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$osp(1,2)$–COVARIANT LAGRANGIAN QUANTIZATION OF GENERAL GAUGE THEORIES*

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An $osp(1,2)$–covariant Lagrangian quantization of general gauge theories is introduced which also applies to massive fields. It generalizes the Batalin-Vilkovisky and the $Sp(2)$–covariant field–antifield approach and guarantees symplectic invariance of the quantized action. Massive gauge theories with closed algebra are considered as an example.

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1. Introduction

Recently, a general method for quantizing general gauge theories in the Lagrangian formalism has been proposed [1] which is based on simultaneous invariance under both, BRST and antiBRST transformations, as well as symplectic transformations of (anti)ghost fields and antifields. It is characterized by a quantum action functional $S = S(\phi^A, \phi^*_{Aa}, \bar{\phi}_A, \eta_A)$ depending, besides on the dynamical fields $\phi^A = (A^i, B^\alpha, C^{\alpha a})$, where A^i, B^α and $C^{\alpha a}$ are the gauge, the auxiliary and the (anti)ghost fields, respectively, also on related external antifields or sources ϕ^*_{Aa}, $\bar{\phi}_A$ and η_A. Here a indicates the members of $Sp(2)$–doublets. To guarantee their (anti)BRST symmetry and symplectic invariance the action S (and the gauge fixed extended action $S_{\rm ext}$)

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is required to satisfy a set of quantum master equations being generated by second order differential operators, $\bar{\Delta}^a (a = 1, 2)$ and $\bar{\Delta}_k$ $(k = 0, \pm)$. The algebra of these generating operators may be chosen such as to obey the orthosymplectic superalgebra $osp(1, 2)$. Moreover, if also *massive* fields should be considered – as is the case in the BPHZL renormalization scheme to circumvent possible infrared singularities occuring in the process of subtracting ultraviolet divergences — without breaking the extended BRST symmetry, then this algebra appears necessarily. The method applies to irreducible as well as reducible, complete gauge theories with either open or closed gauge algebra. However, for the sake of simplicity in this review we restrict to the case of irreducible (or zero-stage) complete gauge theories.

We use the condensed notation introduced by DeWitt [2] and conventions adopted in Ref. [3]. If not otherwise specified, derivatives with respect to the antifields are the (usual) left ones and that with respect to the fields are *right* ones (left derivatives with respect to the fields are labeled by L).

2. General gauge theories

In general gauge theories the classical action $S_{\rm cl}(A)$, depending on gauge and matter fields A^i with Grassmann parity $\varepsilon(A^i) = \varepsilon_i$, is invariant under the gauge transformations $A^i \rightarrow A^i_\xi = \exp(i\xi^\alpha \varGamma_\alpha)^i_j A^j$ where ξ^α are the parameters of gauge transformations and $\varGamma_\alpha(A) = -iR^i_\alpha(A)\delta/\delta A^i$ are the gauge generators having Grassmann parities $\varepsilon(\xi^\alpha) = \varepsilon_\alpha$ and $\varepsilon(R^i_\alpha) = \varepsilon_i + \varepsilon_\alpha$, respectively. The Noether identities, being the first class constraints of the classical action, are given by

$$S_{{\rm cl},i}\, R^i(A) = 0 \quad \text{with} \quad R^i = R^i_\alpha \xi^\alpha + T^i, \tag{1}$$

where the trivial generators $T^i = M^{ij}(A) S_{{\rm cl},j}$ contain arbitrary, graded antisymmetric matrices $M^{ij} = -(-1)^{\varepsilon_i \varepsilon_j} M^{ji}$. Herewith, we also introduced the convention $X_{,j} = \delta X / \delta A^j$. Restricting to irreducible theories, we assume the set of generators $R^i_\alpha(A)$ to be linearly independent and complete. The (open) algebra of gauge generators has the general form [3, 4]:

$$R^i_{\alpha,j}\, R^j_\beta - (-1)^{\varepsilon_\alpha \varepsilon_\beta} R^i_{\beta,j}\, R^j_\alpha = -R^i_\gamma F^\gamma_{\alpha\beta} - M^{ij}_{\alpha\beta} S_{{\rm cl},j}\,, \tag{2}$$

where $F^\gamma_{\alpha\beta}$ and $M^{ij}_{\alpha\beta}$, in general, are field-dependent structure functions being graded antisymmetric with respect to $(\alpha\beta)$ and (ij). The algebra is (off-shell) closed if $M^{ij}_{\alpha\beta} = 0$ and, in that case, it defines a Lie algebra if the structure functions $F^\gamma_{\alpha\beta}$ are independent on A. In general, higher order structure functions occure by taking into account also the Jacobi identity.

