ACCURATE PREDICTION OF ELECTROWEAK OBSERVABLES AND IMPACT ON THE HIGGS MASS BOUND*

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I discuss the importance of the $O(g^4 m_t^2/M_W^2)$ corrections to the effective electroweak angle and M_W in the indirect determination of the Higgs mass. I emphasize the rôle of a very precise M_W measurement on the $M_{\rm H}$ estimate.

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1. Introduction

One of the greatest achievements of the program of accurate verification of the Standard Model of electroweak interaction (SM) carried out at LEP and SLC during the last decade has been the prediction of the top mass. After the experimental discovery of the top quark by the CDF collaboration (with a mass at the right place indicated by the electroweak fits) the challenge of precision physics has moved towards the only remaining unknown particle of the SM, namely the Higgs. However, in this case the game is much harder. The reason is clearly connected to the different behavior of the virtual effects of the two particles in the relevant electroweak corrections: power-like for the top, much milder and just logarithmic for the Higgs. To appreciate how much this logarithmic behavior makes hard the game for the theorists (and the experimentalists also) I consider the effective electroweak mixing angle, $\sin^2\theta_{\rm eff}^{\rm lept} \equiv s_{\rm eff}^2$, that is the most important quantity in the determination of $M_{\rm H}$, and write it as

$$s_{\rm eff}^2 \sim (c_1 + \delta c_1) + (c_2 + \delta c_2) \log y; \quad y \equiv (M_{\rm H}/100\ \text{GeV}). \tag{1}$$

In Eq. (1) I identify the l.h.s. with the experimental result that, I assume, carries no error. In the r.h.s. δc_i represent the theoretical uncertainty in

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the corresponding coefficients connected to the fact that we have computed c_i in perturbation theory through certain order in the perturbative series and therefore we do not know their exact values because of higher order contributions. From Eq. (1) one obtains

$$y = y_0 \exp\left[-\frac{\Delta_{\rm th}}{c_2}\right] \quad \Delta_{\rm th} = \delta c_1 + \delta c_2 \log y \tag{2}$$

where y_0 is the value corresponding to $\delta c_1 = \delta c_2 = 0$. To see the effect of Δ_{th} in extracting $M_{\rm H}$ I take

$$c_2 \sim \frac{\alpha}{2\pi(c^2 - s^2)}\left(\frac{5}{6} - \frac{3}{4}c^2\right) \sim 5.5 \times 10^{-4}; \quad \Delta_{\rm th} \sim \pm 1.4 \times 10^{-4} \tag{3}$$

where $s^2 \sim 0.23$, $c^2 = 1 - s^2$. In Eq. (3) I estimate c_2 through the leading Higgs behavior of the correction $\Delta\hat{r}$ relevant for $s^2_{\rm eff}$ [1, 2] while for $\Delta_{\rm th}$, I take the value estimated in the 1995 CERN report on 'Precision calculation for the Z resonance' [3]. The latter has been obtained comparing the output of five different codes that implement different renormalization schemes and have built in several options for resumming known effects. At the time of the report, the knowledge of the electroweak part of the radiative corrections included, besides the complete one-loop order, the leading logarithms of $O(\alpha^n \log^n M_Z/m_f)$ (here m_f is a generic fermion mass) [4, 5], the $O(\alpha^2 \log M_Z/m_f)$ [5] term while for the two-loop top contribution only the leading $O(g^4 m_t^4/M_W^4)$ correction was known [6]. Therefore the comparison of the various codes was mainly measuring the scheme-dependence error induced by the ignorance of the next term in the two-loop top contribution, namely the $O(g^4 m_t^2/M_W^2)$ corrections. Inserting the values of Eq. (3) into Eq. (2) yields $y \sim 1.29\, y_0$. We see that a theoretical uncertainty coming from two-loop unknown contributions (that are supposed to be not even the dominant part) makes an error in the indirect determination of the Higgs mass of 29 %!

