

SINGLE- AND MULTI-PHOTON EVENTS  
WITH MISSING ENERGY AT LEP\*

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To avoid loss of sensitivity in the search for new physics in single- and multi-photon final states with large missing energy at LEP, precise predictions for the Standard Model irreducible background are required. At LEP1 the theoretical situation is satisfactory. Going to LEP2, some improvements are necessary. To this aim, the matrix elements for the processes  $e^+e^- \rightarrow \nu\bar{\nu}n\gamma$ , with  $n = 1, 2, 3$ , are exactly computed in the Standard Model, including the possibility of anomalous couplings for single-photon production. Due to the presence of observed photons in the final state, particular attention is paid to the treatment of higher-order QED corrections. Comparisons with existing calculations are shown and commented. An improved version of the event generator NUNUGPV is presented.

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**1. Introduction**

The production of one or more photons and missing energy in high energy  $e^+e^-$  collisions is a process of great interest for the scientific programme of LEP [1]. The events with single- and multi-photon final states plus missing energy ( $\cancel{E}$ ) play an important role in the search for new phenomena beyond

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the SM [2]. Actually, the SM processes  $e^+e^- \rightarrow \nu\bar{\nu}n\gamma$ , with  $n = 1, 2, \dots$ , are the largely dominating irreducible backgrounds to a New Physics (NP) signature consisting of one or more photon(s) and nothing else seen in the detector. Such events can indeed originate from various mechanisms, both in gravity- and gauge-mediated supersymmetric models [3] as well as in scenarios with strong electroweak symmetry breaking [4]. Furthermore, this signature can be useful to study anomalous couplings or put constraints on a fourth generation of heavy neutrinos.

The present situation of the theoretical calculations for the above quoted processes can be considered as satisfactory for the purposes of data analysis at LEP1. Going to LEP2, the typical SM cross section is of the order of a few picobarn, yielding thousands of events collected in the four LEP experiments. Hence there is now a need for theoretical predictions with an accuracy of the order of 1% for the rate of  $e^+e^- \rightarrow \nu\bar{\nu}n\gamma$  events in the SM. Furthermore, the situation concerning the theoretical calculations is not completely satisfactory. In particular, a careful treatment of the higher-order QED corrections to processes with detected photons in the final state becomes mandatory for a meaningful comparison between data and theory.

Given the above physics motivations, an *exact* tree-level calculation in the SM of the  $e^+e^- \rightarrow \nu\bar{\nu}n\gamma$  cross sections, with  $n = 1, 2, 3$ , is done and supplemented with the phenomenologically most relevant and presently controlled radiative corrections. A related improved version of the event generator for data analysis NUNUGPV is presented.

## 2. Existing calculations and generators

Concerning the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  several calculations, with a different degree of accuracy, are known in the literature (see Refs. [5, 6] for reviews).

Besides the first attempts in the so-called Point Interaction Approximation [7], other approximate calculations are available. In such approaches the invisible neutrino-pair cross section is dressed with some (universal) radiation factor to attach one external photon to the charged fermion legs, *e.g.* by using an angular dependent radiator [8, 9], a parton shower (PS) algorithm (as in the program PYTHIA) [10] or the Yennie–Frautschi–Suura (YFS) exclusive exponentiation (as done in KORALZ) [11]. By construction, these calculations need to be corrected for the effect of sub-leading terms and/or internal photon radiation from the off-shell  $W$  boson that are contained in the exact matrix element. The first complete calculation of the matrix element of the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  was done in Ref. [12]. The corresponding exact matrix element is implemented in the event generator MMM [13]. In Ref. [12], by working in the approximation of neglecting terms with at least three boson propagators in the squared matrix element, also a compact, an

analytical expression for the differential spectrum in the energy and angle of the observed photon is obtained, yielding the result

$$\frac{d\sigma}{d\cos\vartheta_\gamma dk} = \frac{\alpha}{12\pi^2} G_F^2 M_W^4 \frac{s'k}{sk_+k_-} [\eta_+^2 F(\eta_+) + \eta_-^2 F(\eta_-)], \quad (1)$$

where the meaning and explicit expression of the symbols entering Eq. (1) can be found in Ref. [12]. The photon spectrum of Eq. (1) contains the bulk of the contributions due to  $W$ -boson exchange and agrees within 1% with approximate calculations discussed above for center of mass (c.m.) energies around the  $Z$  resonance [9]. The photon spectrum of Eq. (1) is implemented in the released version of the event generator NUNUGPV [14].

