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WARM NUCLEI: THE TANSITION FROM INDEPENDENT PARTICLE MOTION TO COLLISIONAL DOMINANCE* **

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We study large scale collective dynamics of isoscalar type and examine the influence of interactions residual to independent particle motion. It is argued that for excitations which commonly are present in experimental situations such interactions must not be neglected. With respect to dissipation, our results are contrasted with those of wall friction.

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1. Introduction

After the discovery of the shell model it has become customary to base the description of collective motion on the picture of single particles moving independently within a deformed mean field. This approach was introduced in the early 50–thies by A. Bohr and B. Mottelson to portray low energetic collective excitations, and to the present day there can be little doubt that this approximation is adequate for that regime. It is somewhat astonishing, however, that this picture still is vindicated by many groups even for situations where the nucleons are heated up to considerable amount, say to temperatures of a few MeV. After all, in the very early days of nuclear physics collective motion of large scale was considered to be governed by

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dissipative processes, which in turn imply the presence of fluctuating forces. Such a picture may be condensed into the one equation, which was suggested by Kramers [1] already in 1940 to describe nuclear fission. It reads

$$\frac{\partial}{\partial t}f(Q,P,t) = \left[-\frac{\partial}{\partial Q}\frac{P}{M}+\frac{\partial}{\partial P}\left(\frac{dV(Q)}{dQ}\right)+\frac{\partial}{\partial P}\frac{P}{M}\gamma+D_{pp}\frac{\partial^2}{\partial P\partial P}\right]f(Q,P,t)\,, \tag{1}$$

and has the structure typical of a Fokker–Planck equation. As is well known, Kramers has used this equation to calculate the decay rate for a metastable situation like fission, in generalization of the famous Bohr–Wheeler formula. In these days the origin of dissipation was attributed to the strong "correlations" among the nucleons, as they can be understood within or follow from N. Bohr's compound nucleus.

In this talk we want to look at this transition from "independent particle motion to collisional dominance" in the view of the "linear response approach" (LRT), a complete version of which can be found in [2]. This discussion will be complemented by presenting new aspects in the relation to wall friction, following the more recent considerations of the group of W.J. Swiatecki, J. Blocki and others (see [3]).

2. Linear response theory for collective motion

In the sequel let's suppose to be given a Hamiltonian $\hat{H}(\hat{x}_i,\hat{p}_i,Q)$ for the nucleons' dynamics in a deformed mean field, with the deformation being parameterized by the shape variable Q, whose average $\langle\hat{H}(\hat{x}_i,\hat{p}_i,Q)\rangle$ represents the total energy of the system $E_{\rm tot}$. The equation of motion for $Q(t)$ can then be constructed from energy conservation

$$0=\frac{d}{dt}E_{\rm tot}=\dot{Q}\left\langle\frac{\partial\hat{H}(\hat{x}_i,\hat{p}_i,Q)}{\partial Q}\right\rangle_t\equiv\dot{Q}\left\langle\hat{F}(\hat{x}_i,\hat{p}_i,Q)\right\rangle_t\,. \tag{2}$$

All one needs to do to get the equation of motion for $Q(t)$ is to express the average $\left\langle\hat{F}(\hat{x}_i,\hat{p}_i,Q)\right\rangle_t$ as a functional of $Q(t)$. Following the scheme of the locally harmonic approximation one may expand the $\hat{H}(Q)$ around any given Q_0 to have:

$$\hat{H}(Q(t))=\hat{H}(Q_0)+(Q(t)-Q_0)\hat{F}+\frac{1}{2}(Q(t)-Q_0)^2\left\langle\frac{\partial^2\hat{H}}{\partial Q^2}(Q_0)\right\rangle_{Q_0,T_0}^{\rm qs}\,. \tag{3}$$

The effects of the coupling term $(Q(t)-Q_0)\hat{F}$ may now be treated by LRT, exploiting as a powerful tool the causal response function $\tilde{\chi}$

$$\tilde{\chi}(t-s)=\Theta(t-s)\frac{i}{\hbar}{\rm tr}\left(\hat{\rho}_{\rm qs}(Q_0,T_0)[\hat{F}^I(t),\hat{F}^I(s)]\right)\equiv 2i\Theta(t-s)\tilde{\chi}''(t-s)\,. \tag{4}$$

