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SEMICLASSICAL MODEL OF LOW-ENERGY REACTIONS WITH LOOSELY BOUND PROJECTILES* **

V.P. Aleshin and B.I. Sidorenko

Institute for Nuclear Research
252028 Kiev, Ukraine

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We develop a microscopic classical trajectory approach to low energy reactions induced by projectiles which are loosely bound towards decay in two or three particles. The reactions are assumed to proceed as dissociation of the projectile into its constituent particles, each of which may be absorbed by the target or bypass it. The model is applied to $d-^{93}$Nb collision at $E = 15$–25 MeV. The calculated values of (d,p), (d,n), (d,np), and complete fusion cross sections are in reasonable agreement with quantum results and available experimental data. As another application we calculate the integrated cross sections for $(^6\mathrm{He},n)$, $(^6\mathrm{He},\alpha)$, $(^6\mathrm{He},nn)$, $(^6\mathrm{He},\alpha n)$, $(^6\mathrm{He},\alpha nn)$, and complete fusion reactions following the $^6\mathrm{He}+^{232}\mathrm{Th}$ collision at few MeV above the Coulomb barrier.

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1. Introduction

In recent years, experiments have been performed on the emission of loosely bound particles, such as d, t [1, 2] and radioactive isotopes of He, Li and Be [3] from hot nuclei. In the theoretical description of the emission of such particles a fundamental ingredient is provided by the corresponding transmission coefficients, usually taken from the reversed reaction of the complete fusion of the emitted particle with the residual nucleus.

For loosely-bound projectiles, the complete fusion proceeds in a strong competition with noncomplete fusion reactions, in which breakup of the projectile is followed by fusion of the 'core' particle with the target nucleus.

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The integrated cross sections for such processes could be described within the so-called breakup-fusion (BF) approach [4, 5].

It should be realized, however, that the BF method requires knowledge of projectile–target optical–model (OM) potentials which are not well known for neutron drip line nuclei. Moreover the OM description of both elastic scattering and reaction cross section for halo–nuclei may be not possible by same OM parameters [6]. Besides, the BF approach is restricted to two-particle decays, whereas a typical halo nucleus breaks up into three or more particles [7].

In this work we attempt a microscopic–semiclassical (MS) description of the reactions involving loosely bound nuclei, which avoids using the optical projectile-target potential and is applicable to projectiles consisting of two or three subsystems which can be treated as structureless.

2. MS approach

To explain the idea of the MS model [8] consider the deuteron-induced reactions. Let us introduce an ensemble of N deuterons moving towards a heavy nucleus. At time $t = 0$, all deuterons $s = 1, 2, \ldots N$ are assumed to have the same position $\boldsymbol{R}$ and momentum $\boldsymbol{P}$ but may differ by the intrinsic $n - p$ coordinate $\boldsymbol{r} = r_p - r_n$ and corresponding momentum $\boldsymbol{k} = \frac{1}{2}(\boldsymbol{p}_p - \boldsymbol{p}_n)$.

The value of R is arbitrary, much larger than the radius of the target. The values of $\boldsymbol{R}/R$ and $\boldsymbol{P}$ should be consistent with the fact that all deuterons have energy E and angular momentum $\boldsymbol{L}$. The absolute values of $\boldsymbol{r}$ are distributed in accordance with $\varphi_d^2(r) = r^2 \left|\psi(r)\right|^2$, where $\psi(r)$ is the deuteron intrinsic wave function, the directions $\hat{\boldsymbol{r}}$ are distributed isotropically and the intrinsic momenta are taken from the classical expression

$$\boldsymbol{k}^{(s)} = \pm\sqrt{2\mu\left(-B_d - V(r^{(s)})\right)}\,\hat{\boldsymbol{r}}^{(s)}, \tag{1}$$

where μ is the reduced mass of the $n-p$ pair, B_d is the binding energy of the deuteron, $V(r)$ is the $n-p$ interaction potential. Eq. (1) takes into account that the intrinsic deuteron orbital momentum is zero.