Concerning radiative corrections, the exact one-loop electroweak corrections to  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  process are not yet available. However, in order to take care of the most sizeable higher-order corrections, the lowest-order calculations are typically improved by the inclusion of the (large) effects due to initial-state radiation (ISR). In mostly used computational tools such a contribution is taken into account via traditional algorithms for computing QED radiative corrections in the leading logarithmic (LL) approximation, such as the PS algorithm [15] (as in PYTHIA), the Structure Function (SF) approach [16] (as in MMM and NUNUGPV, but also in PYTHIA) and YFS exclusive exponentiation [17] (as in KORALZ). As it will be discussed later, different variations of the SF method are implemented in the programs.

The previously quoted programs KORALZ, MMM and NUNUGPV are the standard Monte Carlo generators used by the LEP collaborations for the analysis of the data relative to the events  $e^+e^- \rightarrow \nu\bar{\nu}\gamma(\gamma)$ . By looking at the level of agreement between the above generators [18], the present status points out the need of improving theoretical predictions to avoid a loss of sensitivity to NP searches in radiative events at LEP2.

With regard to the theoretical predictions for the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ , which is the most relevant SM background to a signature with two acoplanar photons and large missing energy, dedicated calculations appeared very recently in the literature. A complete diagrammatic calculation, using the helicity amplitude technique, supplemented with collinear SFs to account for ISR, was done in Ref. [19], confirming a previous evaluation in Born approximation [20]. Approximate predictions for the process of interest are obtained by the LEP collaborations by using the above quoted programs with QED “dressing” of the neutrino-pair cross section, namely PYTHIA (via the PS) and KORALZ (via the YFS method). Further, modern packages for the automatic calculation of Feynman amplitudes, such as GRACE [21] and CompHEP [22], are used by the experiments to calculate the cross section and generate events. Both packages implement collinear SFs for ISR, with

an option for PS in GRACE. An extensive comparison of all available calculations [23] shows a not completely satisfactory situation about the software for the analysis of the data with acoplanar photons.

### 3. Details of the calculation

#### 3.1. Tree-level cross sections

In order to approach the aimed theoretical precision and improve the predictions of the earlier version of the program NUNUGPV, the lowest-order matrix elements associated to the processes  $e^+e^- \rightarrow \nu\bar{\nu}n\gamma$ , with  $n = 1, 2, 3$ , have been *exactly* calculated in the SM.

The matrix element for single-photon production has been computed by means of helicity amplitude techniques [24], including the possibility of anomalous  $\Delta k_\gamma$  and  $\lambda_\gamma$  contributions to the  $WW\gamma$  coupling. The diagrammatic calculation has been cross-checked by using the algorithm ALPHA [25] and found to be in perfect agreement. The three-body phase space has been generated recursively, via standard decomposition of the phase-space.

