

$O(\alpha_s)$  CORRECTIONS TO THE PRODUCTION OF ISOLATED PHOTONS IN LARGE- $Q^2$   $ep$  SCATTERING\*

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I summarize the calculation of  $O(\alpha_s)$  corrections to the production of a hard and isolated photon accompanied by one or two jets in deep inelastic lepton nucleon scattering at HERA.

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**1. Introduction**

The production of isolated photons in hadronic processes is an important testing ground for QCD. Since the photon does not take part in the strong interaction, it is a ‘direct’ probe of the hard scattering process and provides a means to determine the strong coupling constant  $\alpha_s$  or to extract information on the parton distributions, in particular the gluon density in the proton. Moreover, a good knowledge of the standard model predictions for direct photon production is required since it is an important background for many searches for new physics.

At HERA, with increasing luminosity, the measurement of isolated photon production is expected to contribute information on the parton content of the proton and, at  $Q^2 = 0$ , also on the parton distributions in the photon. The information obtained in this way would be complementary to the  $F_2$  measurement from inclusive deep inelastic scattering, since *up* and *down* quarks contribute with different weights<sup>1</sup>. Typical cross sections for the production of hard photons in deep inelastic scattering with  $Q^2 > 10 \text{ GeV}^2$  are

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<sup>1</sup> At LEP1, the fact that *up* and *down* quarks couple with different weights to photons has been used to separate their electroweak couplings by combining measurements of photon radiation in hadronic  $Z$  decays with that of the full hadronic width of the  $Z$  [1,2].

of the order of 10 pb. With a luminosity of  $50 \text{ pb}^{-1}$  one thus expects statistical uncertainties of the order of 5% and a measurement of differential cross sections seems feasible.

The next-to-leading order calculation for direct photon production in deep inelastic  $ep$  scattering, *i.e.*  $ep \rightarrow e\gamma X$  at large  $Q^2$ , has been reported recently [3]<sup>2</sup>. Since hard photon production is a process of relative order  $\alpha_e = 1/137$  with respect to the total deep inelastic scattering cross section, one expects sizable event rates only at moderately large  $Q^2$  and one can restrict the calculation to pure photon exchange, *i.e.* neglect  $Z$ -exchange. In the calculation of Ref. [3], the hadronic final state is separated into  $\gamma + (1+1)$ -jet and  $\gamma + (2+1)$ -jet topologies (the remnant being counted as “+1” jet, as usual). The approach is thus analogous to that in calculations of  $(2+1)$ - and  $(3+1)$ -jet cross sections where a gluon is replaced by a photon [4].

In addition to perturbative direct production, photons are also produced through the ‘fragmentation’ of a hadronic jet into a single photon carrying a large fraction of the jet energy [5]. This long-distance process is described in terms of the quark-to-photon and gluon-to-photon fragmentation functions<sup>3</sup> which absorb singularities showing up in a perturbative calculation. In [3] the fragmentation contributions were ignored and the photon-quark collinear singularities have been removed by explicit parton-level cutoffs. A more complete calculation including parton-to-photon fragmentation is in preparation [8].

## 2. The leading-order process

In leading order, the production of photons in deep inelastic electron (positron) proton scattering is described by the quark (antiquark) subprocess

$$e(p_1) + q(p_3) \rightarrow e(p_2) + q(p_4) + \gamma(p_5), \quad (1)$$

where the definition of the particle momenta is given in parentheses. The momentum of the incoming quark is a fraction  $\xi$  of the proton momentum  $p_P$ :  $p_3 = \xi p_P$ . The proton remnant  $r$  carries the momentum  $p_r = (1 - \xi)p_P$  and hadronizes into the remnant jet so that the process (1) gives rise to  $\gamma + (1+1)$ -jet final states. To remove direct photon production in photoproduction and restrict to the case where a scattered electron is observed, one can apply cuts on the usual deep inelastic scattering variables  $x$ ,  $y$  and  $Q^2$ . In addition, an explicit cut on the invariant mass  $W$  of the hadronic final state is required

<sup>2</sup> For literature on related calculations, see [3].

<sup>3</sup> First measurements of the quark-to-photon fragmentation function in  $e^+e^- \rightarrow \gamma + 1$ -jet are described in Refs. [6], see also the contribution of A. Gehrmann-De Ridder in these proceedings [7].

since due to the presence of the photon in the final state, large  $Q^2$  does not guarantee large  $W$ .

Both leptons and quarks emit photons. The subset of Feynman diagrams where the photon is emitted from the lepton (“leptonic radiation”) is gauge invariant and can be treated separately. Similarly, the Feynman diagrams with a photon emitted from the quark line is called “quarkonic radiation”. There is also a contribution from the interference of these two parts. For tests of QCD the interest is in those contributions where the photon is emitted from quarks and leptonic radiation is viewed as a background. Leptonic radiation can easily be suppressed by a cut on the photon emission angle.

At lowest order, each parton is identified with a jet and photon-parton isolation corresponds to the isolation of the photon from an observable jet. With isolation cuts, parton-to-photon fragmentation does not contribute at this order.