2. Recent progress in higher order calculations

The above example clearly tells us that to extract accurate indirect information on the Higgs one needs not only very precise experiments but also a very good control of the theory side. This brings in the issue of what error we can associate to our theoretical predictions. They are affected by uncertainties coming from two different sources: one that is called parametric and it is connected to the error in the experimental inputs used in our predictions. The second one is called intrinsic and it is related to the fact that our knowledge of the perturbative series is always limited, usually to the first few terms. Concerning parametric uncertainties, $\alpha(0)$, G_μ and M_Z are very well measured, m_t and α_s are not so precisely known while for $M_{\rm H}$ there is not at all direct evidence. The scale of the weak interactions is given by the mass of the intermediate vector bosons, so what actually matters in our predictions is not $\alpha(0)$ but $\alpha(M_Z)$. The latter contains the hadronic contribution to the photon vacuum polarization, $(\Delta\alpha)_h$, that cannot be evaluated in perturbation theory. Fortunately, one can use a dispersion relation to relate it to the experimental data on the cross section for e^+e^- annihilation into hadrons. In the recent years there has been a lot of activity on this subject. Several new analyses appeared that differ in the treatment of the experimental data [7, 8] and in the amount of theoretical input used to evaluate them [9, 10]. The situation is not yet settle down (and probably will not be till new experimental data on the e^+e^- cross section in the low and intermediate energy region are available), so a conservative approach is still to use the value given by the most phenomenological analyses [7], $\alpha(M_Z)^{-1} = 128.90 \pm 0.09$.

The status of the intrinsic uncertainties has actually improved since the CERN report. A sizeable amount of work on radiative corrections has been completed in the recent past. In this talk, I will discuss only the information that is now available on the $O(g^4 m_t^2/M_W^2)$ corrections.

The fact that the top is heavier than the other known particles suggests to organize its two-loop contribution to the various radiative parameters as a series in m_t. The first two terms of this series are enhanced by factors $(m_t^2/M_W^2)^n$ $(n = 1, 2)$ while the remaining ones are at most logarithmic in nature. The leading contribution that scales as m_t^4 is completely available for arbitrary value of the Higgs mass since few years [6]. The next term, i.e. the $O(g^4 m_t^2/M_W^2)$ correction, has been recently incorporated in the theoretical calculation of M_W [11], $s^2_{\rm eff}$ [12, 13] and the partial widths of the Z into fermions but the b quark [14]. Indeed in the case of the b there are specific vertex corrections of the same order not yet computed. To gauge the residual scheme dependence, $O(g^4)$, this incorporation has been performed in three electroweak resummation approaches and two different ways of implementing the relevant QCD corrections [12]. One of the approaches ($\overline{\rm MS}$) employs $\hat\alpha(M_Z)$ and $\sin^2\hat\theta_W(M_Z) \equiv \hat s^2$, the $\overline{\rm MS}$ QED and electroweak mixing parameters evaluated at the scale $\mu = M_Z$, while the other two (OSI and OSII) make use of the on-shell parameters α and $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$. As expected, the dependence on the electroweak scale μ cancels through $O(g^4 m_t^2/M_W^2)$. However, because complete $O(g^4)$ corrections have not been evaluated, the $\overline{\rm MS}$ and OSI formulations contain a residual $O(g^4)$ scale dependence. On the other hand OSII is, by construction, strictly μ-independent. In Table I the predictions for $s^2_{\rm eff}$ and M_W in this three different frameworks are shown. The QCD corrections are implemented on the base of a top pole mass pa-

TABLE I

Predicted values of M_W and $s^2_{\rm eff}$ in different frameworks for $m_t = 175$ GeV with QCD corrections based on pole top-mass parameterization. The first row of each $M_{\rm H}$ entry is obtained including only the $O(g^4 m_t^4/M_W^4)$. The $O(g^4 m_t^2/M_W^2)$ result is presented in the second row (only the last two different digits are shown).