In order to quantize these theories having orbits of gauge equivalent configurations A^i_ξ an appropriate gauge condition $G_\alpha(A) = 0$ is introduced. Using the definition $G_{\alpha,i}\, R^i_\beta = G_{\alpha\beta}$ the theory is characterized by an effective action, depending also on the auxiliary field B^α, the ghost and antighost field C^α and $\bar{C}^\alpha$ (later on unified as $C^{\alpha a}$),

$$S_{\rm eff}(\phi) = S_{\rm cl}(A) + B^\alpha G_\alpha(A) - \bar{C}^\alpha G_{\alpha\beta}(A) C^\beta = S_{\rm cl}(A) + \boldsymbol{s}\varPsi(\phi), \qquad (3)$$

with the gauge fixing fermion $\varPsi(\phi) = \bar{C}^\alpha G_\alpha(A)$ and the nilpotent BRST–operator $\boldsymbol{s}$ defined through the following BRST–transformations

$$\begin{aligned} \boldsymbol{s}A^i &= R^i_\alpha(A) C^\alpha, & \boldsymbol{s}C^\alpha &= -\tfrac{1}{2} F^\alpha_{\beta\gamma}(A) C^\beta C^\gamma, \qquad (4)\\ \boldsymbol{s}\bar{C}^\alpha &= B^\alpha, & \boldsymbol{s}B^\alpha &= 0. \qquad (5) \end{aligned}$$

The first doublet contains the minimal set of fields, transforming nonlinear, and the second one is its trivial extension. In addition, an antiBRST operator $\bar{\boldsymbol{s}}$ can be defined by exchanging C^α and $\bar{C}^\alpha$. However, then the equations corresponding to (5) are more complicated. The nilpotence of the BRST–operator, $\boldsymbol{s}^2 = 0$, completely encodes the gauge algebra and, together with the first of Eqs. (4), it ensures the BRST–invariance of $S_{\rm eff}(\phi)$.

3. Batalin-Vilkovisky field–antifield approach

Let us now shortly review the Batalin-Vilkovisky(BV) field–antifield approach [4] of quantizing general gauge theories. There, the total configuration space of fields $\phi^A = (A^i, B^\alpha, C^{\alpha a}), \varepsilon(\phi^A) \equiv \varepsilon_A = (\varepsilon_i, \varepsilon_\alpha, \varepsilon_\alpha + 1)$ is extended to the field–antifield "phase space" by introducing the antifields $\phi^*_A = (A^*_i, B^*_\alpha, C^*_{\alpha a})$, $\varepsilon(\phi^*_A) = \varepsilon_A + 1$, which are the sources of the BRST-transforms $\boldsymbol{s}\phi^A$ and behave trivially under BRST, *i.e.* $\boldsymbol{s}\phi^*_A = 0$. In terms of the extended action, $S_{\rm ext}(\phi, \phi^*) = S_{\rm eff}(\phi) + \phi^*_A(\boldsymbol{s}\phi^A)$, the BRST–symmetry is expressed as $\boldsymbol{s}S_{\rm ext}(\phi, \phi^*) \equiv (\delta S_{\rm ext}/\delta\phi^A)(\delta S_{\rm ext}/\delta\phi^*_A) = 0$.

This formalism allows for a more general setting. First, for any functionals of ϕ, ϕ^* the antibracket, a graded symplectic structure, is introduced:

$$(F, G) := \frac{\delta F}{\delta\phi^A}\frac{\delta G}{\delta\phi^*_A} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}\frac{\delta G}{\delta\phi^A}\frac{\delta F}{\delta\phi^*_A}. \qquad (6)$$

It is graded antisymmetric and fulfills the graded Jacobi identity. Obviously, the antibrackets with only either ϕ^A or ϕ^*_B vanish, but $(\phi^A, \phi^*_B) = \delta^A_B$.

