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VARIOUS PARAMETRIZATIONS OF THE WOODS–SAXON POTENTIAL* **

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The neutron and proton separation energies and mean square charge radii are evaluated within the Woods–Saxon plus BCS model for even–even nuclei with $40 \leq A \leq 256$. The various parametrizations of the Woods–Saxon potential are examined. The improved values of the constants of the central part of Woods-Saxon potential are determined from a least-square adjustment to the existing experimental data.

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1. Introduction

Mean field theory with phenomenological single–particle potential is a powerful tool for the description of low energy nuclear phenomena. Among existing in literature phenomenological potentials the realistic single-particle Woods–Saxon [1] potential is the best to describe the many properties of nuclei such as the nuclear equilibrium deformations and moments, the mean square radii, the nucleon binding energies, the structure of the high-spin isomers, the fission barriers and the number of the single-particle effects for strongly deformed and fast rotating nuclei.

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To determine the Woods–Saxon (W–S) potential twelve constants should be given. There are for neutrons: depth of the central potential (V_0), radius parameter (r_0), diffuseness parameter (a), strength of the spin–orbit interaction (λ), radius parameter of the spin–orbit potential ($r_{0-\mathrm{so}}$), and diffuseness parameter of the spin–orbit part (a_{so}). The similar set of six parameters there are for the protons.

The several parameter sets have been proposed for the Woods–Saxon potential. They are usually determined by a global fit to various ground state nuclear properties of β stable nuclei. Among the earlier known parameter sets are those of Blomquist and Wahlborn [2], the parameters of Chepurnow [3], given by Rost [4] and the "new" parameters [5]. All these parameters are fitted to the contemporarily existing experimental data on spherical nuclei, notably ^{208}Pb.

The latest, so called "universal" [1] parametrization was adjusted to all experimental known odd-mass nuclei (both spherical and deformed) for $A \geq 40$. The scope is always a better description of the available data and a hope to achieve in this way a higher predictive power for nuclei far away from stability. The various sets of Woods–Saxon potential parameters are presented in Table I.

TABLE I

Different sets of parameters of the Woods–Saxon potential

Parameter	units	Universal	Wahlborn	Rost	Chepurnov	New
V_0	MeV	49.6	51.0	49.6	53.3	49.6
κ	-	0.86	0.67	0.86	0.63	0.86
a	fm	0.70	0.67	0.70	0.63	0.70
r_n	fm	1.347	1.27	1.347	1.24	1.347
λ_n	-	35.0	32.0	31.5	23.8·(1+2I)	*
r_n^{so}	fm	1.31	1.27	1.280	1.24	*
r_p	fm	1.275	1.27	1.275	1.24	1.275
λ_p	-	36.0	32.0	17.8	23.8·(1+2I)	*
r_p^{so}	fm	1.32	1.27	0.932	1.24	*

* The radius constant and the strength of the spin-orbit potential are deformation dependent as described in Ref. [5].

The aim of this work is to examine the existing sets of parameters of the Woods–Saxon potential for the description of the properties of the β–stable even-even nuclei in a wide range of the mass numbers. A reliable description of the nuclear ground state properties along the β–stability line is essential for a successful extrapolation to exotic nuclei as well as to superheavy nuclei.

In our investigations we restricted ourselves only to the analyze of the proton S_p and neutron S_n separation energies and the mean square charge radii $\langle r_{\rm ch}\rangle$, because these observables depend mainly on the potential structure.

On the other hand the experimentally known values of S_p and S_n as well as the average mean square charge radii $\langle r_{\rm ch}\rangle$ of nuclei can be used to establish the depth and size of the average potential well. By the least square fit to the experimental values of the above quantities we have obtained the "optimal" parameters of the Woods–Saxon potential. We show that "universal" parameters V_0, r_{0p}, r_{0n} and κ must be changed.