	$\sin^2\theta^{\rm lept}_{\rm eff}$			M_W (GeV)		
$M_{\rm H}$	OSI	OSII	$\overline{\rm MS}$	OSI	OSII	$\overline{\rm MS}$
65	.23131	.23111	.23122	80.411	80.422	80.420
	32	34	30	05	04	06
100	.23153	.23135	.23144	80.388	80.397	80.396
	53	55	52	82	81	83
300	.23212	.23203	.23203	80.312	80.316	80.319
	10	14	10	08	06	08
600	.23251	.23249	.23243	80.256	80.257	80.263
	49	52	49	54	52	54
1000	.23280	.23282	.23272	80.215	80.213	80.221
	77	79	77	14	13	14

rameterization (for results with QCD corrections implemented in terms of running $\overline{\rm MS}$ top mass see Ref. [12]). For each entry of the Higgs mass the first row corresponds to the value obtained including only the $O(g^4 m_t^4/M_W^4)$ contribution while the second one contains also the $O(g^4 m_t^2/M_W^2)$ part.

I will not discuss in detail the effect of the $O(g^4 m_t^2/M_W^2)$ corrections in the electroweak fits (see Bob Clare's talk [15]) but I would like to point out few things that can be easily read from Table I.

(i) The incorporation of the $O(g^4 m_t^2/M_W^2)$ corrections reduces the scheme dependence to the level of 4×10^{-5} in $s^2_{\rm eff}$ and 2 MeV in M_W.

(ii) The $O(g^4 m_t^2/M_W^2)$ values for $s^2_{\rm eff}$ (M_W) are generally higher (lower) than the corresponding $O(g^4 m_t^4/M_W^4)$ results. In the indirect determination of $M_{\rm H}$ this fact favors a lighter value of the mass.

(iii) In general the $O(g^4 m_t^2/M_W^2)$ OSI and $\overline{\rm MS}$ results are very close.

The OSI resummation is actually the natural generalization to $O(g^4 m_t^2/M_W^2)$ of the one proposed by Consoli, Hollik, and Jegerlehner [16] for the reducible $O(g^4 m_t^4/M_W^4)$ term and it is the one presently implemented in ZFITTER [17]. On the other side our $\overline{\rm MS}$ approach [2] is quite similar to the one implemented in TOPAZ0 [18]. This explains why in the new version of the famous LEPEWWG $\Delta\chi^2$ *vs.* $M_{\rm H}$ blue-band plot [15] the ZFITTER and TOPAZ0 lines are very close especially for large values of $M_{\rm H}$ and the blue band seems to have disappeared. With respect to this a comment is in order. The new $\Delta\chi^2$ curve obtained including $O(g^4 m_t^2/M_W^2)$ corrections is not enclosed in the old blue-band representing the $O(g^4 m_t^2/M_W^2)$ scheme-dependence uncertainty [19]. There is nothing wrong with it. Indeed the comparison of results obtained in different schemes of calculation that contain all the available theoretical information at a given order of accuracy gives us a guess of the size of the reducible contribution, namely the part due to resummation or iteration of lower order effects. It does not tell us anything about the exact size of higher order one-particle irreducible contributions. This way of estimating the intrinsic uncertainty should be taken as giving just the order of magnitude of it and moreover can be realistic only if the irreducible part is comparable or smaller than the reducible one. But we have no way to know it before actually performing the calculation of the irreducible part.

3. Importance of a precise M_W measurement

The precise electroweak measurements allow to constrain significantly the value of the Higgs mass. A global fit to all data gives a strong indication for a light Higgs with an upper limit at 95 % C.L. $M_{\rm H} < 215$ GeV [15]. However, the current estimates of $M_{\rm H}$ depend crucially on the world average $s^2_{\rm eff} = 0.23149 \pm 0.00021$, and this follows from a combination of experimental results that are not always in good harmony. The data presented at the recent Winter conferences [15] show a better agreement than the previous ones [19] but still the most precise LEP result ($s^2_{\rm eff} = 0.23213 \pm 0.00039$ from $A^{0,b}_{fb}$) and the SLAC data ($s^2_{\rm eff} = 0.23084 \pm 0.00035$) are quite far apart. To show how much the low value of SLAC is important for a light $M_{\rm H}$ determination I consider $s^2_{\rm eff}$ and use the parameterization [20]