Now, the set of *all* classical actions $S(\phi, \phi^*)$ is considered which satisfy the following requirements: They are bosonic functionals having ghost number zero and fulfil the *classical master equation* $(S, S) = 0$ with the boundary condition $S_{|\phi^*=0} = S_{\rm inv}(A)$. Also the antibracket encodes the whole algebraic structure of the theory because, for any functional $X(\phi, \phi^*)$, it holds $\boldsymbol{s}X := (X, S)$ with $\boldsymbol{s}^2 X = ((X, S), S) = 0$.

Then the following nilpotent second order operator (read off from (6))

$$\Delta := (-1)^{\varepsilon_A} \frac{\delta_L}{\delta \phi^A} \frac{\delta}{\delta \phi^*_A} \qquad \text{with} \qquad \Delta^2 = 0 \tag{7}$$

is introduced. The quantum actions, being given as power series in Planck's constant, are required to fulfil the following *quantum master equation*

$$\Delta \exp\{\tfrac{i}{\hbar} S\} = 0 \qquad \Longleftrightarrow \qquad \tfrac{1}{2}(S, S) = i\hbar \Delta S \tag{8}$$

with the boundary condition $S_{|\phi^*=\hbar=0} = S_{\rm inv}(A)$. The set of quantum actions, being solutions of the (equivalent) equations (8), suffers from gauge degeneracy. This degeneracy is resolved by choosing an admissible gauge fixing fermionic functional $\Psi(\phi)$ and defining the gauge fixed, effective action according to $S_{\rm eff}(\phi) = S(\phi, \phi^* = \delta\Psi/\delta\phi)$, which also fulfills Eqs. (8).

The generating functional of Green's functions is introduced by

$$Z(J, \phi^*) = \int \mathcal{D}\phi \exp\left\{ \tfrac{i}{\hbar} \left(S(\phi, \phi^* + \delta\Psi/\delta\phi) + J_A \phi^A \right) \right\}$$

and the generating functional of 1PI–vertex functions $\Gamma(\phi, \phi^*)$ is obtained through the Legendre transformation $\Gamma(\phi, \phi^*) = \frac{\hbar}{i} \ln Z(J, \phi^*) - J_A \phi^A$. Then, through the first Eq. (8), the Slavnov-Taylor identities are obtained as

$$J_A \delta Z / \delta \phi^* = 0 \qquad \Longleftrightarrow \qquad (\Gamma, \Gamma) = 0. \tag{9}$$

As is well known Eq. (9) is the starting point to prove renormalizability, existence of solutions of the quantum master equation, gauge independence of observables, unitarity of S–matrix *etc.* However, this formalism appears to be in some sense incomplete as long as not also the antiBRST–transformations, generated by $\bar{\boldsymbol{s}}$, are considered.

4. Osp(1,2)–covariant quantization

The BV–approach has been generalized by introducing additional antifields as sources of $\bar{\boldsymbol{s}}\phi^A$ and $\boldsymbol{s}\bar{\boldsymbol{s}}\phi^A$ thus leading to a $Sp(2)$–symmetric extension [3]: $\phi^*_{Aa} = (A^*_{ia}, B^*_{\alpha a}, C^*_{\alpha ab})$, $\varepsilon(\phi^*_{Aa}) = \varepsilon_A + 1$ and $\bar{\phi}_A = (\bar{A}_i, \bar{B}_\alpha, \bar{C}_{\alpha a})$, $\varepsilon(\bar{\phi}_A) = \varepsilon_A$. a labels the fundamental doublets of (anti)BRST operators, $\boldsymbol{s} = \boldsymbol{s}^1, \bar{\boldsymbol{s}} = \boldsymbol{s}^2$, as well as of ghost/antighost fields. Raising and lowering of $Sp(2)$–indices is obtained by the invariant antisymmetric tensor $\varepsilon^{ab}, \varepsilon^{12} = 1, \varepsilon^{ac}\varepsilon_{cb} = \delta^a_b$.