The exact treatment of the single-photon matrix element upgrades the released version of NUNUGPV, based on the photon spectrum of Eq. (1) of Ref. [12], to include previously neglected  $W$ -boson effects relative to contributions with at least three boson propagators. The size of such previously neglected effects is shown in Fig. 1, for typical event selections used by LEP experiments [18]. The calculation of the single-photon cross section obtained with the exact matrix element is compared with the cross section resulting from the integration of the photon spectrum of Eq. (1). The relative difference between the two calculations is at a few per cent level, both without and with a cut on the missing mass, in agreement with the degree of approximation stated in Ref. [12]. However, it should be noticed that an exact treatment of the lowest-order matrix element is actually mandatory at LEP2 if a theoretical accuracy of the order of 1% is aimed at. The already quoted algorithm ALPHA, that is conceived for the automatic computation of tree-level multi-particle production amplitudes without any need of Feynman graphs expansion, has been employed for the calculation of the matrix elements with two and three photons in the final state. For the process  $\nu\bar{\nu}\gamma\gamma\gamma$  the calculation here presented is the first one appearing in the literature.

Numerical results obtained in our study show that the cross section for the signature  $\nu\bar{\nu}\gamma\gamma$  is about a factor 10–100 smaller than the cross section for  $\nu\bar{\nu}\gamma$ , the reduction factor being strongly dependent, as expected, on the imposed photon cuts.

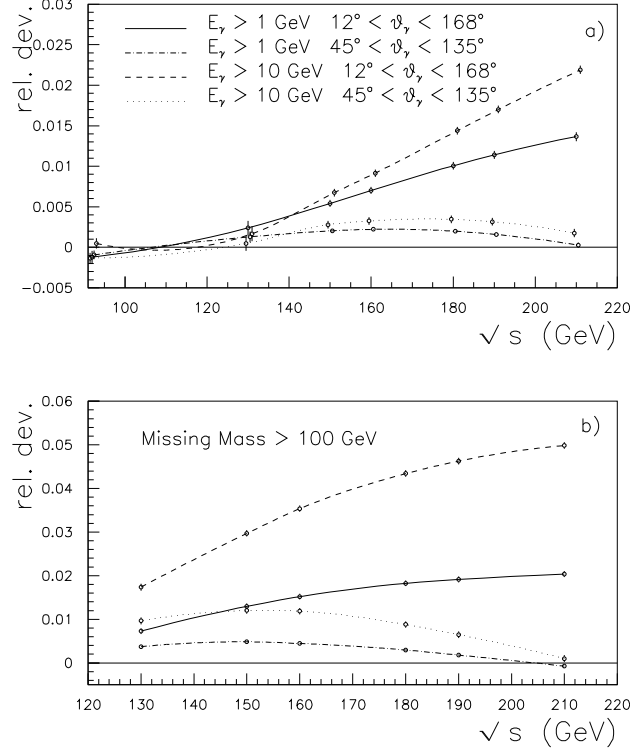


Fig. 1. The relative deviation between the exact  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  cross section and the approximated one, as obtained via Eq. (1) implemented in the earlier version of NUNUGPV. The differences are shown for the different event selections given in (a), with an additional cut on the missing mass in (b).

### 3.2. Treatment of the initial-state radiation

As discussed in Sect. 2, the implementation of ISR is generally achieved by using standard algorithms for universal photonic corrections [15–17]. In particular, the SF approach is certainly the most widely used algorithm, implemented in many generators of interest here, such as CompHEP, GRACE, MMM and NUNUGPV, just to cite a few. More precisely, all the programs make use of SFs in strictly collinear approximation, while in NUNUGPV  $p_T$ -dependent SFs [8, 26] are implemented to improve the treatment of ISR by including  $p_T/p_L$  effects. Because of the presence of photons among the observed final-state products, the inclusion of ISR requires a particular care. This caution is further motivated by the very large enhance-