$$\frac{\sin^2\theta^{\rm lept}_{\rm eff}}{0.23151} - 1 = b_1 \ln\left(\frac{M_{\rm H}}{100~{\rm GeV}}\right) + b_2\left[\frac{(\Delta\alpha)_h}{0.0280} - 1\right] + b_3\left[\left(\frac{m_t}{175~{\rm GeV}}\right)^2 - 1\right] + b_4\left[\frac{\alpha_s(M_Z)}{0.118} - 1\right] \quad (4)$$

that in the range 75 GeV $\leq M_{\rm H} \leq$ 350 GeV, with the other parameters within their $1-\sigma$ errors, approximates the detailed calculations of Ref. [12] with average absolute deviations of $\approx 4 \times 10^{-6}$ and maximum absolute deviations of $(1.1 - 1.3) \times 10^{-5}$ depending on the scheme while outside the above range, the deviations increase reaching $(2.6-2.8)\times 10^{-5}$ for $M_{\rm H} = 600$ GeV (the values of the b_i coefficients for the $\overline{\rm MS}$ scheme are presented in

Table II). Employing in Eq. (4) $m_t = 174.1 \pm 5.4$ GeV, $\alpha_s(M_Z) = 0.118 \pm 0.003$, $(\Delta\alpha)_h = 0.0280 \pm 0.0007$ and the LEP average for $s^2_{\rm eff}$ ($s^2_{\rm eff} = 023186 \pm 0.00026$) I obtain a 95 % C.L. upper bound $M_{\rm H} < 610$ GeV. For the same values of m_t, $\alpha_s(M_Z)$ and $(\Delta\alpha)_h$ the use of the SLAC value for $s^2_{\rm eff}$ in Eq. (4) gives instead a 95 % C.L. upper bound $M_{\rm H} < 110$ GeV.

TABLE II

Values in the $\overline{\rm MS}$ scheme of b_i ($i = 1 \div 4$) in Eq. (4) and d_i ($i = 1 \div 5$) in Eq. (5) and their ratio.

	b_i	d_i	$\lvert b_i/d_i \rvert$
$i = 1$	2.26×10^{-3}	-7.2×10^{-4}	~ 3.1
2	4.26×10^{-2}	-6.4×10^{-3}	~ 6.6
3	-1.20×10^{-2}	6.7×10^{-3}	~ 1.8
4	1.94×10^{-3}	-1.1×10^{-3}	~ 1.8
5		-1.0×10^{-4}	

Clearly it is the SLAC result that mainly pushes the electroweak fit towards a light Higgs mass. Notice that a fit to LEP data alone (excluding the direct determination of m_t) gives a light Higgs ($M_{\rm H} = 56^{+101}_{-31}$) but at the price of a low top ($m_t = 156^{+12}_{-10}$) [15]. There is another observation to be made with respect to $s^2_{\rm eff}$. This observable is very sensitive to $(\Delta\alpha)_h$. As I said, the accuracy we know this quantity is presently under discussion. The most conservative error [7] ($\delta(\Delta\alpha)_h = 7 \times 10^{-4}$) makes it the bottleneck in the improvement of the $M_{\rm H}$ determination. The recent more theoretically oriented analyses [10] give an error on $(\Delta\alpha)_h$ ranging from $\delta(\Delta\alpha)_h = 1.6 \times 10^{-4}$ to $\delta(\Delta\alpha)_h = 4.5 \times 10^{-4}$. Using a smaller error for $(\Delta\alpha)_h$ implies to weight more $s^2_{\rm eff}$ in the $M_{\rm H}$ fit that means we have to trust more the $s^2_{\rm eff}$ results.