Here, the time evolution in $\hat{F}^I(t)$ as well as the density operator $\hat{\rho}_{\rm qs}$ are determined by $H(Q_0)$. The $\hat{\rho}_{\rm qs}$ is meant to represent thermal equilibrium at Q_0. The transport coefficients for average motion can be introduced by approximation of the $\chi_{\rm coll}(\omega)$ by an oscillator response function $\chi_{\rm osc}(\omega)$

$$\begin{aligned}(\chi_{\rm coll}(\omega))^{-1}\ \delta\langle\hat{F}\rangle_\omega &\simeq (\chi_{\rm osc}(\omega))^{-1}\ \delta\langle\hat{F}\rangle_\omega \\ &\equiv \left(-M\omega^2 - \gamma i\omega + C\right)\ \delta\langle\hat{F}\rangle_\omega = -f_{\rm ext}(\omega)\,. \end{aligned} \tag{5}$$

As shown first in [4] (for the damped self-consistent case) the equation $dE_{\rm tot}/dt = 0$ can be rewritten as

$$-\frac{d}{dt}E_{\rm coll} \equiv -\frac{d}{dt}\left(\frac{M(\omega_1)}{2}\dot{q}^2 + \frac{C(\omega_1)}{2}q^2\right) = \gamma(\omega_1)\dot{q}^2 \equiv T\frac{d}{dt}S \tag{6}$$

which correctly expresses the exchange between collective motion into heat. (The ω_1 represents one of the possible (complex!) frequencies of the system, as determined from the secular equation).

3. Forced energy transfer to a system of independent particles

As is clearly seen from (6), the friction force parameterizes that energy which is transferred *irreversibly* to the intrinsic system. Let us study this feature within a simple model, with the simplifications consisting first of all in neglecting self-consistency. This means that we take a nucleus at given deformation Q_0 which is *exposed to a time dependent external field.* We may thus use a Hamiltonian of the type given in (3). The $Q(t) - Q_0$ is then a truly *external* quantity, which shall be called $q(t)$ in the sequel. As another important simplification we will assume the $\hat{H}(Q_0)$ to represent the ensemble of *independent particles* as given by the deformed shell model at *zero temperature.*

Such a system has been studied in a series of papers which aimed at a new understanding of the physics of wall friction (see [3] and references given there). The time dependence of the $q(t)$ was assumed to be of the form $q(t) = q_0 \sin(\Omega t)$ and the system was followed for one period simulating the solutions of the Schrödinger equation numerically.

Let us examine this problem within LRT. The energy transferred to the intrinsic degrees of freedom within one cycle may be evaluated from the following well known formula:

$$\overline{\Delta E_{\rm int}} = -\int\limits_{-T/2}^{T/2} dt \int\limits_{-\infty}^{\infty} ds\, \dot{q}(t)\tilde{\chi}(t-s)q(s) = \pi q_0^2 \chi''(\Omega)\,. \tag{7}$$

This result may be compared with those of [3], we simply need to identify $\overline{\Delta E_{\rm int}}$ with their ΔE and calculate the total unperturbed energy E_0 as the sum of single particle energies.

In Figs. 1 and 2 we present numerical results for the quantity $\Delta E/E_0 = \overline{\Delta E_{\rm int}}/E_0$ for the case of quadrupole excitations. They were calculated on the basis of our formula (7) but for the same system as in [3], namely independent particles in a Woods–Saxon potential (of an un-physically large depth to decrease the escape probability). All parameters are chosen like there. The frequency of oscillations ω is expressed in terms of ratio η of the highest wall speed to the speed of fastest particle. Approximately $\eta \approx 0.02269\hbar\omega$. The most striking is difference to the (quantal) results presented in Fig. 1 of [3]. In our case we observe strong oscillations with η, which represent nothing else but the typical strength function behavior. In both figures we show as the straight line marked by dots the result one gets in case that this energy transfer is calculated with wall friction. Apparently the latter result can be obtained at best after performing some averages.