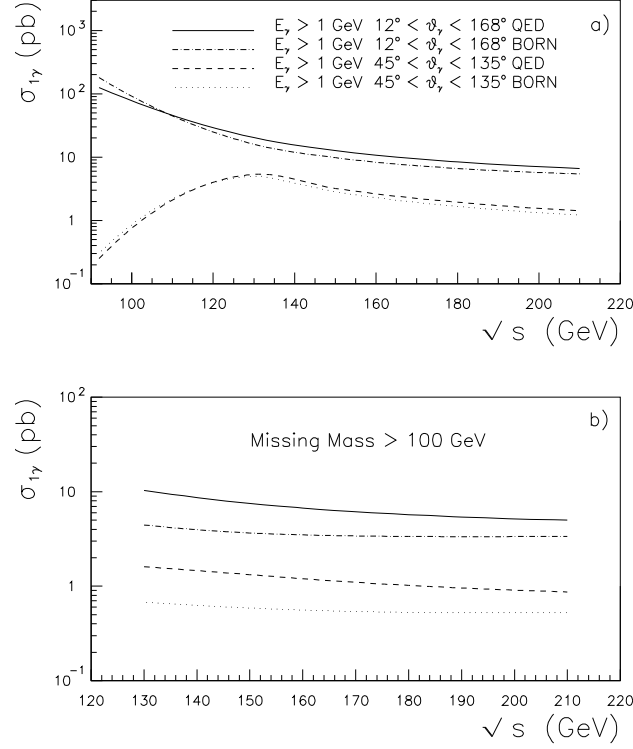


Fig. 2. The tree-level cross section for the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$  as compared with the cross section with higher-order QED corrections, obtained by using collinear SFs. Two typical selection criteria, specified in Fig. 2a, are considered, including (2a) and excluding, via a cut on the missing mass (b), the  $Z$  radiative return.

ment of the lowest-order cross sections as due to ISR and clearly visible in Fig. 2. Actually, as can be seen from Fig. 2, the enhancement factor is about 1.3 when including the  $Z$  return and about 2 when excluding it, both for single- and double-photon cross sections. As done in many practical applications [9,13,19], the above results can be simply obtained by convoluting the hard-scattering cross section with collinear SFs, according to the factorized formula

$$\sigma_{\text{coll}} = \int dx_1 dx_2 D(x_1, s) D(x_2, s) d\sigma \Theta(\text{cuts}). \quad (2)$$

It allows to take into account the impact of higher-order QED corrections, due to photon emission before the hard-scattering reaction (pre-emission), at the LL level. Equation (2) is a good approximation to QED radiative cor-

rections in the LEP1 energy regime at the LL level, since, with the standard selection criteria, hard “pre-emission” photons are inhibited by the finite  $Z$ -boson width. On the contrary, going to LEP2 it can be easily realized that the implementation of ISR as given by Eq. (2) is an approximation that deteriorates whenever the photonic degrees of freedom of the pre-emission and hard-scattering process, respectively, overlap in the same phase space region. Actually, because the collinear SFs can be seen as the result of an integration over the angular variables of the photon radiation, Eq. (2) does not take into account the correct statistical factor to be included in the presence of identical particles in the final state. Furthermore, if the pre-emission photon is detectable, the reconstruction of the event via Eq. (2) is only approximate and this might imply an additional inaccuracy. Therefore, one should expect that the implementation of ISR as given by Eq. (2) leads to an overestimate of the higher-order QED corrections in the LEP2 energy regime. This effect is clearly dependent on the photon(s) detection criteria and can be expected to be not negligible with respect to a  $O(1\%)$  theoretical accuracy. An estimate of the effects due to the phase space overlapping of the IS pre-emission photons with the observed ones can be obtained by supplying the QED SFs with the transverse degrees of freedom. Actually, the generation of the angular variables at the level of ISR gives the possibility of rejecting in the event sample those pre-emission photons above the minimum detection angle, thus avoiding “overlapping effects”.