This state of affairs strongly suggests the desirability of obtaining constraints on $M_{\rm H}$ derived from future precise measurements of M_W. Similarly to Eq. (4) I parameterize the result for M_W as [20]

$$\frac{M_W}{80.383} - 1 = d_1 \ln\left(\frac{M_{\rm H}}{100\ \text{GeV}}\right) + d_2\left[\frac{(\Delta\alpha)_h}{0.0280} - 1\right] + d_5 \ln^2\left(\frac{M_{\rm H}}{100\ \text{GeV}}\right) + d_3\left[\left(\frac{m_t}{175\ \text{GeV}}\right)^2 - 1\right] + d_4\left[\frac{\alpha_s(M_Z)}{0.118} - 1\right], \quad (5)$$

where the d_i coefficients are shown in Table II and notice that to obtain an accuracy in the parameterization similar to that of Eq. (4) I need to introduce an extra term proportional to $\ln^2(M_{\rm H}/100\ \ {\rm GeV})$. Comparing the coefficients of the Eq. (4) and Eq. (5) we see that at equal level of experimental accuracy (which is, in fact, the current situation) $s^2_{\rm eff}$ is more sensitive than M_W by a factor ≈ 2.7 in $\ln(M_{\rm H}/100)$ (taking also into account the $\ln^2(M_{\rm H}/100)$ term of Eq. (5)). On the other side, M_W has the welcome characteristic to be not so sensitive to $(\Delta\alpha)_h$. Let us now consider future scenarios where the experimental errors in the various quantities that enter in Eq. (4) and Eq. (5) are somewhat reduced and compare the indirect determination of $M_{\rm H}$ from M_W and $s^2_{\rm eff}$, separately. To make a simple comparison I use central values that give the same Higgs mass, so I choose $s^2_{\rm eff} = 0.23151$, $M_W = 80.383\ \ {\rm GeV}, (\Delta\alpha)_h = 0.0280$, $m_t = 175$ GeV, $\alpha_s(M_Z) = 0.118$ that correspond to $M_{\rm H} = 100$ GeV. Table III presents 3 possible scenarios in all of which I assume no improvement in the $s^2_{\rm eff}$ and $\alpha_s(M_Z)$ determination (*i.e.* $\delta s^2_{\rm eff} = 0.00021$ and $\delta\alpha_s(M_Z) = 0.003$) while the errors in m_t, M_W and in the last case also in $(\Delta\alpha)_h$ get reduced. One sees that a determination of M_W at the level of 35 Mev together with an improvement in m_t to $\delta m_t = 3$ GeV gives an information on $M_{\rm H}$ competitive with the one that is presently obtained from $s^2_{\rm eff}$. Such a scenario is consistent with the expectation of Tevatron Run 2. A further reduction in δM_W and δm_t, that can be foreseen at LHC, will make M_W more effective than $s^2_{\rm eff}$ in determining $M_{\rm H}$ even in a situation in which the error on $(\Delta\alpha)_h$ will be significantly reduced.

TABLE III

Errors on $\ln(M_{\rm H}/(100\ \ {\rm GeV}))$ determined from M_W (Eq. (5)) and $s^2_{\rm eff}$ (Eq. (4)) for $M_{\rm H} = 100\ \ {\rm GeV}$, $\delta s^2_{\rm eff} = 0.00021$, $\delta\alpha_s(M_Z) = 0.003$ and different values of δm_t, δM_W and $\delta(\Delta\alpha)_h$.

$\ln(M_{\rm H}/(100\ \ {\rm GeV}))$	M_W determination	$s^2_{\rm eff}$ determination
$\delta m_t = 3\ \ {\rm GeV}$, $\delta M_W = 35$ MeV $\delta(\Delta\alpha)_h = 0.0007$	$0^{+0.663}_{-0.815}$	0 ± 0.647
$\delta m_t = 1\ \ {\rm GeV}$, $\delta M_W = 20$ MeV $\delta(\Delta\alpha)_h = 0.0007$	$0^{+0.404}_{-0.455}$	0 ± 0.623
$\delta m_t = 1\ \ {\rm GeV}$, $\delta M_W = 20$ MeV $\delta(\Delta\alpha)_h = 0.0002$	$0^{+0.352}_{-0.390}$	0 ± 0.428

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