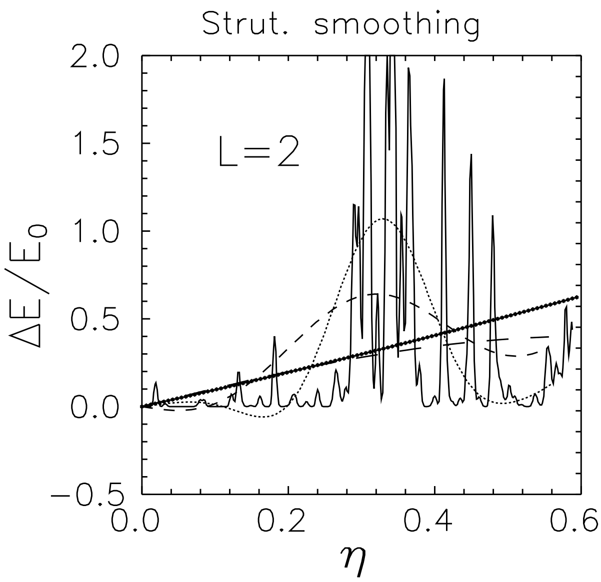

Fig. 1. Average energy transfer to a spherical system of independent particles by an external quadrupole perturbation, calculated within linear response. Otherwise the same picture is adopted as in [3].

Indeed, it has been shown in [5] that the friction coefficient obtained within LRT (in the zero frequency limit) becomes close to the one of the wall formula after applying smoothing procedures in the sense of the Strutinsky method. This features goes along very nicely with the claim that wall friction represents the "macroscopic limit" for a system of independent particles (for an extensive discussion of this topic see [2]). In Figs. 1 and 2 we present curves obtained from applying Strutinsky smoothing to the microscopic evaluations: For the dotted lines the averaging interval was 5 MeV, for the short dashed ones 10 MeV and for the long dashed ones 20 MeV. From Fig. 1 it is seen that and how smoothing leads to results similar to that of the wall formula. This calculation corresponds to the case where Q_0 stands for a spherical deformation, the same situation which has been considered also in [5]. The one presented in Fig. 2 corresponds to a case where the unperturbed system has a sizable octupole deformation of $\alpha = 0.3$. In this case the wall formula is not recovered. We may say that similar results are obtained for vibrations of other multi-polarity.

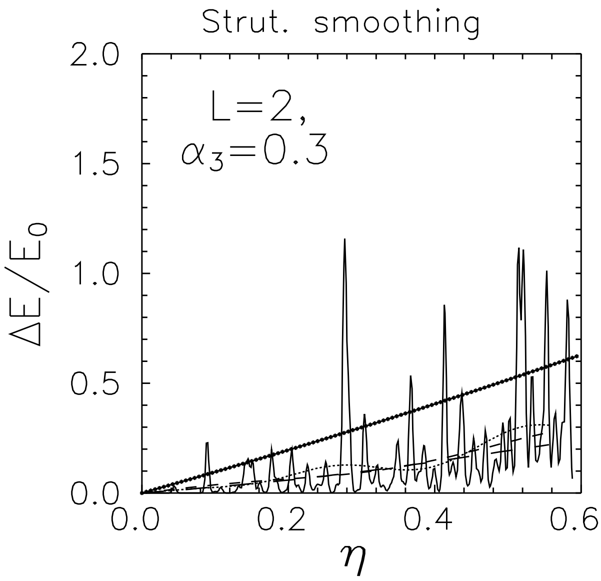

Fig. 2. The same as for Fig. 1 but for a system with octupole deformation.