Assuming the target nucleus to be so heavy that recoil is negligible we can characterize the whole system by the positions of neutron $\boldsymbol{r}_n^{(s)}(t)$ and proton $\boldsymbol{r}_p^{(s)}(t)$, and corresponding momenta $\boldsymbol{p}_n^{(s)}(t)$ and $\boldsymbol{p}_p^{(s)}(t)$. These are obtained from Hamiltonian equations containing besides $V(r)$ the real nucleon-target interaction potentials V_n and V_p.

The most important quantities to be found for each phase space trajectory are the current intrinsic energies of $n - p$ pairs $\varepsilon_t^{(s)}$, and the survival factors ($i = n$ or p)

$$P_i^{(s)} = \exp\left(\frac{2}{\hbar}\int\limits_0^{\infty} \Theta(\varepsilon_t^{(s)}) W_i[r_i^{(s)}(t)]dt\right),$$

where $W_n(r_n)$, $W_p(r_p)$ are the imaginary nucleon–nucleus potentials.

The dissociation probability $F_d(t)$ is determined as the relative number of the members of the ensemble for which $\varepsilon_t^{(s)}$ is positive

$$F_d(t) = \frac{1}{N}\sum_{s=1}^{N} \Theta(\varepsilon_t^{(s)}).$$

The asymptotic value of $F_d(t)$ at $t \to \infty$ provides the total reaction transmission coefficient $T_R(L)$

$$T_R(L) = F_d(\infty). \tag{2}$$

The survival factors give the probability for the particle (n or p) to avoid absorption by the target nucleus, the complementary factors $Q_i^{(s)} = 1 - P_i^{(s)}$ being the probability to be absorbed. The step function $\Theta(\varepsilon_t^{(s)})$ entering $P_i^{(s)}$ ensures that the absorption gains its contributions from that part (or parts) of the trajectory when n and p are not bound to each other.

The factors $P_i^{(s)}$, $Q_i^{(s)}$ are utilized to calculate the partial probabilities ('transmission factors') of (d,p), (d,n), and (d,np)–reactions

$$\begin{aligned}
T_{d,p}(L) &= \frac{1}{N}\sum_{s=1}^{N} P_p^{(s)} Q_n^{(s)}\,, \\
T_{d,n}(L) &= \frac{1}{N}\sum_{s=1}^{N} P_n^{(s)} Q_p^{(s)}\,, \\
T_{d,pn}(L) &= \frac{1}{N}\sum_{s=1}^{N} P_p^{(s)} P_n^{(s)} \Theta(\varepsilon_\infty^{(s)})\,,
\end{aligned}$$

which combined with (2) allow one to find the complete fusion transmission coefficient

$$T_{\rm fus}(L) = T_R(L) - T_{d,p}(L) - T_{d,n}(L) - T_{d,pn}(L)\,.$$

In the following calculations, we use the Hulthen-like wave function $\psi_{\rm H}$. The Hulthen potential has singularity at $r = 0$ which causes difficulties in the trajectory calculations. To avoid these, we replaced it with the exponential potential $-V_0 \exp\left(-\frac{r}{r_0}\right)$, where V_0 =125 MeV, $r_0 = 0.87$ fm. It provides the correct binding energy of the deuteron and the wave function very close to that in the Hulthen potential.

The optical potentials for nucleons are taken from [9]. The Coulomb potential is that of a uniformly charged sphere with the radius parameter $r_c = 1.25$ fm. The statistical ensemble consisted of $N = 100$ deuterons. The random numbers distributed according to $r^2\psi_H(r)^2$ were generated by the acceptance-rejection method.

In Table I we compare the cross sections

$$\sigma_c = \frac{2\pi\hbar^2}{k^2}\sum_L (2L+1)T_c(L)\,,$$

where $c = (d,p)$, (d,n), (d,np), fus and R with the BF calculation by Mastroleo *et al.* [5]. The comparison is shown for $d-^{93}$Nb collision at 15 and 25 MeV. One observes a very good overall agreement between two methods. The only serious deviation (about 30%) is detected for $\sigma_{d,np}$ at $E = 15$ MeV. This cross section, however, comprises a small part of the non-fusion reactions. The experimental data on $\sigma_{d,p} + \sigma_{d,np}$ from [10] are 436 and 514 mb at E =15 and 25 MeV, respectively. The corresponding MS results (440 and 501mb, respectively) are in close agreement.

