A STUDY OF THE ${ }^{90} \mathrm{Zr}(\mathrm{p}, \mathrm{t}){ }^{88} \mathrm{Zr}$ REACTION *<br>M. JASKÓŁA ${ }^{\dagger}$<br>Sołtan Institute for Nuclear Studies, Świerk 05-400 Otwock, Poland<br>P. Guazzoni, L. Zetta<br>Dipartimento di Fisica dell'Università and I.N.F.N<br>I-20133 Milano, Italy<br>J.N. GU ${ }^{\ddagger}$, A. Vitturi<br>Dipartimento di Fisica dell'Università and I.N.F.N<br>I-35100 Padova, Italy<br>G. Graw, R. Hertenberger, B. Valnion<br>Sektion Physik der Universität München<br>D-85748 Garching, Germany<br>F. Nuoffer and G. Staudt<br>Physikalisches Institut der Universität<br>D-72076 Tübingen, Germany

(Received November 24, 1997)
The ${ }^{90} \mathrm{Zr}(\mathrm{p}, \mathrm{t})^{88} \mathrm{Zr}$ reaction has been studied in a high resolution experiment at an incident energy of 25 MeV . Angular distributions for transitions to the levels of ${ }^{88} \mathrm{Zr}$ up to an excitation energy of $\sim 3.1 \mathrm{MeV}$ have been measured. The data have been analyzed by means of the DWBA theory using double folding triton potential. The energy spectrum of ${ }^{88} \mathrm{Zr}$ has been studied in the framework of shell model.

PACS numbers: 25.40. Hs, 27.50. +e, 21.60. Cs

[^0]
## 1. Experimental procedure and results

The investigation of the ${ }^{90} \mathrm{Zr}(p, t)^{88} \mathrm{Zr}$ reaction has been performed to obtain accurate experimental data of differential cross sections. These data are needed for both spectroscopic studies of the nucleus ${ }^{88} \mathrm{Zr}$ and for comparison with the angular distributions obtained in an investigation of the ${ }^{91} \mathrm{Zr}(p, t){ }^{89} \mathrm{Zr}$ reaction in order to study if the two neutrons in the $(p, t)$ reactions are transferred pairwise from the same orbit or from different orbits.

A 25 MeV proton beam from the Munich University MP tandem accelerator, bombarded a $50 \mu \mathrm{~g} / \mathrm{cm}^{2}$ target of ${ }^{90} \mathrm{Zr}$ enriched to about $97 \%$, on a $12 \mu \mathrm{~g} / \mathrm{cm}^{2}$ carbon backing. Outgoing tritons have been detected in the focal plane of the Q3D magnetic spectrometer by the light-ion focal plane detector with periodic readout [1].

Absolute cross sections are estimated with an uncertainty of $\pm 15 \%$. The energy calibration was carried out using the following peaks from Ref. [2]: $1.818,2.140,2.456,2.539,2.605,2.888$ and 3.033 MeV . The estimated energy accuracy is $\pm 3 \mathrm{keV}$.

In Table I the energies of the levels of ${ }^{88} \mathrm{Zr}$ and the attributed spins and parities are given together with the energies, spins and parities of ${ }^{88} \mathrm{Zr}$ levels adopted so far [2] and with the energies, spins and parities of the levels observed in previous $(p, t)$ reactions [3, 4].