4. The influence of collisional damping on transport properties

From the discussion of the last section it is clearly seen that for nuclear collective motion it is *not possible to justify a local friction force within the mere picture of independent particles.* For such a model one has to employ averaging procedures of one kind or other. Moreover, we have observed that quite large intervals in the averaging parameters are involved if for the latter one chooses energy. This fact clearly hints to an inherent deficiency of the underlying model: At the excitations which are at stake in common experimental situations the picture of particles moving in a mean field *without "collisions" does not apply*! For this reason the notion of the $\hat{H}(Q_0)$ to be simply given by the deformed shell model has been given up a long time ago whenever transport properties where calculated within the linear response approach (see [2] for a detailed discussion). Instead it was assumed that the particles are dressed by self-energies having both real and imaginary parts: $\Sigma(\omega \pm i\epsilon, T) = \Sigma'(\omega, T) \mp i\Gamma(\omega, T)/2$. The intrinsic response functions are then calculated after replacing the single particle strength $\varrho_k(\omega) = 2\pi\, \delta(\hbar\omega - e_k)$ by

$$\begin{aligned} \varrho_k &= \frac{\Gamma(\omega,T)}{(\hbar\omega - e_k - \Sigma'(\omega))^2 + \Gamma^2(\omega,T)/4}\,, \\ \Gamma(\omega,T) &= \frac{1}{\Gamma_0}\,\frac{(\hbar\omega-\mu)^2+\pi^2T^2}{1+\left[(\hbar\omega-\mu)^2+\pi^2T^2\right]/c^2} \end{aligned} \tag{8}$$

with the μ being the chemical potential.

In Fig. 3 we present the reduced friction coefficient (friction over inertia) calculated along a fission path of ^{224}Th for different temperatures (given in MeV). All curves except the one marked by triangles are identical to those of Fig. 13 of [6], where for the deformed shell model the Pashkevich code has been employed; details can be found in [6]. It is seen (*i*) that this ratio γ/M does not change very much with the collective variables as soon as T is of the order of 2 MeV or larger, and (*ii*) that it *increases with* T (for reasons given below, the "heat pole" contribution has been removed in this calculation). For larger T the ratio is of the order as predicted by the wall formula (for γ) plus the one of irrotational flow for the inertia. The reason for this behavior is due to the fact that with increasing T the residual interactions become more and more important, with the two implications of a) smoothing out details of shell structure and b) making the microscopic mechanism of dissipation more effective.

For $T = 1$ we have included a preliminary result (marked by triangles) of an extension of our theory to the inclusion of pairing correlations. As expected the latter reduce the influence of shell effects, albeit details still will have to be clarified further [7].

5. The role of symmetries and the heat pole for nuclear friction

Above it has been indicated that for the calculations presented in Fig. 3 a particular contribution to friction was discarded. This shows up at finite T and is related to an interesting quasi-static property which in turns is dominated by the influence of symmetries (for a detailed discussion see [5, 2]). Let us demonstrate these features with the help of the zero frequency limit

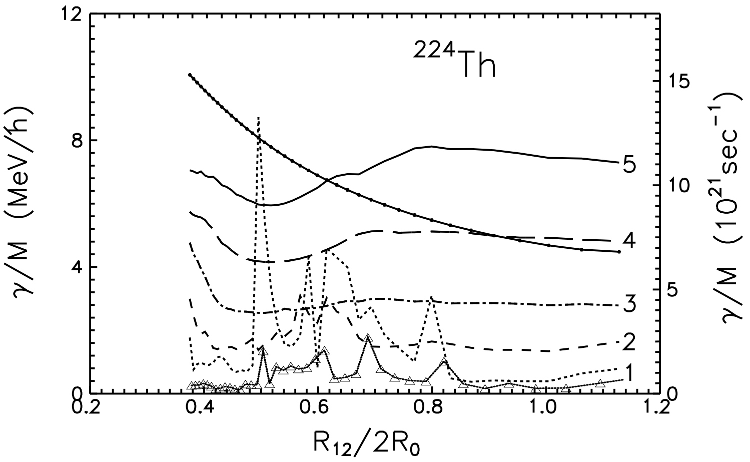

Fig. 3. Ratio of friction to inertia along the fission path of ^{224}Th (for details see text). The deformation parameter is the distance R_{12} between left and right centers of mass divided by the diameter $2R_0$ of the sphere of identical volume.

