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HIGHER ORDER QCD CORRECTIONS TO FRAGMENTATION FUNCTIONS IN $e^+ \, e^-$ ANNIHILATION*

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We analyze the second order QCD corrections to the fragmentation functions $F_k^{\rm H}(x,Q^2)$ $(k=T,L,A)$ which are measured in $e^+ \, e^-$ annihilation From these fragmentation functions one can derive the integrated transverse (σ_T), longitudinal (σ_L) and asymmetric (σ_A) cross sections. The sum $\sigma_{\rm tot}=\sigma_T+\sigma_L$ corrected up to order α_s^2 agrees with the well known result in the literature. It turns out that the order α_s^2 corrections to the transverse and asymmetric quantities are small in contrast to our findings for $F_L^{\rm H}(x,Q^2)$ and σ_L where they turn out to be large. Therefore in the latter case one gets a better agreement between the theoretical predictions and the data obtained from he LEP experiments.

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1. Introduction

Semi inclusive hadron production in $e^+ \, e^-$ annihilation into a vector boson V $(V=\gamma,Z)$ proceeds via the reaction (see Fig. 1)

$$e^-(l_1,\sigma_1)+e^+(l_2,\sigma_2)\rightarrow V(q)\rightarrow H(p,s)+\text{“}X\text{”}\,. \qquad (1.1)$$

In the process above “X” denotes any inclusive final hadronic state and H represents either a specific charged outgoing hadron or a sum over all charged hadron species. The unpolarized differential cross section of the above process is given by

$$\frac{d^2\sigma^H(x,Q^2)}{dx\,d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_T^H(x,Q^2)}{dx}+\frac{3}{4}\sin^2\theta\frac{d\sigma_L^H(x,Q^2)}{dx} + \frac{3}{4}\cos\theta\frac{d\sigma_A^H(x,Q^2)}{dx}. \qquad (1.2)$$

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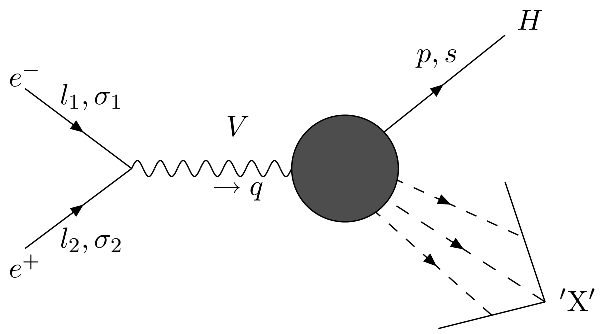

Fig. 1. Kinematics of electron positron annihilation $e^- + e^+ \rightarrow H +' \mathrm{X}'$

The transverse, longitudinal and asymmetric cross sections are given by σ_T^H, σ_L^H, and σ_A^H respectively. The latter, which is due to parity violation, only shows up if the intermediate vector boson is given by the Z-boson and is absent when $V = \gamma$. The cross sections depend in addition to the CM energy Q also on the Bjørken scaling variable x defined by

$$x = \frac{2pq}{Q^2}, \qquad q^2 = Q^2 > 0, \qquad 0 < x \leq 1\,. \tag{1.3}$$

In the centre of mass (CM) frame of the electron-positron pair this variable can be interpreted as the fraction of the beam energy carried away by the hadron H. The variable θ denotes the angle of emission of particle H with respect to the electron beam direction in the CM frame. Before the advent of LEP1 the CM energies were so low ($Q \ll M_Z$) that σ_A could not be measured and no effort was made to separate σ_T from σ_L so that only data for $\sigma_T + \sigma_L$ were available. After LEP1 came into operation one was able to measure σ_T and σ_L separately [1, 2]. Moreover σ_A could be determined for the first time [2, 3]. The separation of σ_T and σ_L is important because the latter cross section enables us to extract the strong coupling constant α_s and allows us to determine the gluon fragmentation density $D_g^H(x, \mu^2)$ with a much higher accuracy as could be done before. Furthermore the measurement of σ_A provides us with information on hadronization effects since the QCD corrections are very small. In the QCD improved parton model, which describes the production of the parton and its subsequent fragmentation into a hadron H, the cross sections σ_k^H ($k = T, L, A$) can be expressed as follows:

$$\frac{d\sigma_k^H(x, Q^2)}{dx} = \int_x^1 \frac{dz}{z} \Big[\sigma_{\mathrm{tot}}^{(0)}(Q^2) \Big\{ D_S^H(\frac{x}{z}, \mu^2) \mathcal{C}_{k,q}^S(z, Q^2/\mu^2) + D_g^H(\frac{x}{z}, \mu^2) \cdot$$

$$\cdot \mathcal{C}^{S}_{k,g}(z, Q^2/\mu^2)\Big\} + \sum_{f=1}^{n_f} \sigma_f^{(0)}(Q^2)\, D^H_{NS,f}\left(\frac{x}{z}, \mu^2\right) \mathcal{C}^{NS}_{k,q}(z, Q^2/\mu^2)\Big], \tag{1.4}$$

for $k = T, L$. In the case of the asymmetric cross section we have

$$\frac{d\sigma^H_A(x, Q^2)}{dx} = \int\limits_x^1 \frac{dz}{z}\Big[\sum_{f=1}^{n_f} A_f^{(0)}(Q^2) D^H_{A,f}(\frac{x}{z}, \mu^2) \mathcal{C}^{NS}_{A,q}(z, Q^2/\mu^2)\Big]. \tag{1.5}$$

In the formulae above, which only hold for massless quarks, we have introduced the following notations. The functions $D^H_l(z, \mu^2)$ $(l = q, \bar{q}, g)$, which depend on the factorization/renormalization scale μ, stand for the parton fragmentation densities. The singlet (S) and non-singlet (NS,A) combinations with respect to the flavour group SU(n_f) are defined by

$$\begin{aligned}
D^H_S(z, \mu^2) &= \frac{1}{n_f} \sum_{q=1}^{n_f} \Big(D^H_q(z, \mu^2) + D^H_{\bar{q}}(z, \mu^2)\Big)\,, \\
D^H_{NS,q}(z, \mu^2) &= D^H_q(z, \mu^2) + D^H_{\bar{q}}(z, \mu^2) - D^H_S(z, \mu^2)\,, \\
D^H_{A,q}(z, \mu^2) &= D^H_q(z, \mu^2) - D^H_{\bar{q}}(z, \mu^2)\,.
\end{aligned} \tag{1.6}$$

The index q stands for the quark species and n_f denotes the number of light flavours. The pointlike cross section $\sigma_q^{(0)}$ and the asymmetry factor $A_q^{(0)}$ of the process $e^+ + e^- \to q + \bar{q}$ can be found in [4, 5]. The total cross section, summed over all flavours is given by $\sigma^{(0)}_{\rm tot}(Q^2) = \sum_{q=1}^{n_f} \sigma_q^{(0)}(Q^2)$. The QCD corrections in Eqs. (1.4), (1.5) are described by the coefficient functions $\mathcal{C}^r_{k,l}$ $(k = T, L, A;\ l = q, g)$. Like the fragmentation densities they depend on the scale μ and they can be split into a singlet $(r = S)$ and a non-singlet part $(r = NS)$. From (1.2) we can derive the total hadronic cross section

$$\begin{aligned}
\sigma_{\rm tot}(Q^2) &= \frac{1}{2}\sum_H \int\limits_0^1 dx \int\limits_{-1}^1 d\cos\theta \left(x \frac{d^2\sigma^{\rm H}(x, Q^2)}{dx\, d\cos\theta}\right) \\
&= \sigma_T(Q^2) + \sigma_L(Q^2)\,,
\end{aligned} \tag{1.7}$$

with

$$\sigma_k(Q^2) = \frac{1}{2}\sum_H \int\limits_0^1 dx\, x\, \frac{d\sigma^{\rm H}_k(x, Q^2)}{dx}\,, \qquad k = T, L\,, \tag{1.8}$$

where one has summed over all types of outgoing hadrons H. From the momentum conservation sum rule given by

$$\sum_H \int_0^1 dx\, x\, D_l^{\rm H}(x,\mu^2) = 1\,, \qquad l = q, \bar{q}, g\,, \tag{1.9}$$

and Eqs. (1.4), (1.8) one can derive

$$\sigma_k(Q^2) = \sigma_{\rm tot}^{(0)}(Q^2) \int_0^1 dx\, x \Big[\mathcal{C}_{k,q}^{\rm S}(x, Q^2/\mu^2) + \frac{1}{2}\mathcal{C}_{k,g}^{\rm S}(x, Q^2/\mu^2)\Big]\,. \tag{1.10}$$

Finally we also define the transverse, longitudinal and asymmetric fragmentation functions $F_k^H(x, Q^2)$

$$F_k^{\rm H}(x,Q^2) = \frac{1}{\sigma_{\rm tot}^{(0)}(Q^2)} \frac{d\sigma_k^{\rm H}(x,Q^2)}{dx}\,, \quad k = (T, L, A)\,. \tag{1.11}$$

One observes that the above fragmentation functions[1] are just the timelike analogues of the structure functions measured in deep inelastic electron-proton scattering.