TABLE I
${ }^{88} \mathrm{Zr}$ levels

| Present work |  | ( $p, t$ ) reaction [3] |  | ( $p, t$ ) reaction [4] |  | Adopted levels [2] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E_{x}$ | $J^{\pi}$ | $E_{x}$ | $J^{\pi}$ | $E_{x}$ | $J^{\pi}$ | $E_{x}$ | $J^{\pi}$ |
| 0 | $0^{+}$ | 0 | $0^{+}$ | 0 | $0^{+}$ | 0 | $0^{+}$ |
| 1.057 | $2^{+}$ | 1.055 | $2^{+}$ | 1.057 | $2^{+}$ | 1.057 | $2^{+}$ |
| 1.521 | $0^{+}$ | 1.520 | $0^{+}$ | 1.517 | $0^{+}$ | 1.521 | $0^{+}$ |
| 1.818 | $\left(2^{+}\right)$ | 1.820 | $\left(2^{+}, 4^{+}\right)$ | 1.816 |  | 1.818 | $\left(2^{+}\right)$ |
| 2.140 | $4^{+}$ | 2.130 | $4^{+}$ | 2.134 | $4^{+}$ | 2.140 | $4^{+}$ |
| 2.225 | $0^{+}$ | 2.225 | $0^{+}$ | 2.225 | $0^{+}$ | 2.225 | $0^{+}$ |
| 2.456 | $3^{-}$ | 2.445 | $3^{-}$ | 2.446 | $3^{-}$ | 2.456 | $3^{-}$ |
| 2.539 | $5^{-}$ | 2.52 | $\left(4^{+}\right)$ |  |  | 2.539 | (5-) |
| 2.570 | $2^{+}$ | 2.57 | $2^{+}$ |  |  | 2.570 | $2^{+}$ |
| 2.605 | $4^{+}$ | 2.60 | $\left(4^{+}, 6^{+}\right)$ |  |  | 2.605 | $\left(4^{+}\right)$ |
|  |  |  |  |  |  | 2.674 |  |
| 2.801 | $5^{-}$ | 2.795 | $5^{-}$ | 2.793 | $5^{-}$ | 2.801 | $5^{-}$ |
| 2.811 | $6^{+}$ |  |  |  |  | 2.811 | $\left(6^{+}\right)$ |
| 2.888 | $\left(2^{+}\right)$ | 2.89 | $(4,6,8)$ | 2.875 | $\left(8^{+}, 6^{+}\right)$ | 2.888 | $\left(8^{+}\right)$ |
| 2.928 | $3^{-}$ |  |  |  |  |  |  |
| 2.990 | $5^{-}$ |  |  |  |  | 2.990 | $\left(3^{-}, 4^{-}, 5^{-}\right)$ |
|  |  |  |  |  |  | 2.998 |  |
| 3.027 | $2^{+}$ |  |  |  |  |  |  |
| 3.033 | $3^{-}$ | 3.02 | $2^{+},\left(4^{+}\right)$ |  |  | 3.033 | $\left(3,4^{+}\right)$ |
|  |  | 3.06 | $\left(4^{+}\right)$ |  |  | 3.060 | $\left(4^{+}\right)$ |
| 3.092 | $5^{-}$ |  |  |  |  |  |  |

## 2. Distorted wave analysis

The experimental reaction data have been analyzed using a doublefolding procedure to calculate the real part of the triton-nucleus potential in the framework of the model of Kobos et al. [5].

The potential is described by

$$
U_{F}(\vec{r})=\lambda \int d \vec{r}_{1} \int d \vec{r}_{2} \rho_{T}\left(\vec{r}_{1}\right) \rho_{\alpha}\left(\vec{r}_{2}\right) t\left(E, \rho_{T}, \rho_{\alpha}, \vec{s}\right),
$$

where $\vec{s}=\vec{r}+\overrightarrow{r_{2}}-\overrightarrow{r_{1}}, \vec{r}$ is the separation of the centers of mass of the target nucleus and the triton, $\rho_{T}\left(\vec{r}_{1}\right)$ and $\rho_{t}\left(\vec{r}_{2}\right)$ are the respective nucleon densities, and $\lambda$ is an overall normalization factor. The effective nucleonnucleon interaction $t$ calculated in nuclear matter from the Bonn potential and parameterised in terms of a local density- and energy-dependent twobody interaction [6] is used. The target nucleus density distribution $\rho_{T}$ was the experimental one [7] obtained from electron scattering and unfolded from the finite-charge distribution of the proton. A Gaussian form [8] for the density distribution of the triton, and a volume Woods-Saxon potential for the imaginary part have been used.

The DWBA calculations have been made in the finite-range approximation with a cluster form-factor, using the computer code DWUCK5 [9]. The angular distribution of the ground-state transition was fitted with the code TROMF [10], which allows a simultaneous fit to both the elastic-scattering data in the entrance and exit channels and the reaction data [11].

For the proton channel the experimental data from Van Der Bijl et al. [12] at $\mathrm{E}_{p}=25.05 \mathrm{MeV}$ have been used, and for the triton channel the experimental data from Hardekopf et al. [13] for 15 MeV triton elastic scattering from ${ }^{90} \mathrm{Zr}$ have been used. In Table II the parameters resulting from this analysis are reported.

TABLE II
Optical model parameters

|  | $V_{r}$ | $r_{r}$ | $a_{r}$ | $\lambda$ | $W_{d}$ | $r_{d}$ | $a_{d}$ | $V_{\text {so }}$ | $r_{\text {so }}$ | $a_{\text {so }}$ | $r_{c}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p$ | 50.2 | 1.20 | 0.69 |  | 6.76 | 1.3 | 0.69 | 5.9 | 1.072 | 0.63 | 1.264 |
|  |  |  |  |  | $W_{v}$ | $r_{v}$ | $a_{v}$ |  |  |  |  |
| $p$ |  |  |  |  | 1.56 | 1.236 | 0.69 |  |  |  |  |
|  | $V_{r}$ | $r_{r}$ | $a_{r}$ | $\lambda$ | $W_{v}$ | $r_{v}$ | $a_{v}$ | $V_{s o}$ | $r_{s o}$ | $a_{\text {so }}$ | $r_{c}$ |
| $t$ |  |  |  | 1.12 | 16.6 | 1.598 | 0.527 | 7.1 | 1.126 | 0.537 | 1.30 |
| B.S. | $r$ | $a$ |  |  |  |  |  |  |  |  |  |
|  | 1.30 | 0.60 |  |  |  |  |  |  |  |  |  |