In Section 2 a brief discussion of the formalism used in our investigations is presented. In Section 3 we give some details about the calculations and results. In Section 4 the adjusted parameters of W–S potential are presented and discussed.

2. The theoretical formalism

The deformed Woods–Saxon potential is widely described in the literature [1] and we restrict ourselves to represent only the basic formulae. The potential consists of the central part $V_{\rm cent}$, the spin–orbit part $V_{\rm so}$ and the Coulomb potential $V_{\rm Coul}$ for the protons:

$$V^{\rm WS}(\vec{r},\vec{p},\vec{s};\beta) = V_{\rm cent}(\vec{r};\beta) + V_{so}(\vec{r},\vec{p},\vec{s};\beta) + V_{\rm Coul}(\vec{r};\beta) \tag{1}$$

with

$$V_{\rm so}(\vec{r},\vec{p},\vec{s};\beta) = -\lambda(\nabla V_{\rm cent}\times\vec{p})\cdot\vec{s}. \tag{2}$$

The central part is defined by:

$$V_{\rm cent}(\vec{r};\beta) = \frac{V_0\left[1 \pm \kappa\frac{N-Z}{N+Z}\right]}{\left[1+\exp\left(\frac{l(\vec{r};\beta)}{a}\right)\right]}, \tag{3}$$

where a is the diffuseness of the nuclear surface. The set of deformation parameters β_λ, which characterize the nuclear shape, is denoted by β. The function $l(\vec{r},\beta)$, describing the distance between the given point $\vec{r}$ and the nuclear surface has been determined numerically [1]. For spherical nuclei we have $l(\vec{r},\beta=0) = r - R_0$, where $R_0 = r_0 A^{1//3}$, is the radius of the corresponding spherical nucleus. In the above formulas β stands for the parameters characterizing the nuclear shape:

$$R(\theta) = c(\beta)R_0\left[1+\sum_\lambda \beta_\lambda Y_{\lambda 0}(\cos(\theta)\right]. \tag{4}$$

The function $c(\beta)$ insures the conservation of the nuclear volume with a change of the nuclear shape.

Since most of the nuclei considered here are open shell nuclei, pairing has been included using the BCS formalism. We have used constant pairing gaps for protons and neutrons which have been obtained from the empirical particle separation energies by the formulae:

$$\begin{aligned}\Delta_p(Z,N) &= \tfrac{1}{4}(B(Z-2,N)-3B(Z-1,N)\\ &\quad +3B(Z,N)-B(Z+1,N))\,, \qquad (5)\\ \Delta_n(Z,N) &= \tfrac{1}{4}(B(Z,N-2)-3B(Z,N-1)\\ &\quad +3B(Z,N)-B(Z,N+1))\,. \qquad (6)\end{aligned}$$

We have chosen nuclei with the smallest mass for a given nucleon number A. Obviously these nuclei are stable against β–decay. We have considered in our analysis the even–even nuclei with $40 \le A \le 256$.

For an estimate of the proton (neutron) separation energies an approximate method is proposed [6]. It is known from the BCS theory that in order to separate a nucleon from an even–even nucleus one has to break the Cooper pair and to move this nucleon from the Fermi level (λ) to the continuum limit. This experimental relation can be written as follows:

$$\begin{aligned}S_p(Z,N) &= \Delta_p(Z,N)-\lambda_p(Z,N)\,, \qquad (7)\\ S_n(Z,N) &= \Delta_n(Z,N)-\lambda_n(Z,N)\,. \qquad (8)\end{aligned}$$

The charge radii were calculated from the corresponding proton radii taking into account the correction due to finite proton size:

$$\langle \vec{r}^{\,2}\rangle_{\rm ch} = \langle \vec{r}^{\,2}\rangle_p + 0.64\ {\rm fm}^2\,. \qquad (9)$$

We have neglected here only the small contributions to the mean charge square radius originating from the electric neutron form factor and the electromagnetic spin–orbit coupling [9,10].

