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RESONANCE-LIKE BEHAVIOUR IN TWO-NUCLEON ABSORPTION/EMISSION MECHANISM OF DCX REACTION* **

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We have studied the effects of pion wave-function distortion on the cross-section of the absorption/emission mechanism of the pionic double charge exchange reaction on ^{56}Fe, ^{76}Ge and ^{128,130}Te. We are using the pion-nucleus optical potential and the quasiparticle p-n random phase approximation formalisms, considering only the contribution of pion absorption/emission (p-wave mechanism) on the correlated nucleon pair. We confirm the resonant behaviour of the forward cross-section at around 50 MeV, opposite to the plane wave-function results.

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1. Introduction

There are many reasons to consider pionic DCX (double charge exchange) reactions on nuclei in spite of its very small cross–section. The main interest has always been related to the essential two-body nature of this reaction. Recently the observation of a resonance-like behaviour of the DCX cross-section around the pion kinetic energy $T_\pi = 50\,\mathrm{MeV}$ raised the question of the possible existence of a d' dibaryon [1], proposed to explain

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this phenomenon. There is still, however, a chance to improve on the theory within the *conventional* two-nucleon mechanism.

We used the framework of Quasiparticle Random Phase Approximation, very successful in describing the nucleon–nucleon correlations, and performed the calculations on ^{56}Fe, ^{76}Ge and ^{128,130}Te, for several reasons: they are heavy enough to expect that the QRPA gives reliable results, recent experimental data for ^{56}Fe and ^{128,130}Te are available [2] and last but not least ^{76}Ge and ^{128,130}Te are double-beta decay isotopes so the DCX reaction can serve as a testbed for the nuclear structure functions to be applied in calculations of $\beta\beta$-decay matrix elements.

2. Formalism

We follow closely the formalism developed and presented in a series of previous papers [3], taking QRPA to describe the DCX process and extending it to account for distortion effects. The wave-function of the incoming and outgoing pions are the solutions of the Klein–Gordon equation [4] with the pion–nucleus optical potential consisting of nuclear (V_{N}) and Coulomb (V_{C}) parts. Making the partial-wave expansion of the pion wave-function we find the following equation for each partial wave u_l:

$$\left[-\frac{\partial^2}{\partial r^2}+\frac{l(l+1)}{r^2}\right]u_l(r)=\left[k^2-2\xi(U_{\mathrm{C}}+U_{\mathrm{N}})+U_{\mathrm{C}}^2\right]u_l(r). \tag{1}$$

From the pion–nucleon scattering amplitude one derives the first-order pion–nucleus optical potential in the so-called Kisslinger form. Taking into account the effects of Pauli blocking, the polarization of the nucleonic medium by pions (Lorentz–Lorenz–Ericson–Ericson correction) and true pion absorption on a pair of nucleons the complete second-order optical potential emerges [5, 6] and equation (1) is to be solved numericaly.

The DCX reaction takes place generally on the two correlated nucleons. Because of the weakness of this reaction we assume that all other nucleons play an indirect role through wave distortion and other medium effects. The differential cross-section of the DCX reaction is defined by the total amplitude:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\vartheta_{kk'})=\left|\frac{1}{4\pi}\sum_{J^\pi}F^p_{J^\pi}(\boldsymbol{k},\boldsymbol{k}')\right|^2, \tag{2}$$

where $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the momenta of the incoming and outgoing pions, respectively. The partial amplitude $F^p_{J^\pi}(\boldsymbol{k},\boldsymbol{k}')$ is connected with p-wave pion–nucleon effective Hamiltonian of the form:

$$h_p(\boldsymbol{k})=-\sqrt{48\pi}\mathrm{i}\frac{f}{m_\pi}\sum_{pn}\sum_{JM}(-1)^{\frac{l_n+l_p+J}{2}}Y^*_{JM}(\vartheta_k,\varphi_k)G^J_{pn}\mathcal{R}^{JM}_{pn}, \tag{3}$$

where $\mathcal{R}^{JM}_{pn} = [\hat{c}^{\dagger}_{p}\hat{c}_{n}]_{JM}$ is the transition density operator and f is the constant determined to reproduce the experimental data for pion–nucleon elastic scattering. The explicit form of the nuclear form-factor G^{J}_{pn} can be found in [3]:

Finally, the DCX transition amplitude can be expressed as follows:

$$F^{p}_{J^{\pi}}(\boldsymbol{k},\boldsymbol{k}') = \sum_{M,mm'} \frac{\langle 0^{+}_{\rm f};\pi^{-}(\boldsymbol{k}')|h_{p}(\boldsymbol{k}')|m',J^{\pi}M\rangle\langle m',J^{\pi}M|m,J^{\pi}M\rangle\langle m,J^{\pi}M|h_{p}(\boldsymbol{k})|0^{+}_{\rm i};\pi^{+}(\boldsymbol{k})\rangle}{E_{i}+\omega-\frac{1}{2}(E^{m}_{J^{\pi}}+E^{m'}_{J'^{\pi'}})} + \{\text{crossed term}\}, \tag{4}$$