${ }^{90} \mathbf{Z r}(\mathbf{p}, \mathrm{p})$

${ }^{90} \operatorname{Zr}(\mathbf{t}, \mathbf{t})$


$$
{ }^{90} \mathrm{Zr}(\mathbf{p}, \mathbf{t})^{88} \mathrm{Zr}
$$




Fig. 1. The results of the fitting procedure with respect to the elastic scattering data, for the entrance and exit channels, are shown together with the angular distributions for ${ }^{88} \mathrm{Zr}$ levels up to 2.539 MeV state, compared with the calculations performed using Double-Folded triton potential.


Fig. 2. Angular distributions for ${ }^{88} \mathrm{Zr}$ levels up to 3.092 MeV state, compared with the calculations performed using Double-Folded triton potential.

Since only natural-parity states are allowed in the one-step $(p, t)$ reaction process, each final level excited by pick-up of a neutron pair from the eveneven target ${ }^{90} \mathrm{Zr}\left(J^{\pi}=0^{+}\right)$, will be populated with a unique $L$-transfer. Assignment of $L$-transfer values then yields directly spin-parity assignments for the observed states in ${ }^{88} \mathrm{Zr}$.

As shown in Figs 1 and 2, the fit to the experimental data is rather good.
The $L=0$ DWBA calculations for the G.S. and for the 2.225 MeV level and $L=2$ for levels at $1.057 \mathrm{MeV}, 2.570 \mathrm{MeV}$ and 3.027 MeV particularly well reproduce the experimental data, while the $L=2$ theoretical curve reproduces the mean slope of the measured angular distribution for the 1.818 MeV and 2.888 MeV levels.

The levels at 2.140 and 2.605 MeV are well described by the $L=4$ transfer.

The main maximum of the angular distribution for the level at 2.811 MeV is reproduced by the $L=6$ transfer, while the part at forward angles systematically deviates from the experimental values.

For negative-parity states, the levels at $2.456,2.928$ and 3.033 MeV are quite well reproduced by the $L=3$ transfer and the levels at 2.539, 2.801 and 3.092 MeV by the $L=5$ transfer.

## 3. Shell model calculations

In connection with the experimental work here presented, we will add some preliminary theoretical predictions of the energy spectrum in the framework of shell model, using the OXBASH code [14].

For the calculations, the shell model Hamiltonian can be written as

$$
H=\sum_{i} \epsilon_{i} a_{i}^{\dagger} a_{i}+\sum_{i j k l} V_{i j k l} a_{i}^{\dagger} a_{j}^{\dagger} a_{k} a_{l}
$$

For calculating the two body matrix elements $V_{i j k l}$, the PMM90 interaction [14], which was introduced by B.A. Brown, is used, while the model space has been constructed from the $1 f_{5 / 2}, 2 p_{3 / 2}, 2 p_{1 / 2}$ and $1 g_{9 / 2}$ orbitals for both neutrons and protons.

The allowed occupation number, in square brackets, for each orbital and the corresponding total single particle energies $\epsilon_{i}(\mathrm{MeV})$ used to generate both positive (PPS) and negative (NPS) parity states are given in the following:

PPS: $1 f_{5 / 2}[10-12] 2.30 \mathrm{MeV}, 2 p_{3 / 2}[6-8] 3.40 \mathrm{MeV}, 2 p_{1 / 2}[2-4] 5.13$ $\mathrm{MeV}, 1 g_{9 / 2}[8-10] 2.75 \mathrm{MeV}$;
NPS: $1 f_{5 / 2}[12] 2.30 \mathrm{MeV}, 2 p_{3 / 2}[7-8] 3.40 \mathrm{MeV}, 2 p_{1 / 2}[2-4] 5.13 \mathrm{MeV}$, $1 g_{9 / 2}[9-11] 2.75 \mathrm{MeV}$.

The calculated level energies are compared in Fig. 3 with the experimental ones seen in the present experiment. For positive parity states the number of predicted states is correct and the calculated energies are in acceptable agreement with the experiment. On the contrary, the $2^{+}$states are systematically predicted at higher energies and the $4^{+}$and $6^{+}$at energies lower than the experimental ones. A similar agreement is found for the negative parity states, with the exception of the first $5^{-}$state, predicted at 1.497 MeV of excitation energy and not seen experimentally.