In the $Sp(2)$–approach a doublet of graded symplectic structures $(F, G)^a$ is introduced by replacing the antifields ϕ^*_A in Eq. (6) by ϕ^*_{Aa}, and a doublet of (relative) nilpotent generating operators is defined through

$$\bar{\Delta}^a = \Delta^a + \tfrac{i}{\hbar} V^a \quad \text{with} \quad \Delta^a = (-1)^{\varepsilon_A} \frac{\delta_L}{\delta \phi^A} \frac{\delta}{\delta \phi^*_{Aa}} \quad \text{and} \quad V^a = \varepsilon^{ab} \phi^*_{Ab} \frac{\delta}{\delta \bar{\phi}_A}. \tag{10}$$

The relative nilpotency of the (anti)BRST–operators, $\{\boldsymbol{s}^a, \boldsymbol{s}^b\} = 0$, repeats itself in the important relations $\{\bar{\Delta}^a, \bar{\Delta}^b\} = 0$.

Now, the quantum actions are required to satisfy the master equations:

$$\bar{\Delta}^a \exp\{\tfrac{i}{\hbar} S\} = 0 \quad \Longleftrightarrow \quad \tfrac{1}{2}(S, S)^a + V^a S = i\hbar \Delta^a S, \tag{11}$$

with the boundary condition $S|_{\phi^*_a = \bar{\phi} = \hbar = 0} = S_{\rm inv}(A)$. The degeneracy of the action S is removed by the help of a gauge-fixing bosonic functional $F(\phi^A)$, and defining the extended action $S_{\rm ext}$ according to

$$\exp\{\tfrac{i}{\hbar} S_{\rm ext}\} = \exp\{\tfrac{\hbar}{i} \hat{T}(F)\} \exp\{\tfrac{i}{\hbar} S\} \quad \text{with} \quad \hat{T}(F) = \tfrac{1}{2} \varepsilon_{ab} \{\bar{\Delta}^b, [\bar{\Delta}^a, F]\}.$$

This formalism, however, despite appearing manifest $Sp(2)$–symmetric leads to solutions of the quantum master equations which also may be $Sp(2)$–nonsymmetric. The reason for this can be traced back to the fact that the general transformation properties of the solutions of (11) do not restrict F to be a $Sp(2)$–scalar. Therefore, to ensure $Sp(2)$–symmetry the extended quantum action has to be subjected to additional requirements. In addition, the formalism may be generalized to contain also *massive* gauge (and ghost) fields, which are necessary at least intermediatly in the BPHZL renormalization procedure to avoid unwanted infrared singularities. Of course, then the (anti)BRST transformations, and the operators $\bar{\Delta}^a$, must be generalized to include also mass terms, thus leading to an $osp(1, 2)$-symmetric formalism.

Let us now state the essential modifications of the $Sp(2)$–formalism to obtain the $osp(1, 2)$–covariant quantization of irreducible complete gauge theories with massive fields whose action S_m also depends on the mass m. In addition to the *m–extended* quantum master equations (12) which ensure (anti)BRST invariance, the action S_m is required to satisfy the generating equations (13) of $Sp(2)$–invariance, too:

$$\bar{\Delta}^a_m \exp\{\tfrac{i}{\hbar} S_m\} = 0 \quad \text{with} \quad \bar{\Delta}^a_m = \Delta^a + \tfrac{i}{\hbar} V^a_m, \tag{12}$$

$$\bar{\Delta}_k \exp\{\tfrac{i}{\hbar} S_m\} = 0 \quad \text{with} \quad \bar{\Delta}_k = \Delta_k + \tfrac{i}{\hbar} V_k. \tag{13}$$

$\bar{\Delta}^a_m$ and $\bar{\Delta}_k$ are odd and even (second–order) differential operators, respectively, whose linear parts V^a_m and V_k depend and act only on the antifields.