The global measure of the deformation of the neutron (or proton) distribution in the case of the microscopic theories can be expressed by the corresponding quadrupole moment

$$\langle Q_2\rangle_{n,p} = \langle 2r^2 P_2(\cos\theta)\rangle\,. \qquad (10)$$

The reduced electric quadrupole transition between the rotational 2^+ and 0^+ states are proportional to the square of the proton quadrupole moment

$$B(E2) = \frac{5}{4\pi}\langle Q_2\rangle_p^2\,. \qquad (11)$$

3. Numerical results and discussion

The calculations were done for the frequently used sets of the parameters of the W–S potential (see Table I). The proton (S_p) and neutron (S_n) separation energies, the mean-square charge radii ($\langle r_{\rm ch}\rangle$), electric quadrupole moments (Q_2) and the reduced electric quadrupole transition probabilities ($B(E2)$) have been systematically investigated.

We have taken into account the nuclei along the β-stability line and selected, experimentally known chains of isotopes or isobares (in order to examine the properties of nuclei far from the β-stability line). Fig. 1 shows the schematic diagram of the considered nuclei.

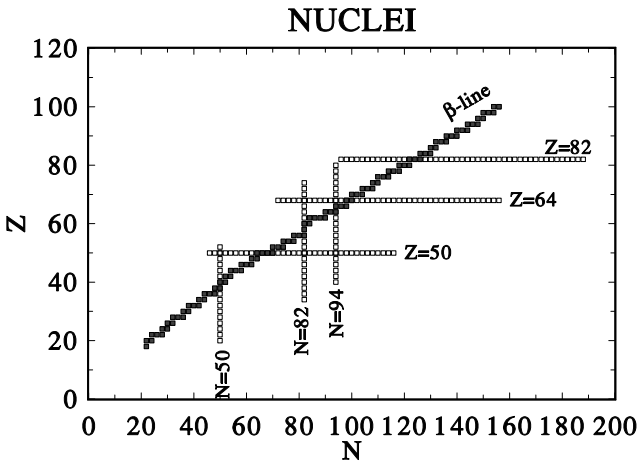

Fig. 1. The schematic diagram of the considered nuclei

The selected results are presented in Fig. 2, where the differences between the theoretical and the experimental [7] separation energies of protons and neutrons are plotted. The best theoretical estimates of S_n and S_p were obtained with the Woods–Saxon potential of Chepurnov [3].

The mean square deviations between the theoretical and experimental values of S_n, S_p, $B(E2)$ and $\langle r_{\rm ch}\rangle$ for all investigated Woods–Saxon models are plotted in Fig. 3. It is seen that the Chepurnov set apart from the charge radii gives the smallest deviations from the experiment. For the charge radii $\langle r_{\rm ch}\rangle$ the other sets are somewhat better than Chepurnov parametrization.