### 3. $O(\alpha_s)$ corrections

At next-to-leading order, processes with an additional gluon, either emitted into the final state or as incoming parton, have to be taken into account:

$$e(p_1) + q(p_3) \rightarrow e(p_2) + q(p_4) + \gamma(p_5) + g(p_6), \quad (2)$$

$$e(p_1) + g(p_3) \rightarrow e(p_2) + q(p_4) + \gamma(p_5) + \bar{q}(p_6), \quad (3)$$

where the definition of momenta is again shown in parentheses (see Fig. 1). In addition, virtual corrections (one-loop diagrams at  $O(\alpha_s)$ ) to the process (1) have to be included. The corresponding complete matrix elements are given in [9]. The processes (2, 3) contribute both to the  $\gamma + (1 + 1)$ -jet cross section, as well as to the cross section for  $\gamma + (2 + 1)$ -jets, depending on whether the quark-gluon or quark-antiquark pair in the final state appears as one single jet or as two separated jets. The two cases can be identified by comparing the scaled invariant masses,

$$y_{ij} = \frac{s_{ij}}{W^2}, \quad s_{ij} = (p_i + p_j)^2, \quad (4)$$

( $W^2 = (p_P + p_1 - p_2 - p_5)^2$ ) of parton pairs with a jet resolution parameter  $y^J$ : two partons ( $i, j$ ) with  $i, j = 4, 6, r$  are supposed to lead to 2 jets if

$$y_{ij} > y^J. \quad (5)$$

Otherwise the two partons are recombined into one jet. Also the remnant  $r$  is treated as a parton and a quark, antiquark, or gluon in the final state is recombined with the remnant into one jet if  $y_{ir} = \frac{1-\xi}{\xi} y_{i3}$  is smaller than

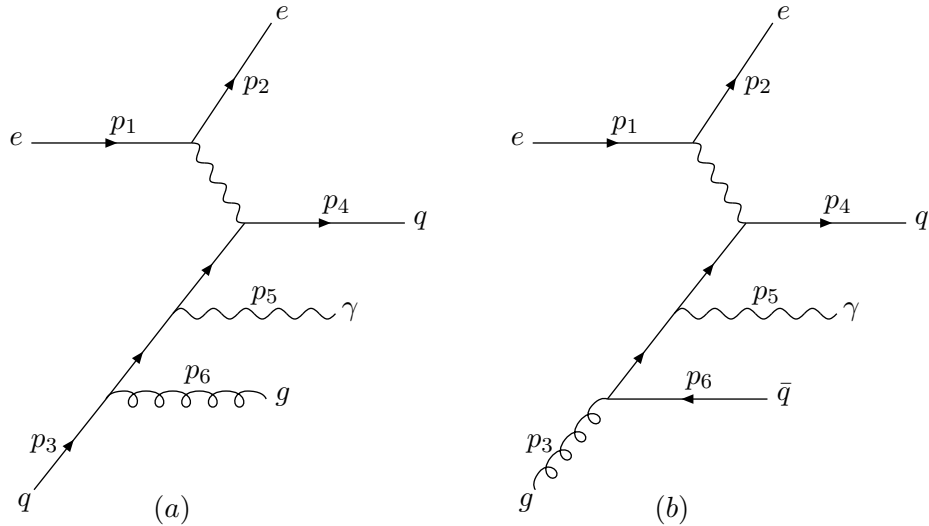


Fig. 1. Examples of Feynman diagrams for  $eq \rightarrow eqg\gamma$  (a) and  $eg \rightarrow eq\bar{q}\gamma$  (b) with the definition of momenta.

$y^J$ . Similarly, photon isolation can be imposed with the help of cuts on the scaled invariant masses of photon-parton or photon-jet pairs.

In the phase space region where two partons cannot be separated, the matrix elements become singular. These singularities appear when one of the partons becomes soft or when two partons become collinear to each other. The singularities can be assigned either to the initial state or to the final state (ISR: initial-state radiation, FSR: final-state radiation). There are contributions involving the product of an ISR and a FSR factor which first have to be separated by partial fractioning. The FSR singularities cancel against singularities from virtual corrections to the lower-order process. For the ISR singularities, this cancellation is incomplete and the remaining singular contributions have to be factorized and absorbed into renormalized parton distribution functions.

To accomplish this procedure, the singularities have to be isolated in an analytic calculation, *e.g.* with the help of dimensional regularization. The evaluation of dimensionally regularized phase space integrals is, however, very difficult for the complete cross section of the higher-order processes. Therefore we use the so-called phase-space slicing method [10] to separate those regions in the 4-particle phase space which give rise to singular contributions. A separation cut  $y_0^J$  is applied to the scaled invariant masses  $y_{ij}$  and chosen small enough, such that the calculation can be simplified by neglecting terms of the order  $O(y_0^J)$ . Contributions from phase space regions where one of the  $y_{ij}$  is smaller than  $y_0^J$  are singular and have to be combined

with the one-loop corrections to obtain a finite result. The sum of these two contributions defines the cross section for events where two partons are recombined into a parton-level jet (parton-level (1+1)-jet events). The contributions where all  $y_{ij}$  are bigger than  $y_0^J$  are related to final states with three separate partons (parton-level (2+1)-jet events). The latter are free of singularities and can be calculated in 4 dimensions with the help of Monte Carlo techniques.