TABLE I

Calculated cross sections (mb) in the $d-^{93}$Nb collision

	MS	BF [5]	MS	BF [5]
E_d, MeV	15	15	25	25
σ_R	1497	1505	1867	1806
$\sigma_{d,p}$	369	392	397	356
$\sigma_{d,n}$	227	240	304	267
$\sigma_{d,np}$	71	49	104	91
$\sigma_{\rm fus}$	830	824	1062	1092

3. Three-particle projectiles

In the extension of the MS model to 3-particle projectiles, the most difficult problem is the construction of a classical ensemble, compatible with the appropriate ground state wave function Ψ. To solve this problem it is convenient to use the hyperspherical harmonic method. To be specific we take an example of ^{6}He projectile, a recognized halo nucleus with the $2n$-separation energy $B = 0.97$ MeV, which will be treated as consisting of α, $n1$, and $n2$.

The translationally invariant normalized Jacobi coordinates $\boldsymbol{x}$, $\boldsymbol{y}$ and the momenta $\boldsymbol{k}_x$, $\boldsymbol{k}_y$ conjugate to them are determined according to [11]. In its simplest form, the hyperspherical harmonic method assumes that the potential $V(\boldsymbol{x}, \boldsymbol{y})$ can be represented by an 'average' field $V(\rho)$ depending only on the hyperradius $\rho = \sqrt{x^2 + y^2}$.

As a first step here and guided by calculations [12] we assume for ^{6}He a pure K=2 state with $\boldsymbol{l}_x$=0, $\boldsymbol{l}_y$=0. Here K is the hypermomentum quantum number, $\boldsymbol{l}_x = \boldsymbol{x} \times \boldsymbol{k}_x$ and $\boldsymbol{l}_y = \boldsymbol{y} \times \boldsymbol{k}_y$. Then, up to normalization factor, the distribution over $\boldsymbol{x}$, $\boldsymbol{y}$ can be written as

$$|\Psi(\boldsymbol{x}, \boldsymbol{y})|^2 \, d\boldsymbol{x} d\boldsymbol{y} \sim \chi^2(\rho) \sin^2 4\theta d\theta d\hat{\boldsymbol{x}} d\hat{\boldsymbol{y}},$$

where $\theta \in \left[0, \frac{\pi}{2}\right]$, is the angular variable defined by $\tan\theta = x/y$ and $\chi(\rho)$ is obtained from the Schrödinger equation

$$\frac{\hbar^2}{2m}\left[-\frac{d^2\chi}{d\rho^2} + \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2}\chi\right] + V(\rho)\chi = -B\chi,$$

where m is the nucleon mass, $\mathcal{L} = K + \frac{3}{2}$.

The specific values of ρ and θ should be sampled in accordance with $\chi^2(\rho)$ and $\sin^2 4\theta$, respectively. Then the initial Jakobi coordinates are given by

$$\boldsymbol{x} = \rho \sin\theta \, \hat{\boldsymbol{x}}, \;\; \boldsymbol{y} = \rho\cos\theta \, \hat{\boldsymbol{y}}\,,$$

where $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$ are isotropic unit vectors.

Given $\boldsymbol{x}$, $\boldsymbol{y}$, the assumption $l_x = l_y = 0$ allows one to determine $\boldsymbol{k}_x$, $\boldsymbol{k}_y$ from the expressions

$$\boldsymbol{k}_x = p_x \hat{\boldsymbol{x}}, \;\; \boldsymbol{k}_y = p_y \hat{\boldsymbol{y}},$$

where

$$p_x = p_\rho \sin\theta + \frac{p_\theta}{\rho}\cos\theta, \;\; p_y = p_\rho\cos\theta - \frac{p_\theta}{\rho}\sin\theta,$$

$$p_\rho = \pm\sqrt{2m\left(-B - V(\rho)\right) - \frac{p_\theta^2}{\rho^2}},\ p_\theta = \pm\hbar\sqrt{\mathcal{L}(\mathcal{L}+1)}.$$

To illustrate this formalism, consider the ^{6}He+^{232}Th collisions at few MeV above the Coulomb barrier. The potential $V(\rho)$ for ^{6}He is taken from Ref. [13]. The neutron optical potential is taken from the same source [9] as in the deuteron case. The parameters of the α–Th optical potential ($r_c = 1.3$ fm, $V_0 = -200$ MeV, $r_0 = 1.3$ fm, $a = 0.6$ fm, $W = -36$ MeV, $r_W = 1.6$ fm, $a_W = 0.44$ fm) are taken from [14]. These were obtained by fitting the experimental angular distribution of α particles elastically scattered on ^{232}Th at 23.5 MeV.