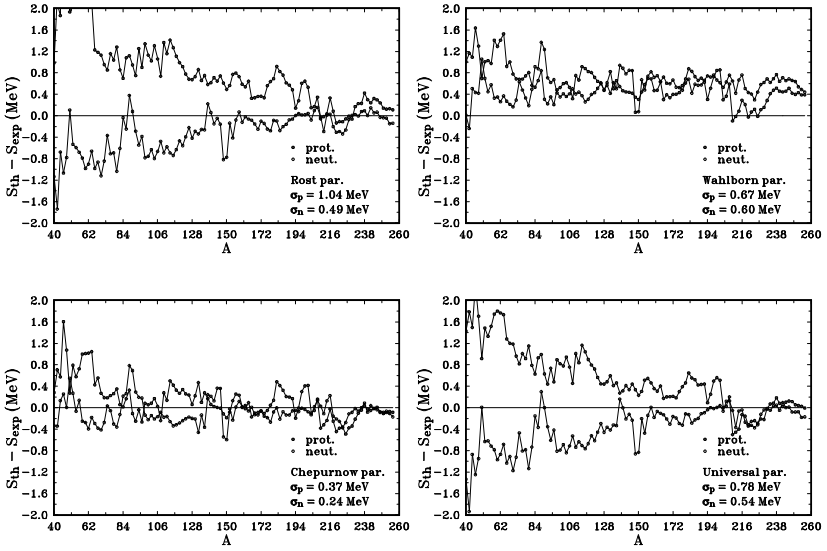

Fig. 2. The differences between the theoretical and experimental separation energies of protons S_p and neutrons S_n. The theoretical estimates were obtained with the frequently used sets of parameters of Woods–Saxon potential, see Tab. I.

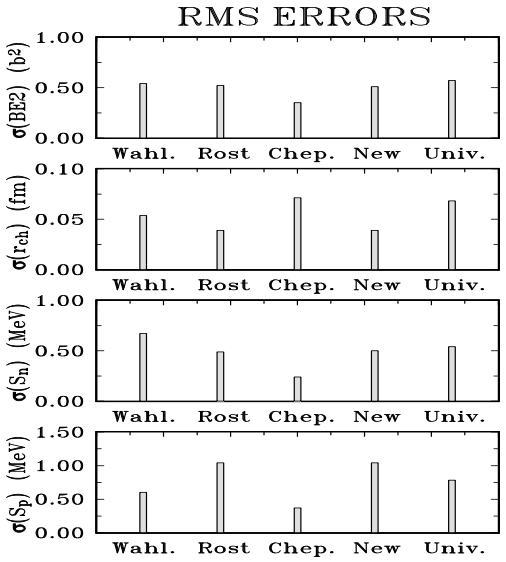

Fig. 3. The root mean square errors for the neutron $\sigma(S_n)$ and proton $\sigma(S_)$ separation energies, the reduced transition probabilities $\sigma(B(E2))$ and the charge radii $\sigma(r_{\rm ch})$ obtained within different Woods–Saxon potentials.

4. Adjustment procedure and model error

The values of S_p and S_n as well as $\langle r_{\rm ch}\rangle$ can be used to establish the depth (V_0) and size parameter (r_0) of the Woods–Saxon potential. The coefficients (V_0) and (r_0) can be obtained by the least square fit of its observables to the experimental data of all β-stable nuclei.

Model error has been defined as a simply the Root-Mean-Square (RMS) deviation, which as usual is given by:

$$\mathrm{RMS} = \left[\frac{1}{n}\sum_{i=1}^{n}(S^i_{\rm exp} - S^i_{\rm th})^2\right]^{1/2} . \tag{12}$$

Here $S^i_{\rm th}$ is the calculated and $S^i_{\rm exp}$ the corresponding measured quantity (in our case proton and neutron separation energy and mean square charge radii).

In Figs 4 and 5 we show the root-mean-square errors for proton (S_p) and neutron (S_n) separation energies for all β-stable nuclei as the function of