Explicit calculations show that the phase space slicing parameter  $y_0^J$  has to be chosen very small, of the order of  $10^{-3}$  or smaller, in order to allow for the neglect of terms of order  $O(y_0^J)$ . Therefore,  $y_0^J$  cannot be identified with the  $y$ -cut of a jet algorithm applied in an experimental analysis. There, due to experimental restrictions,  $y$  cannot be reduced to values below  $O(10^{-3})$ . In addition, a fixed-order calculation may give unphysical, *i.e.* negative, (1+1)-jet cross sections for too small values of  $y$ . The Monte Carlo approach, however, allows to apply a jet algorithm to the parton-level events, *i.e.* to recombine 2 partons in the parton-level (2+1)-jet events according to a jet algorithm using  $y$ -cuts  $y^J$  for the separation of jet pairs (similarly:  $y^\gamma$  for the separation of a jet and a photon) with values as appropriate for the given experimental situation.

The calculation thus proceeds through two subsequent steps: First, phase space slicing is applied with a small  $y$ -cut  $y_0^J$  of the order of  $\lesssim 10^{-3}$  to accomplish the cancellation of singularities. This step relies on analytic calculations. Secondly, a jet algorithm is applied with experimentally feasible, *i.e.* large enough values  $y^J$  and  $y^\gamma$  of the order of 0.01 – 0.1. The second step is performed during the Monte Carlo integration.

For a contribution containing the pole factor  $1/y_{ij}$ , we can separate the phase space into three regions:

- $y_{ij} < y_0^J$ . This region contains the infrared singularity at  $y_{ij} = y_{ik} = 0$ , as well as the collinear singularity at  $y_{ij} = 0, y_{ik} > 0$  ( $j \neq k$ ) and leads to singular contributions, *i.e.*  $1/\epsilon$  and  $1/\epsilon^2$  poles in dimensional regularization. The double-poles  $1/\epsilon^2$  and parts of the single-poles  $1/\epsilon$  cancel with corresponding singular contributions from virtual corrections. The remaining  $1/\epsilon$ -pole contributions are associated to the initial state, can be factorized, and are absorbed by renormalizing the parton distribution functions. The analytic integration over this phase space region is performed with the approximation of small  $y_0^J$ , *i.e.* neglecting terms of  $O(y_0^J)$ .
- $y_{ij} \geq y_0^J$  and  $y_{ik} \geq y_0^J$  with only parton-level  $\gamma + (2+1)$ -jet events.
- $y_{ij} \geq y_0^J$  and  $y_{ik} < y_0^J$ . Here, the result is non-singular, but does not vanish with  $y_0^J \rightarrow 0$ . Its contribution is calculated numerically. It is non-negligible in particular for terms related to ISR singularities.

The remaining phase space integrations are performed with the help of Monte Carlo techniques. The three contributions are treated separately, each with appropriate mappings of the respective integration variables to improve the numerical stability of the calculation.

The necessary analytic calculations are well-suited for an automatic treatment with the help of computer algebra programs, similarly to the case of virtual corrections where already a number of packages exist. The algorithm is straightforward and can be formulated in a general way for any  $2 \rightarrow n$  process, with or without photon production:

- Generate the full set of Feynman diagrams, *e.g.* with the help of one of the existing programs.
- Evaluate color factors, traces of gamma matrices, polarization sums.
- Sort the result according to the singular factors  $1/y_{ij}$ .
- Apply partial fractioning to terms which contain products of singular factors in order to separate ISR and FSR singularities.
- Choose for each term an appropriate parametrization for the phase space integration in dimensional regularization. By choosing an explicitly Lorentz-invariant parametrization, *e.g.* in terms of invariant masses  $s_{ij}$ , the phase space slicing cut ( $y_0$ ) can be applied easily.
- Sort the result to obtain a sum of standard forms of integrals. Here, the fact that  $y_0$  will be chosen small can be used to reduce the number of basic integrals.
- Reduce tensor to scalar integrals which have to be evaluated separately. Since the final result contains poles in  $1/\epsilon^n$  of known degree  $n$ , an expansion in powers of  $\epsilon$  can be applied to simplify the calculation.
- Combine the result with the virtual corrections.
- Instead of explicitly factorizing the remaining ISR singularities, one can repeat the above steps for the corresponding  $2 \rightarrow n - 1$  processes and insert the appropriate subtraction terms (Altarelli–Parisi kernels) to obtain a finite result.
- Finally, an automatic generation of corresponding **Fortran** or **C** code will provide the basic building blocks for the development of a Monte Carlo program which can be used to perform the remaining numerical integrations.

This algorithm has been tested for the present case of photon production in deep inelastic scattering and the corresponding **form** programs (to be described elsewhere) can easily be generalized to be applicable to other processes of interest.

More details of the analytical calculations are given in [9]. Numerical results and details of kinematical cuts, in particular details of the jet algorithm and photon isolation criteria, can be found in [3, 8].

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