As long as $m \neq 0$ the operators $\bar{\Delta}^a_m$ are neither nilpotent nor do they anticommute among themselves; instead, together with the operators $\bar{\Delta}_k$ they generate a superalgebra isomorphic to $osp(1,2)$ [5]:

$$[\bar{\Delta}_k, \bar{\Delta}_l] = \tfrac{i}{\hbar}\varepsilon_{kl}{}^j \bar{\Delta}_j, \ [\bar{\Delta}_k, \bar{\Delta}^a_m] = \tfrac{i}{\hbar}\bar{\Delta}^b_m(\sigma_k)_b{}^a, \ \{\bar{\Delta}^a_m, \bar{\Delta}^b_m\} = -\tfrac{i}{\hbar}m^2(\sigma^k)^{ab}\bar{\Delta}_k. \tag{14}$$

the matrices σ_k $(k = 0, +, -)$ generate the algebra $sl(2,R)$, the even part of $osp(1,2)$, being isomorphic to $sp(2,R)$: $\sigma_k\sigma_l = g_{kl} + \frac{1}{2}\varepsilon_{klj}\sigma^j$, with the metric $g_{00} = 2g_{+-} = 1$, and ε_{klj} being the antisymmetric tensor, $\varepsilon_{0+-} = 1$. Raising and lowering of indices is obtained by ε^{ab} and g^{kl}, respectively.

From (14) it follows that, if and only if the action S_m is $Sp(2)$–invariant, it can be (anti)BRST–invariant as well. Furthermore, in order to express the algebra (14) by *operator identities* and to get nontrivial solutions of the generating equations (12) and (13) one is enforced to enlarge the set of antifields by additional sources, $\eta_A = (D_i \equiv 0, E_\alpha, F_{\alpha a})$ with $\varepsilon(\eta_A) = \varepsilon_A$. In a componentwise notation the operators V^a_m, V_k and Δ^a, Δ_k are given by (Eq. (10))

$$\begin{aligned}
V^a_m &= \varepsilon^{ab}A^*_{ib}\frac{\delta}{\delta\bar{A}_i} + m^2\bar{A}_i\frac{\delta}{\delta A^*_{ia}} + \varepsilon^{ab}B^*_{\alpha b}\frac{\delta}{\delta\bar{B}_\alpha} + (m^2\bar{B}_\alpha - E_\alpha)\frac{\delta}{\delta B^*_{\alpha a}}\\
&\quad + \varepsilon^{ab}C^*_{\alpha bc}\frac{\delta}{\delta\bar{C}_{\alpha c}} + 2m^2\bar{C}_{\alpha b}\frac{\delta}{\delta C^*_{\alpha\{ab\}}} - F_{\alpha b}\frac{\delta}{\delta C^*_{\alpha ab}} + m^2\varepsilon^{ab}C^*_{\alpha[bc]}\frac{\delta}{\delta F_{\alpha c}},\\
V_k &= A^*_{ia}(\sigma_k)^a{}_b\frac{\delta}{\delta A^*_{ib}} + B^*_{\alpha a}(\sigma_k)^a{}_b\frac{\delta}{\delta B^*_{\alpha b}} + \bar{C}_{\alpha a}(\sigma_k)^a{}_b\frac{\delta}{\delta\bar{C}_{\alpha b}}\\
&\quad + (C^*_{\alpha ac}(\sigma_k)^a{}_b + C^*_{\alpha ba}(\sigma_k)^a{}_c)\frac{\delta}{\delta C^*_{\alpha bc}} + F_{\alpha a}(\sigma_k)^a{}_b\frac{\delta}{\delta F_{\alpha b}}
\end{aligned}$$

and

$$\begin{aligned}
\Delta^a &= (-1)^{\varepsilon_i}\frac{\delta_L}{\delta A^i}\frac{\delta}{\delta A^*_{ia}} + (-1)^{\varepsilon_\alpha}\frac{\delta_L}{\delta B^\alpha}\frac{\delta}{\delta B^*_{\alpha a}} + (-1)^{\varepsilon_\alpha+1}\frac{\delta_L}{\delta C^{\alpha b}}\frac{\delta}{\delta C^*_{\alpha ab}},\\
\Delta_k &= (-1)^{\varepsilon_\alpha+1}(\sigma_k)_a{}^b\frac{\delta_L}{\delta C^{\alpha b}}\frac{\delta}{\delta F_{\alpha a}}\,.
\end{aligned}$$