2. Order α_s^2 corrected coefficient functions

The coefficient functions $\mathcal{C}_{k,l}^r$ corrected up to order α_s^2 receive contributions from the following parton subprocesses. In zeroth order we have the Born reaction

$$V \to \text{“}q\text{''} + \bar{q}\,, \tag{2.1}$$

where “l” ($l = q, \bar{q}, g$) denotes the detected parton which subsequently fragments into the hadron of species H. In next-to-leading order (NLO) one has to compute the one-loop virtual corrections to reaction (2.1) and the parton subprocesses

$$V \to \text{“}q\text{''} + \bar{q} + g\,, \tag{2.2}$$

$$V \to \text{“}g\text{''} + q + \bar{q}\,. \tag{2.3}$$

[1] Notice that we make a distinction in nomenclature between the fragmentation densities D_q^H,D_g^H and the fragmentation functions F_k^H.

After mass factorization of the collinear divergences which arise in the above processes one obtains the coefficient functions which are presented in [4]. The determination of the order α_s^2 contributions involves the computation of the two-loop corrections to (2.1) and the one-loop corrections to Eqs. (2.2), (2.3). Furthermore one has to calculate the following subprocesses

$$V \to \text{“}q\text{”} + \bar{q} + g + g\,, \tag{2.4}$$

$$V \to \text{“}g\text{”} + q + \bar{q} + g\,, \tag{2.5}$$

$$V \to \text{“}q\text{”} + \bar{q} + q + \bar{q}\,. \tag{2.6}$$

In reaction (2.6) the two anti-quarks, which are inclusive, can be identical as well as non-identical. Notice that in the above reactions the detected quark can be replaced by the detected anti-quark so that in reaction (2.6) one can also distinguish between the final states containing identical quarks and non-identical quarks. After mass factorization and renormalization for which we have chosen the $\overline{\mathrm{MS}}$-scheme one obtains the order α_s^2 contributions to the coefficient functions which are presented in [5].

3. Review of the most important results

The most important results of our calculations can be summarized as follows. From Eq. (1.10) and the coefficient functions originating from the processes above we can obtain σ_L and σ_T corrected up to order α_s^2

$$\sigma_T(Q^2) = \sigma_{\rm tot}^{(0)}(Q^2)\Big[1 + \Big(\frac{\alpha_s(\mu^2)}{4\pi}\Big)^2\Big(C_F^2\Big\{6\Big\}$$
$$+C_AC_F\Big\{-\frac{196}{5}\zeta(3) - \frac{178}{30}\Big\} + n_fC_FT_f\Big\{16\zeta(3) + \frac{8}{3}\Big\}\Big)\Big]\,, \tag{3.1}$$

$$\sigma_L(Q^2) = \sigma_{\rm tot}^{(0)}(Q^2)\Big[\frac{\alpha_s(\mu^2)}{4\pi}C_F\Big\{3\Big\} + \Big(\frac{\alpha_s(\mu^2)}{4\pi}\Big)^2\Big(C_F^2\Big\{-\frac{15}{2}\Big\}$$
$$+C_AC_F\Big\{-11\ln\frac{Q^2}{\mu^2} - \frac{24}{5}\zeta(3) + \frac{2023}{30}\Big\} + n_fC_FT_f\Big\{4\ln\frac{Q^2}{\mu^2} - \frac{74}{3}\Big\}\Big)\Big]\,. \tag{3.2}$$

Addition of σ_L and σ_T yields the well known answer for $\sigma_{\rm tot}$ (1.7) which is in agreement with the literature [6]. Hence this quantity provides us with a check on our calculation of the longitudinal and transverse coefficient functions. Notice that in lowest order $\sigma_{\rm tot}$ only receives a contribution from the transverse cross section whereas the order α_s contribution can be only attributed to the longitudinal part. In order α_s^2 both σ_L and σ_T contribute to $\sigma_{\rm tot}$.