of friction. To sufficient accuracy the latter can be written as

$$\gamma(0) = \frac{\partial \chi''(\omega)}{\partial \omega}\bigg|_{\omega=0} = \frac{\psi''(\omega=0)}{2T}\,. \tag{9}$$

On the very right the correlation function has been introduced which is related to the response function by the famous fluctuation dissipation theorem (FDT) $\hbar\chi''(\omega) = \tanh(\hbar\omega/2T)\,\psi''(\omega)$. The definition of ψ is similar to that given for χ in (4), with two important exceptions: The commutator is to be replaced by an *anti-commutator* and from the operator $\hat{F}$ one has to subtract its unperturbed average value $\langle\hat{F}\rangle$. The general microscopic expression for $\psi''(\omega)$ is

$$\psi''(\omega) = \psi^0 2\pi\delta(\omega) \;+\; {}_R\psi''(\omega) \qquad \text{with} \qquad \psi^0 = T\Big(\chi^T - \chi(0)\Big) \tag{10}$$

with the ${}_R\psi''(\omega)$ being regular at $\omega = 0$. The χ^T is the *isothermal susceptibility* which measures how the (quasi-)static expectation value $\langle\hat{F}\rangle^{qs}$ changes with Q if the temperature is kept constant. The singularity at $\omega = 0$ is called "heat pole", in analogy to a similar pole in the density density strength distribution for infinite matter being responsible for heat diffusion there. When applied to (9) it turns out that the heat pole implies the following contribution to friction

$${}_0\gamma(0) = \frac{\hbar}{\Gamma(\mu,T)}\frac{\psi^0}{T} = \frac{\hbar}{\Gamma(\mu,T)}\Big(\chi^T - \chi(0)\Big)\,. \tag{11}$$

The "heat pole" contribution to friction (11) is shown in Fig. 4. The fully drawn line and the dashed one correspond to ${}_0\gamma(0)$ of (11) with the c of (8) put equal to $c = 20$ MeV and $c \to \infty$, respectively. The curve with the heavy dots corresponds to the contribution of the remaining part of the correlation function. As demonstrated in [5] (see also [2]), the distinction of the two contributions can simply be made in terms of the matrix elements of the (one-body) operator $\hat{F}$ with the shell model states. The ${}_0\gamma(0)$ is solely to be associated to the *diagonal* elements. That they may lead to dissipation, nevertheless, (and thus to entropy production) is due to the effects of "collisions".

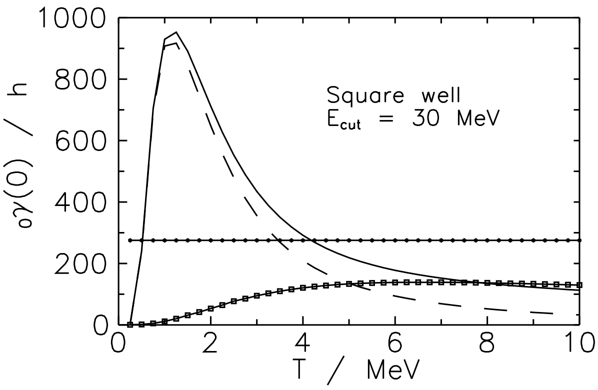

Fig. 4. The contribution of the "heat pole" to friction for collective quadrupole oscillations of a system of particles in a square well potential (see text).

6. Dissipation within Landau theory

As seen above, the friction coefficient tends to decrease with T at larger temperature. This feature is evident for the component ${}_0\gamma(0)$ (see (11) and Fig. 4), but as discussed in [5] it will eventually hold true also for the other component (see also [8]) under certain circumstances (like approximating the imaginary part of the self-energy in "common" relaxation time approximation (with $c = \infty$)).