In order to set up the gauge fixing the m–extended gauge fixed quantum action, $S_{m,\text{ext}} = S_{m,\text{ext}}(\phi^A, \phi^*_{Aa}, \bar{\phi}_A, \eta_A)$, will be introduced according to

$$\begin{aligned}
\exp\{\tfrac{i}{\hbar}S_{m,\text{ext}}\} &= \exp\{-i\hbar\hat{T}_m(F)\}\exp\{\tfrac{i}{\hbar}S_m\},\\
\hat{T}_m(F) &= \tfrac{1}{2}\varepsilon_{ab}\{\bar{\Delta}^b_m, [\bar{\Delta}^a_m, F]\} + (i/\hbar)^2m^2F.
\end{aligned} \tag{15}$$

The gauge fixed action $S_{m,\text{ext}}$ satisfies Eqs. (12) and (13) as well, and the Ward identities related to $osp(1,2)$–symmetry are given by

$$\tfrac{1}{2}(\Gamma_m, \Gamma_m)^a + V^a_m\Gamma_m = 0, \qquad \tfrac{1}{2}\{\Gamma_m, \Gamma_m\}_k + V_k\Gamma_m = 0. \tag{16}$$

Here, a new even graded bracket is defined through $\{F,G\}_k := \Delta_k(FG) - (\Delta_k F)G - F(\Delta_k G)$. It corresponds to the algebraic definition of the antibracket $(F,G)^a := (-1)^{\varepsilon(F)}(\Delta^a(FG) - (\Delta^a F)G) - F(\Delta^a G)$. For Yang-Mills theories Eqs. (16) are the Slavnov-Taylor identities of (anti)BRST–symmetry and, for $k=\pm$, the Delduc-Sorella identities of $Sp(2)$-symmetry [6]. For $k=0$ the ghost number conservation is obtained.

5. Massive theories with a closed algebra

To illustrate the formalism of the $osp(1,2)$-quantization we consider irreducible massive gauge theories where, for the sake of simplicity, we assume that A^i are *bosonic* fields. The closed gauge algebra is characterized by Eq. (2) with $M^{ij}=0$. Furthermore, for field-dependent structure constants, the Jacobi identity looks $F^\delta_{\eta\alpha}F^\eta_{\beta\gamma} - R^i_\alpha F^\delta_{\beta\gamma,i}$ +cyclic$(\alpha,\beta,\gamma)=0$. Additionally, we restrict ourselves to consider only solutions of Eqs. (12), (13) being *linear* in the antifields; they uniquely may be cast into the form

$$S_m = S_{\rm cl}(A) + (\tfrac{1}{2}\varepsilon_{ab}\mathbf{s}^b_m\mathbf{s}^a_m + m^2)\bar\phi_A\phi^A. \tag{17}$$

The (linear) realization of the (anti)BRST– and the $Sp(2)$–operators $\boldsymbol{s}^a_m$ and $\boldsymbol{d}_k$ must satisfy the following $osp(1,2)$–algebra, derived from Eqs. (14):

$$[\boldsymbol{d}_k,\boldsymbol{d}_l] = \varepsilon_{kl}{}^j\boldsymbol{d}_j,\quad [\boldsymbol{d}_k,\boldsymbol{s}^a_m] = \boldsymbol{s}^b_m(\sigma_k)_b{}^a,\quad \{\boldsymbol{s}^a_m,\boldsymbol{s}^b_m\} = -m^2(\sigma^k)^{ab}\boldsymbol{d}_k. \tag{18}$$