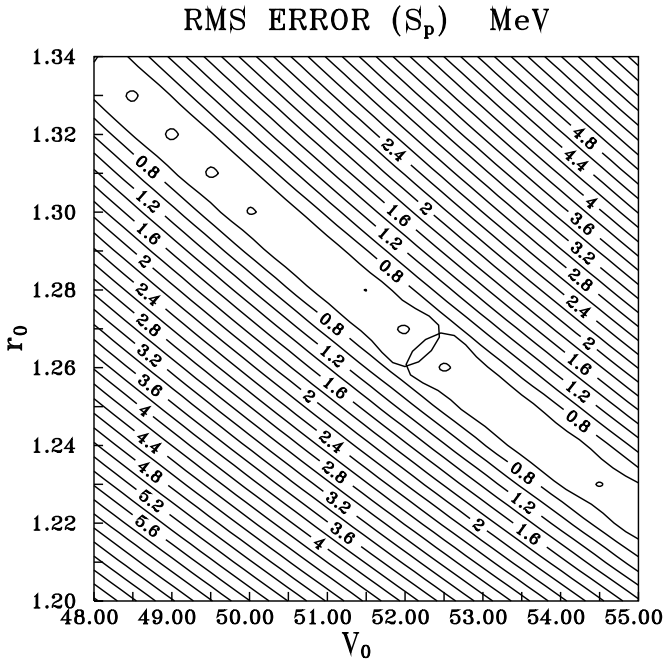

Fig. 4. The root-mean-square errors for proton (S_p) separation energy for all β-stable nuclei as the function of the depth (V_0) and size (r_0) parameters of the Woods–Saxon potential.

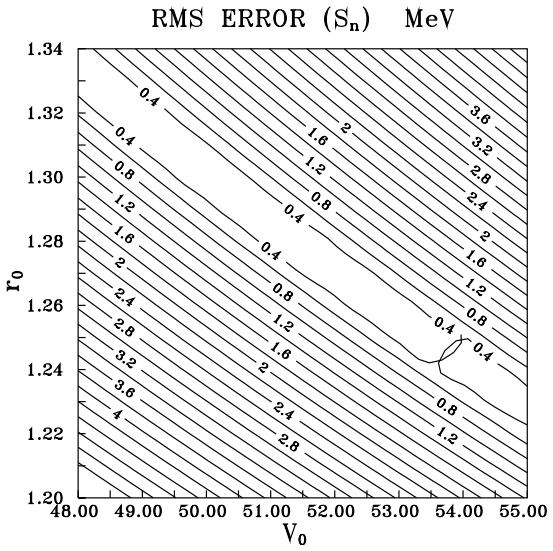

Fig. 5. The same as in Fig. 4 for neutron separation energy S_n

the depth (V_0) and size (r_0) parameters of the Woods–Saxon potential. The remaining are the "universal" parameters. The ranges of parameters V_0 and r_0 are:

- $V_0 \subset$ (48.00 ÷ 55.00) 0.50 MeV (15 points),
- $r_0 \subset$ (1.20 ÷ 1.35) 0.01 fm (12 points).

It is seen that the condition of the minimal value of the RMS error fulfills the simple linear relations:

$$r_0 = aV_0 + b\,. \tag{13}$$

From this relation it is impossible choose the one set of optimal values of depth and size parameters. The additional criterion at choice the "optimal" parameters V_0 and r_0 provides the analyze of the root-mean-square errors (RMS) for mean square charge radii (see Fig. 6). It is clear that the "optimal" values of V_0 and r_{0P} must simultaneously fulfill condition of the least mean square error for the proton separation energy S_p and mean square charge radii $\langle r_{\rm ch}\rangle$. For that reason on Fig. 7 we drown the RMS errors both for S_p and $\langle r_{\rm ch}\rangle$. It is easily seen that "optimal" values of V_0 and proton r_{0p} are:

$$V_0 = 52.4 \ \ \text{MeV}\,, \tag{14}$$

$$r_{0p} = 1.261 \ \ \text{fm}\,. \tag{15}$$

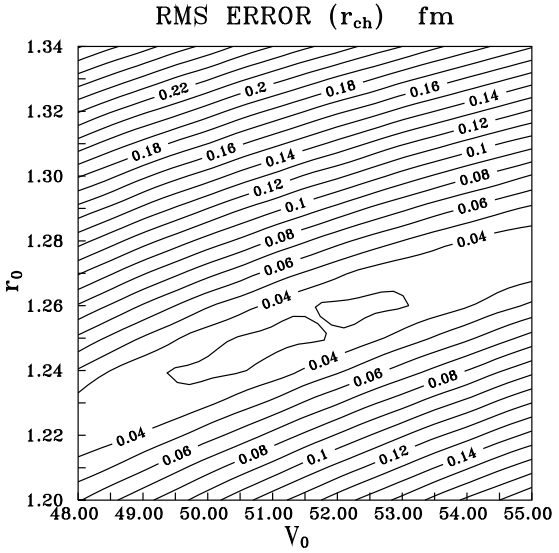

Fig. 6. The same as in Fig.4 for mean square charge radii

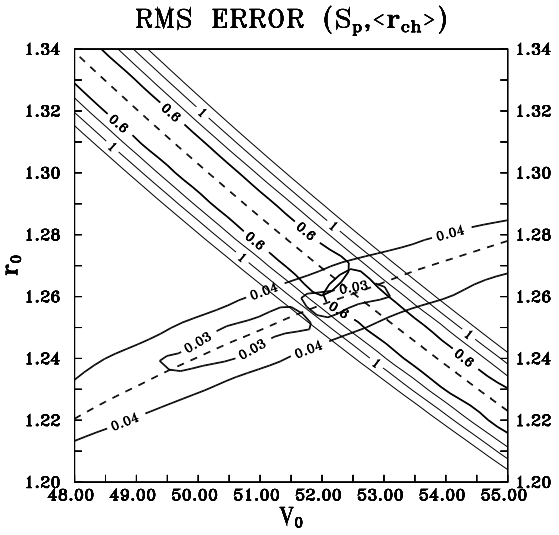

Fig. 7. The same as in Fig. 4 for proton separation energy and mean square charge radii.

The neutron parameter r_{0n} we have obtained on condition that the depth parameter V_0 is the same for protons and neutrons [1].

From the Fig. 5 we can see that:

$$r_{0n} = 1.264 \ \text{ fm}\,. \tag{16}$$

The real depth of the W–S potential for nucleus with N neutrons and Z protons depends on the isospin of nuclei and is adjusted to the experimental data by the parameter κ (see Eq. 3). The correct value of its parameter is very important, especially for description of the nuclei far from the β-stability line.

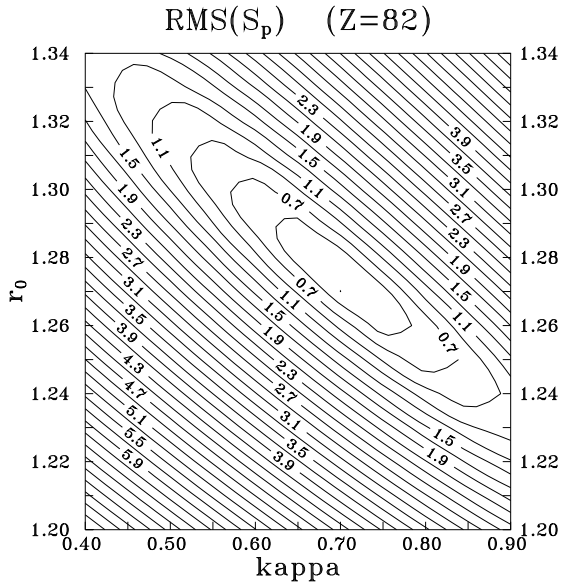

Fig. 8. The root-mean-square errors for proton (S_p) separation energy for Pb isotopes as the function of the (κ) and size (r_0) parameters of the Woods–Saxon potential.

Fig. 8 shows the root-mean-square errors of the proton separation energy (S_p) for the chain of the Pb isotopes as the function of r_0 and κ parameters. It is seen that the "optimal" value of the κ parameter (minimum of the RMS error) is equal $\kappa = 0.70$. The exact analyze for the all isotopes and isotones used in our calculations gives:

$$\kappa = 0.68\,.$$

The obtained "optimal" values of the V_0, r_{0p}, r_{0n} and κ parameters give the proper values of proton and neutron separation energies and the mean square charge radii for all experimentally known nuclei from the β-stability line and also far from its line. It is worth noticed that these parameters differ considerably from the so called "universal" set.

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