Such a behavior with T reminds one of the two body viscosity of hydrodynamics. In [9] a model has been suggested in which the intrinsic dynamics is described by the Landau–Vlasov equation and where the finiteness of the system is considered in terms of special boundary conditions. In Fig. 5 we present a calculation of the friction coefficient (as function of T) for quadrupole vibrations around a sphere, done within an extension of this model to higher temperatures and lower frequencies, [10]. The dashed and the fully drawn lines correspond to the hydrodynamical limit, for two different choices of the parameter c entering the relaxation time used in the collision term of the Landau-Vlasov equation. The squares correspond to contributions from different peaks in the correlation function, where the full ones are supposed to correspond to the analog of the "heat pole". Similarities with the behavior shown in Fig. 4 are evident. So far however, it is yet unclear where exactly the contribution comes from. Further studies are under way.

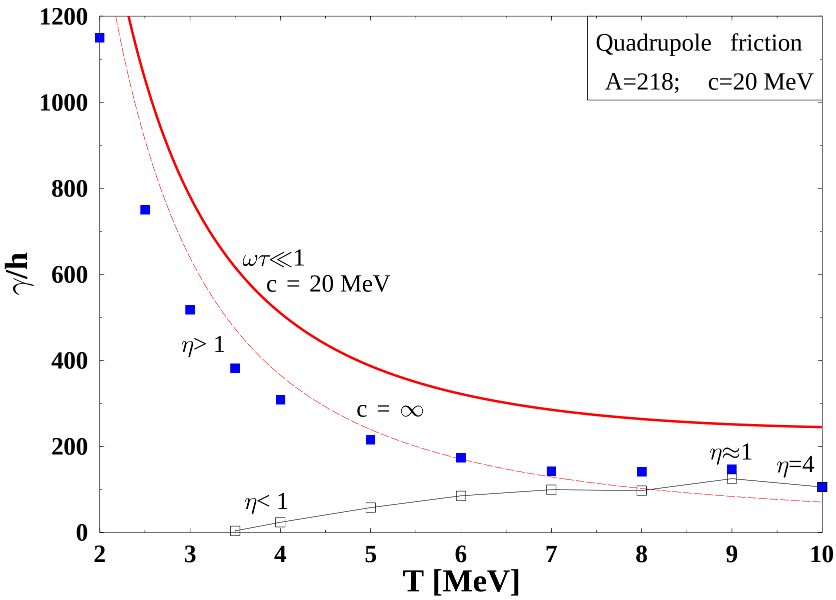

Fig. 5. Friction for quadrupole oscillations calculated from a Landau–Vlasov approach to a finite nucleus.

7. Summary

To describe collective motion as a Markovian transport process one needs to be able to define transport coefficients which vary smoothly with the macroscopic variables, which by the way have to include the parameter which measures the thermal excitation. We have demonstrated that such a condition can hardly be fulfilled within the picture of the deformed shell model. On the other hand, we have shown that residual interactions may do the job, the better the larger is the thermal excitation. At present the situation is less clear at smaller temperatures. Whether or not pairing alone will do is currently under investigation. It may well be, however, that even in this regime one may want to include more of the configurations as given by the nuclear compound model.

REFERENCES

[1] H.A. Kramers, *Physica* **7**, 284 (1940).

[2] H. Hofmann, *Phys. Rep.* **284**, 137 (1997).

[3] J. Blocki, J. Skalski, W.J. Swiatecki, *Nucl. Phys.* **A594**, 137 (1995).

[4] D. Kiderlen, H. Hofmann, F.A. Ivanyuk, *Nucl. Phys.* **A550**, 473 (1992).

[5] H. Hofmann, F.A. Ivanyuk, S. Yamaji, *Nucl. Phys.* **A598**, 187 (1996).

[6] F.A. Ivanyuk, H. Hofmann, V.V. Pashkevich, S. Yamaji, *Phys. Rev.* **C55**, 1730 (1997).

[7] F.A. Ivanyuk, H. Hofmann, to be published.

[8] S. Yamaji, F.A. Ivanyuk, H. Hofmann, *Nucl. Phys.* **A612**, 1 (1997).

[9] A.G. Magner, H. Hofmann, V.M. Kolomietz, S. Shlomo, *Phys. Rev.* **C51**, 2457 (1995).

[10] A.G. Magner, H. Hofmann, to be published.