The (anti)BRST–transformations in Eq. (17) for this setting are realized by

$$\begin{aligned}
\boldsymbol{s}^a_m A^i &= R^i_\alpha C^{\alpha a}, \qquad \boldsymbol{s}^a_m C^{\alpha b} = \varepsilon^{ab}B^\alpha - \tfrac{1}{2}F^\alpha_{\beta\gamma}C^{\beta a}C^{\gamma b} \qquad\qquad (19)\\
\boldsymbol{s}^a_m B^\alpha &= -m^2C^{\alpha a} + \tfrac{1}{2}F^\alpha_{\beta\gamma}B^\beta C^{\gamma a} + \tfrac{1}{12}\varepsilon_{cd}(F^\alpha_{\eta\beta}F^\eta_{\gamma\delta} + 2R^i_\beta F^\alpha_{\gamma\delta,i})C^{\gamma a}C^{\delta c}C^{\beta d},\\
\boldsymbol{s}^a_m \bar A_i &= \varepsilon^{ab}A^*_{ib},\quad \boldsymbol{s}^a_m A^*_{ib} = m^2\delta^a_b\bar A_i,\quad \boldsymbol{s}^a_m\bar B_\alpha = \varepsilon^{ab}B^*_{\alpha b},\quad \boldsymbol{s}^a_m B^*_{\alpha b} = m^2\delta^a_b\bar B_\alpha,\\
\boldsymbol{s}^a_m\bar C_{\alpha c} &= \varepsilon^{ab}C^*_{\alpha bc},\quad \boldsymbol{s}^a_m C^*_{\alpha bc} = m^2\bar C_{\alpha\{b}\delta^a_{c\}} - \delta^a_b F_{\alpha c},\quad \boldsymbol{s}^a_m F_{\alpha c} = m^2\varepsilon^{ab}C^*_{\alpha[bc]}.
\end{aligned}$$

For the particular case $m=0$ these transformations were already obtained earlier [7]. The action of the operators $\boldsymbol{d}_\alpha$ is obtained through the anticommutation relations of Eqs. (18). Let us stress that satisfying the algebra (18) as operator identities is a stronger restriction than solving it only weakly by means of symmetry transformations as (19). The latter can be realized without the introduction of $F_{\alpha c}$, namely by choosing $C^*_{\alpha ab} = C^*_{\alpha ba}$. This has been assumed in [8], where the Curci-Ferrari model in the Delbourgo-Jarvis gauge has been considered.

Substituting the expressions (19) and those following for $\frac{1}{2}\varepsilon_{ab}\mathbf{s}^b_m\mathbf{s}^a_m$ into

$$\begin{aligned}
S_m &= S_{\rm cl} + A^*_{ia}(\mathbf{s}^a_m A^i) + B^*_{\alpha a}(\mathbf{s}^a_m B^\alpha) - C^*_{\alpha ac}(\mathbf{s}^a_m C^{\alpha c}) + (F_{\alpha c} - \tfrac{m^2}{2}\bar C_{\alpha c})C^{\alpha c}\\
&\quad + \bar A_i(\tfrac{1}{2}\varepsilon_{ab}\mathbf{s}^b_m\mathbf{s}^a_m A^i) + \bar B_\alpha(\tfrac{1}{2}\varepsilon_{ab}\mathbf{s}^b_m\mathbf{s}^a_m B^\alpha) + \bar C_{\alpha c}(\tfrac{1}{2}\varepsilon_{ab}\mathbf{s}^b_m\mathbf{s}^a_m C^{\alpha c}) \qquad (20)
\end{aligned}$$

specifies the theory completely. Thereby, $F_{\alpha c}$ is the only nonvanishing component of η_A (irreducible theories allow the choice $E_\alpha = 0$). A direct verification shows that the resulting action S_m satisfies Eqs. (12), (13) identically.

However, we must emphasize that, we ignored the important question whether the action (17) is the most general solution of Eqs. (12), (13), being especially stable against small perturbations. Here, unfortunately this is not the case, because the fields ϕ^A and the antifields $\bar{\phi}_A$ have the same quantum numbers and hence mix under renormalization. Therefore, in order to ensure the required stability the action (17) must be enlarged to depend also *nonlinearly* on the antifields. However, the corresponding altered action cannot be expressed in such a simple form as Eq. (17) (see Ref. [8, 9]).

6. Concluding remarks

We introduced a generalization of the $Sp(2)$–covariant approach to the $osp(1,2)$–covariant quantization for the case of irreducible general gauge theories. It has been extended also to the case of reducible theories (see [1]). The BPHZL renormalization of infrared singularities can be carried out without breaking the extended master equations (12). Unfortunately, the mass–dependence of the gauge and (anti)ghost fields leads to a soft breaking of gauge–independence and possibly also to non-unitarity of the S–matrix. Of course, in the limit $m \to 0$ gauge–independence and unitarity of S–matrix is restored. An open problem is the general proof, analogous to [3], of existence theorems, *i.e.* absence of anomalies of the theory. An extension of that formalism to include also background fields is possible.

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