The partial cross sections of (^{6}He$, n$), (^{6}He$, \alpha$), (^{6}He$, nn$), (^{6}He$, \alpha n$), (^{6}He$, \alpha nn$) and complete fusion reactions on ^{232}Th together with the total reaction partial cross sections are shown in Fig. 1. The calculation is performed at E =25.5 MeV which is about 3.8 MeV higher than the Coulomb barrier estimated assuming point like ^{6}He. The corresponding grazing angular momentum defined in the potential $V_\alpha(R) + 2V_n(R)$ was found to be $L_{gr} = 11.8\hbar$. For each L we sampled $N = 1200$ collisions.

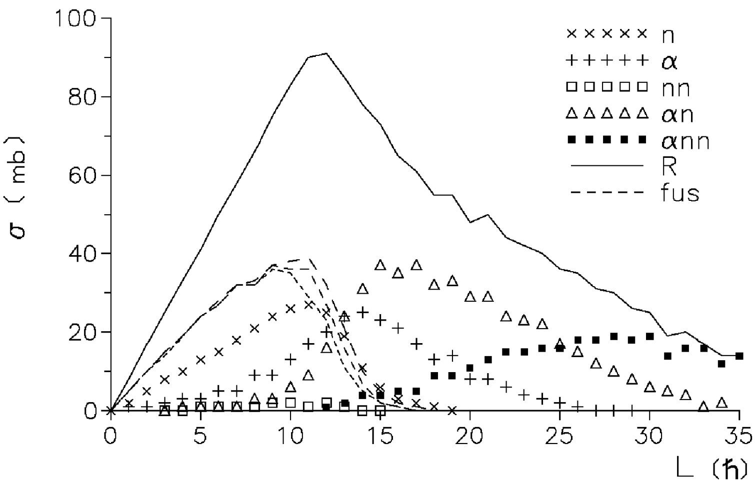

Fig. 1. Partial cross sections for ^{6}He+^{232}Th at 25.5 MeV. Short dashed and long dashed lines represent $\sigma_{\rm fus}$ on ^{222}Th and ^{242}Th, respectively.

Only at low L ($L < L_{gr}$), the $\sigma_{\rm fus}(L)$ comprise an essential part of $\sigma_R(L)$. The (^{6}He$, n$) and (^{6}He$, \alpha$) reactions come into play at very low L. The (^{6}He$, \alpha n$) and especially (^{6}He$, \alpha nn$) cross sections are extending to very large L (corresponding distances of closest approach are two-three times larger than the barrier radius) which is due to Coulomb breakup.

The (^{6}He, nn) reaction is strongly suppressed at all L because of Coulomb repulsion of α–core.

To probe the sensitivity of the complete fusion transmission coefficients to the neutron excess in the target nucleus, we show $\sigma_{\rm fus}$ of ^{6}He with ^{222}Th and ^{242}Th. One can see an increase of $\sigma_{\rm fus}$ as the neutron excess increases. This is in qualitative agreement with an increase of the ratio ^{6}He/^{4,3}He with growing neutron excess of the emitting system observed by Dempsey *et al.* [3] in the emission from the composite systems formed in the $^{124,136}_{54}$Xe+$^{112,124}_{50}$Sn collisions at 55A MeV.

4. Conclusion

To conclude, we formulated a simple semiclassical model for interactions of loosely bound projectiles with heavy nuclei. The model provides the energy-integrated cross sections of incomplete fusion reactions. It can be used for estimating the n and $2n$ removal cross sections in low-energy collisions with halo nuclei.

The present scheme is easy to apply, especially because it does not need the optical potential for projectiles. In this respect it is similar to the Glauber type microscopic models [15–17] used at high energies.

Despite its simplicity the model is in good agreement with the available quantum calculations. The complete fusion transmission coefficients generated within this model can be used in the statistical calculations of the evaporative emission of loosely bound particles from hot nuclei.

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