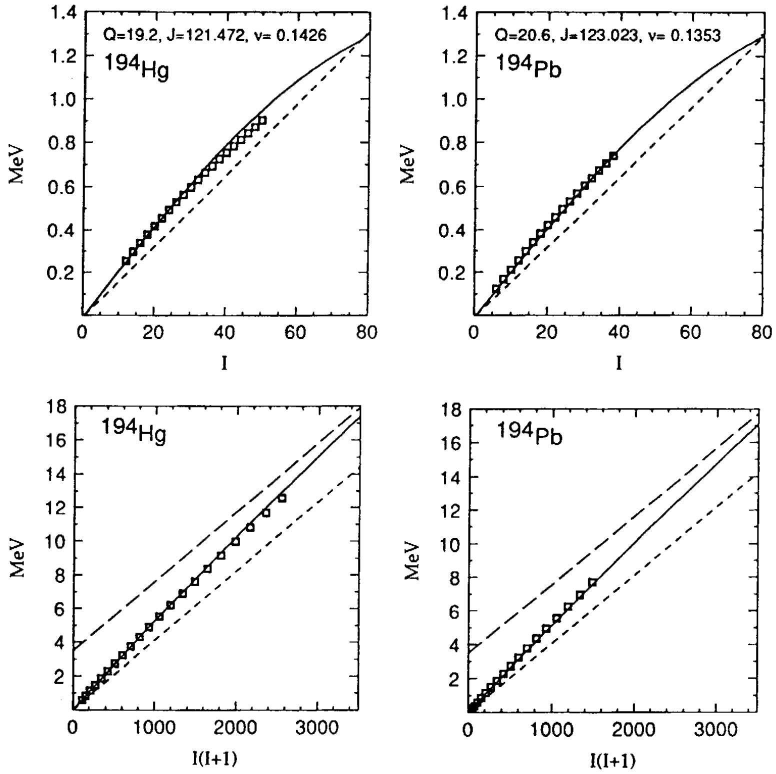

Fig. 5. This is like Fig. 4 but for ^{194}Hg and ^{194}Pb.

7. Discussion

The objective of our model was to approximate the increase in nuclear mass (decrease in binding energy) caused by rotation. Figs. 4 and 5 together with the other cases considered in [6] show that these mass increases (ranging up to 30 MeV) are usually reproduced to better than 1 MeV. Our model achieves this at the price of one freely adjustable parameter κ and a second parameter $\delta = 0.5$ fm chosen to slightly reduce the size of the inscribed sphere. From a practical point of view this seems like an acceptable performance for a macroscopic model of nuclear masses and deformations.

We would like to thank M.A. Deleplanque-Stephens, P. Fallon, I-Y. Lee, A. Machiavelli, L. Moretto, M. Redlich, G. Wozniak and, especially, F. Stephens for illuminating discussions on many aspects of rotational spectra and fission barriers. The beautifully produced compilations of nuclear data in Ref. [10] were of great help and we thank R. Firestone and F. Chu for making them available before publication.

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