Because of the high sensitivity of expression (3.2) to the value of α_s, the longitudinal cross section provides us with an excellent tool to measure the running coupling constant. To illustrate the importance of the order α_s^2 contribution to σ_L we have computed the ratio

$$R_L(Q^2) = \frac{\sigma_L(Q^2)}{\sigma_{\rm tot}(Q^2)}\,, \tag{3.3}$$

for $\mu = Q = M_Z$ and $\alpha_s(5, M_Z) = 0.126$. The result is

$$R_L = 0.040 + 0.014 = 0.054\,, \qquad (0.057 \pm 0.005)\,, \tag{3.4}$$

where the first and the second number represent the order α_s and order α_s^2 contribution respectively. Between the brackets we have quoted the result from OPAL [2]. Here one observes a considerable improvement when the order α_s^2 contributions are included. Recently DELPHI [3] used Eq. (3.2) to determine the strong coupling constant. Their measurement of R_L yields

$$R_L = 0.051 \pm 0.01\,(\text{stat.}) \pm 0.007\,(\text{syst.})\,, \tag{3.5}$$

from which the strong coupling constant can be extracted. The result is

$$\alpha_s^{\rm NLO}(5, M_Z) = 0.120 \pm 0.002\,(\text{stat.}) \pm 0.013\,(\text{syst.})\,. \tag{3.6}$$

If one includes power corrections to σ_L which are due to higher twist contributions of the order Λ/Q (see [7]) then one obtains

$$\alpha_s^{\rm NLO+POW}(5, M_Z) = 0.101 \pm 0.002\,(\text{stat.}) \pm 0.013\,(\text{syst.}) \pm 0.007\,(\text{scale})\,, \tag{3.7}$$

where the scale uncertainty comes from varying the renormalization scale in he range $0.5\ Q < \mu < 2\ Q$. The result above shows that the power corrections enhance σ_L so that the strong coupling constant decreases. A similar analysis (see [5]) shows that also the longitudinal fragmentation function F_L receives large order α_s^2 contributions. However the corrections to the transverse and asymmetric fragmentation functions are small.

Finally we discuss the order α_s^2 corrections to the asymmetric cross section. The latter is given by

$$\sigma_A(Q^2) = \sum_H \int_0^1 dx\, \frac{d\sigma_A^{\rm H}(x, Q^2)}{dx} = Q_A \int_0^1 dx\, \mathcal{C}_{A,q}^{\rm NS}(x, Q^2/\mu^2)\,, \tag{3.8}$$

Up to NNLO the above expression yields the following result

$$\sigma_A = Q_A\Big[1 + \Big(\frac{\alpha_s(\mu^2)}{4\pi}\Big)^2\Big\{-12\beta_0 C_F\zeta(3)\Big\}\Big], \qquad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_f n_f\,. \tag{3.9}$$

Notice that σ_A is independent of the factorization scale. However it depends like σ_L and σ_A on the renormalization scale via α_s. From Eq. (3.9) we infer that the NLO correction is zero but the NNLO contribution is nonvanishing although numerically it is very small for $\mu = Q = M_Z$. Finally in [5] we have evaluated the first and second moment of the asymmetric structure function and compared it with the data. The NNLO result for the first moment is

$$\int_{0.1}^{1} dx F_A(x, M_Z^2) = -0.016\,, \qquad -0.0229 \pm 0.0044\,(\mathrm{OPAL})\,,$$
$$-0.028 \pm 0.006\,(\mathrm{DELPHI})\,. \tag{3.10}$$

The second moment becomes

$$\int_{0.1}^{1} dx \frac{x}{2} F_A(x, M_Z^2) = -0.0020\,, \qquad -0.00369 \pm 0.00046\,(\mathrm{OPAL})\,,$$
$$-0.0036 \pm 0.0008\,(\mathrm{DELPHI})\,. \tag{3.11}$$

We also computed the above sum rules up to NLO. However there is hardly any difference between NLO and NNLO which could already be expected from the comments made below Eq. (3.9). The numbers presented above reveal that the experimental values are far below the theoretical predictions. Furthermore it turns out that the LO results are much closer to the ones obtained from experiment. In the case of Eq. (3.10) we obtain -0.023 whereas for Eq. (3.11) we get -0.0027. Probably higher twist corrections might become very important as we saw in the case of σ_L below (3.6).

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