where $|0^{+}_{\rm i};\pi^{+}(\boldsymbol{k})\rangle$ and $|0^{+}_{\rm f};\pi^{-}(\boldsymbol{k}')\rangle$ are correspondingly the initial and final ground states of target (A,Z) and daughter $(A,Z+2)$ nuclei together with incoming and outgoing pion states of the momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$, $\omega = \sqrt{k^2+m^2_{\pi}}$ is the incident pion energy, $|m,J^{\pi}M\rangle$ and $|m',J^{\pi}M\rangle$ are the intermediate nuclear states constructed as one-phonon QRPA excitations from the initial and final nuclei, $E^{m}_{J^{\pi}}$ and $E^{m'}_{J'^{\pi'}}$ are their corresponding energies and E_i is the ground-state energy of the initial nucleus. The details of the QRPA formalism and final expressions can be found in [3].

3. Details of the calculations, results and discussion

We assumed the single-particle basis consisting of isotropic harmonic oscillator levels both for protons and neutrons:

^{56}Fe**:** [no core] $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $0d_{3/2}$, $1s_{3/2}$, $0d_{7/2}$, $1p_{1/2}$, $0d_{5/2}$, $1p_{1/2}$, $0f_{9/2}$, $1d_{5/2}$, $2s_{1/2}$

^{76}Ge**:** [core: ^{16}O] $0d_{5/2}$, $0d_{3/2}$, $1s_{1/2}$, $0f_{7/2}$, $1p_{3/2}$, $0f_5/2$, $1p_{1/2}$, $0g_{9/2}$, $1d_{5/2}$, $2s_{1/2}$, $0g_{7/2}$, $1d_{3/2}$, $0h_{11/2}$, $2p_{3/2}$, $1f_{7/2}$, $2p_{1/2}$.

^{128,130}Te**:** [core: ^{40}Ca] $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, $0g_{9/2}$, $0g_{7/2}$, $1d_{5/2}$, $0h_{11/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $0i_{13/2}$, $0h_{9/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$

The single-particle energies were obtained from a Coulomb-corrected Woods–Saxon potential and the nuclear G-matrix elements were calculated by solving the Bethe–Goldstone equation with the realistic Bonn potential with one-boson exchange [8]. We followed the standard procedure [3] renormalizing the bare nuclear matter two-body proton–proton and neutron–neutron G-matrix elements to fit the experimental pairing gaps ($g^{\rm pair}$ factors, for the pairing matrix elements), but did not touch the particle–particle and particle–hole channels of the proton–neutron G-matrix interaction and set $g_{\rm pp} = g_{\rm ph} = 1$. The results of the fitting are listed in Table I.

TABLE I

The experimental pairing gaps calculated using the atomic mass evaluation of Audi and Wapstra [10] by the prescription of Moeller and Nix [11].

	Nucleus							
	^{56}Fe	^{56}Ni	^{76}Ge	^{76}Se	^{128}Te	^{128}Xe	^{130}Te	^{130}Xe
$\Delta_{\rm p}^{\rm exp}$ [keV]	1571.6	2078	1506	1753	1127	1318.6	1056	1306.5
$\Delta_{\rm n}^{\rm exp}$ [keV]	1425.1	2150	1535	1709	1275.4	1266.2	1181.7	1247.3
$g_{\rm p}^{\rm pair}$	1.015	1.038	0.965	0.992	0.918	0.975	0.883	0.967
$g_{\rm n}^{\rm pair}$	1.011	1.070	1.1055	1.138	1.042	1.036	1.005	1.039

For the calculations we used the parameters of the optical potential determined by a fit to data of pionic atoms, to π^+ elastic scattering and $\pi^\pm$ absorption on different targets over a broad energy range [4]. The parameters were linearly interpolated where necessary. The data for the nuclear densities (two–parameter Fermi model) for ^{56}Fe and ^{76}Ge were taken from [9]. Unfortunately the data for ^{128,130}Te are not available so we used those for the neighbouring antimon (Sb) [9].

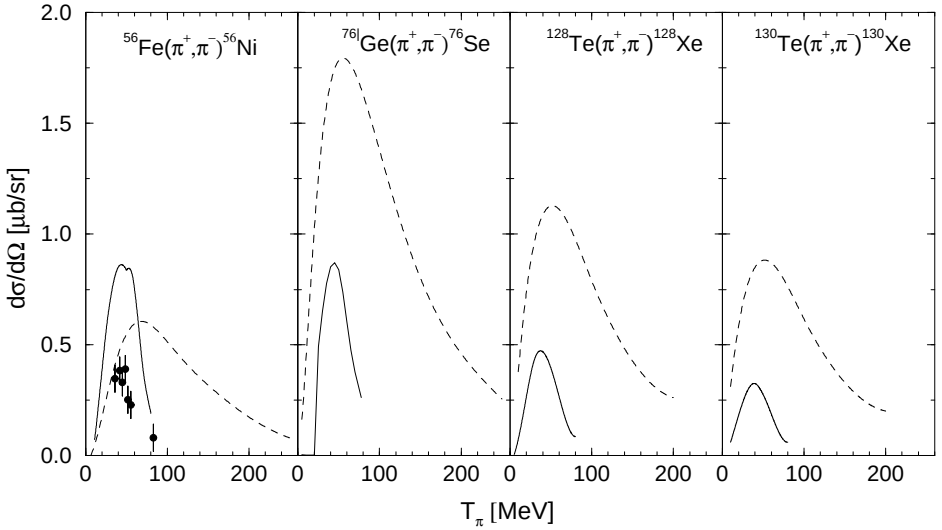

Fig. 1. Differential cross-sections as a function of the pion kinetic energy T_π. Dotted curves result from the plane-wave approximation, solid lines depict calculations with distorted waves.