The main components of the positive and negative parity states are given in the Tables III and IV. Note that, for the lowest $0^{+}, 2^{+}, 4^{+}$and $6^{+}$, the main component always corresponds to having a full shell closure at $Z$, $N=40$ and the remaining 8 neutrons in the $g_{9 / 2}$ shell. Nevertheless, the


Fig. 3. Energy level scheme comparing shell model and experimental results.
part of wave functions corresponding to two particle core excitation across $N=40$ subclosure is rather large and amounts to about $41 \%, 53 \%, 44 \%$ and $41 \%$, respectively.

TABLE III
The wave functions of the first $0^{+}, 2^{+}, 4^{+}$and $6^{+}$states of ${ }^{88} \mathrm{Zr}$. Each line corresponds to a state of the basis, which partition order is reported in the last four columns. [NO] is the number of unpaired nucleons; $p \%$ is the percent occupation contributing more than $0.5 \%$.

| $\begin{gathered} E(\text { th }), J^{\pi} \\ {[\mathrm{NO}]} \\ \hline \end{gathered}$ | $\begin{gathered} 0.000,0^{+} \\ p \% \\ \hline \end{gathered}$ | $\begin{gathered} 1.492,2^{+} \\ p \% \\ \hline \end{gathered}$ | $\begin{gathered} 1.934,4^{+} \\ p \% \\ \hline \end{gathered}$ | $\begin{gathered} 2.110,6^{+} \\ p \% \\ \hline \end{gathered}$ | Orbit partition order |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $f_{5 / 2}$ | $p_{3 / 2}$ | $p_{1 / 2}$ | $g_{9 / 2}$ |
| [2] | 1.50 | 1.66 | 1.88 | 1.82 | 11 | 7 | 4 | 10 |
| [0] | 16.90 | 14.94 | 13.93 | 13.46 | 10 | 8 | 4 | 10 |
| [2] | 2.05 | 3.98 | 2.68 | 2.44 | 11 | 8 | 3 | 10 |
| [2] | 1.94 | 4.09 | 3.00 | 2.72 | 12 | 7 | 3 | 10 |
| [0] | 7.36 | 8.35 | 7.71 | 7.37 | 12 | 6 | 4 | 10 |
| [0] | 11.92 | 19.58 | 15.45 | 13.19 | 12 | 8 | 2 | 10 |
| [0] | 58.32 | 47.41 | 55.34 | 59.01 | 12 | 8 | 4 | 8 |

TABLE IV
The wave functions of the first $3^{-}$and $5^{-}$states of ${ }^{88} \mathrm{Zr}$. Each line corresponds to a state of the basis, which partition order is reported in the last four columns. [ NO ] is the number of unpaired nucleons; $p \%$ is the percent occupation contributing more than $0.5 \%$.

| $E($ th $), J^{\pi}$ | $2.298,3^{-}$ | $1.479,5^{-}$ | Orbit partition order |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[\mathrm{NO}]$ | $p \%$ | $p \%$ | $f_{5 / 2}$ | $p_{3 / 2}$ | $p_{1 / 2}$ | $g_{9 / 2}$ |
|  |  |  |  |  |  |  |
| $[2]$ | 13.74 | 2.27 | 12 | 7 | 2 | 11 |
| $[2]$ | 27.85 | 3.93 | 12 | 7 | 4 | 9 |
| $[2]$ | 58.41 | 93.80 | 12 | 8 | 3 | 9 |

## REFERENCES

[1] E. Zanotti et al., Nucl. Instr. Meth. A310, 706 (1991).
[2] H.W. Müller, Nucl. Data Sheets 54, 1 (1988).
[3] J.B. Ball et al., Phys. Rev. 177, 1699 (1969).
[4] J.B. Ball et al., Phys. Rev. C4, 196 (1971).
[5] A.M. Kobos et al., Nucl. Phys. A425, 205 (1984).
[6] G. Bartnitzky et al., Phys. Lett. B386, 7 (1996).
[7] H.de Vries et al., At. Data Nucl. Data Tables 36, 495 (1987).
[8] G.R. Satchler, W.G. Lowe, Phys. Rep. 55, 183 (1979).
[9] P.D. Kunz, computer code DWUCK5, University of Colorado, unpublished.
[10] M. Walz, computer code TROMF, University of Tübingen, unpublished.
[11] M. Walz et al., J.Phys. G 14, L91 (1988).
[12] L.T. van der Bijl et al., Nucl. Phys. A393, 173 (1983).
[13] R.A. Hardenkopf et al., Phys. Rev. Lett. 35, 1623 (1975).
[14] B.A. Brown et al., MSU-NSCL Report No. 524 (1985).


[^0]:    * Presented at the XXV Mazurian Lakes School of Physics, Piaski, Poland, August 27-September 6, 1997.
    ${ }^{\dagger}$ guest researcher at Istituto Nazionale di Fisica Nucleare - Sezione di Milano.
    $\ddagger$ guest researcher, permanent address: Institute of Modern Physics, Academia Sinica, Lanzhou, P.R.China.