Figure 1 shows the p-wave differential cross-section as a function of the pion kinetic energy dependence and the corresponding data from measurements of the Tübingen experimental group at the Paul Scherrer Institute [1, 2], interpolated to $\vartheta = 0^\circ$. One can see much difference between the

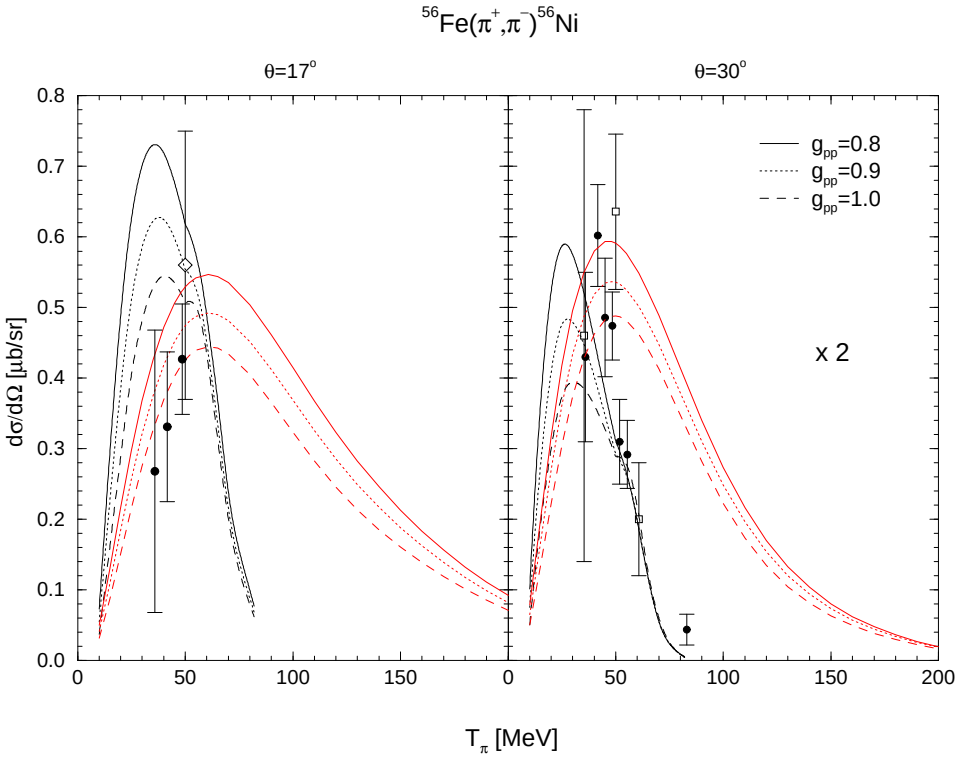

Fig. 2. Differential cross-sections for the plane-wave approximation (red lines) and distortedwave approximation (black lines) as a function of the pion kinetic energy T_π calculated for the different values of the particle–particle strength $g_{\rm pp}$ and different scatering angles. Experimental data are taken from [1,2] (□), [2] (•) and [7] (◇).

plane-wave approximation and distorted-wave approach for all the nuclei under consideration. With plane waves the maximum of the energy distribution appears for $T_\pi \approx 50\,\mathrm{MeV}$ but it is too broad and does not account for the reduction of the cross-section in the high-energy region of 80–100 MeV. With distorted waves a more resonance-like shape is obtained and the maximum is shifted towards lower energies of about 35–40 MeV.

4. Conclusions, remarks and outlook

We have shown the importance of the distortion of the pion wave-functions for DCX reaction cross-section calculated in the QRPA framework. With our model the gross features of the resonance-like shape of the cross-section as a function of pion energy can be reproduced for all four nuclei at least qualitatively in terms of the conventional two-nucleon mechanism without

dibaryons or multiple quark clusters. Nevertheless, the experimental data are so scarce and thus the interpolation procedure to forward angles so uncertain that the question how far dibaryon mechanism competes with the conventional one is still open. Further additions and improvements to the theory can be done, *e.g.* application of full QRPA with proton–neutron pairing [12] and/or proper treatment of Pauli principle within renormalized QRPA (RQRPA) formalism [13]. The corresponding calculations are on the way. We also plan to extend our study onto other cases where experimental data are available (*e.g.* Ca isotopes).

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