According to such a procedure, the cross section with higher-order QED corrections can be calculated as follows (for the realistic data sample of at least one photon)

$$\sigma^{1\gamma(\gamma)} = \int dx_1 dx_2 dc_\gamma^{(1)} dc_\gamma^{(2)} \tilde{D}(x_1, c_\gamma^{(1)}; s) \tilde{D}(x_2, c_\gamma^{(2)}; s) \Theta(cuts) \times (d\sigma^{1\gamma} + d\sigma^{2\gamma} + d\sigma^{3\gamma} + \dots), \quad (3)$$

where  $\tilde{D}(x, c_\gamma; s)$  [14] is a proper combination of the collinear SF  $D(x, s)$  with an angular factor inspired by the leading behaviour  $1/(p \cdot k)$ . The latter is used to generate the angular variables of the pre-emission photons. According to Eq. (3), an “equivalent” photon is generated for each colliding lepton and accepted as a higher-order ISR contribution if:

- the energy of the equivalent photon is below the threshold for the observed photon  $E_{\gamma, \min}$ , for arbitrary angles; or
- the angle of the equivalent photon is outside the angular acceptance for the observed photons, for arbitrary energies.

Within the angular acceptance of the seen photon(s), the cross section is evaluated by summing the exact matrix elements for the processes

$e^+e^- \rightarrow \nu\bar{\nu}n\gamma$ ,  $n = 1, 2, 3$  ( $d\sigma^{1\gamma}, d\sigma^{2\gamma}, d\sigma^{3\gamma}$ ). Notice that from the point of view of computing  $\sigma^{1\gamma(\gamma)}$  the real contributions  $d\sigma^{n\gamma}$ ,  $n \geq 2$ , represent the “hard” radiative corrections to be matched with the soft+virtual ones accounted for by the SFs. Therefore they are in principle necessary at all orders. The truncation of hard radiative corrections at the level of  $d\sigma^{3\gamma}$  introduces a spurious infra-red sensitivity in radiative corrections at the order  $\alpha^4 \ln^4(E/E_{\gamma,\min})$  which, from the practical point of view, is completely negligible at realistic  $E_{\gamma,\min}$ . Since the radiative corrections implemented by means of this procedure are at the LL level, its theoretical error is dominated by missing truly  $O(\alpha)$  corrections.

In order to quantify the overestimate introduced by the collinear SFs in the calculation of ISR via Eq. (2), the relative difference between Eq. (2) and Eq. (3) is shown in Fig. 3, for several photon detection criteria. As can be seen, at LEP1 the overlapping effects are contained within a few per mille, and therefore negligible on the scale of the experimental accuracy. Going to LEP2 energies, the overlapping effects are of the order of 1-4% when including the  $Z$  return and still larger, reaching 10%, when imposing a cut on the missing mass, as usually done in realistic event selections. These effects are therefore important in the light of the aimed theoretical precision. A qualitative explanation of the effects can be given as follows. The overestimate of radiative corrections takes place when the pre-emission photon can reach the observability region for the detected photon. At LEP1, and with standard selection criteria, the emission of multiple detectable photons is inhibited by the finite  $Z$ -boson width and therefore the overlapping effect is naturally suppressed by the dynamics. At LEP2, where this suppression is no longer active, the overlapping effects can become more sizeable, depending on the angular acceptance and minimum energy of the observed photons, and, more generally, in the presence of additional cuts on the four-momenta of the observed photons (as in the case of a cut on the missing mass).

Analogously to Eq. (3), the QED corrected cross section for the signature of at least two photons in the final state can be cast as follows

$$\begin{aligned} \sigma^{2\gamma(\gamma)} = & \int dx_1 dx_2 dc_{\gamma}^{(1)} dc_{\gamma}^{(2)} \tilde{D}(x_1, c_{\gamma}^{(1)}; s) \tilde{D}(x_2, c_{\gamma}^{(2)}; s) \Theta(\text{cuts}) \\ & \times (d\sigma^{2\gamma} + d\sigma^{3\gamma} + \dots). \end{aligned} \quad (4)$$

Numerical results for such a signature indicate that the implementation of ISR via collinear SFs can lead to an overestimate of the corrected cross section of the order of several per cent [27]. Notice that  $d\sigma^{3\gamma}$  in Eq. (4) plays the same role as  $d\sigma^{2\gamma}$  in Eq. (3), and hence is a key ingredient when considering the signature with at least two photons in the final state.

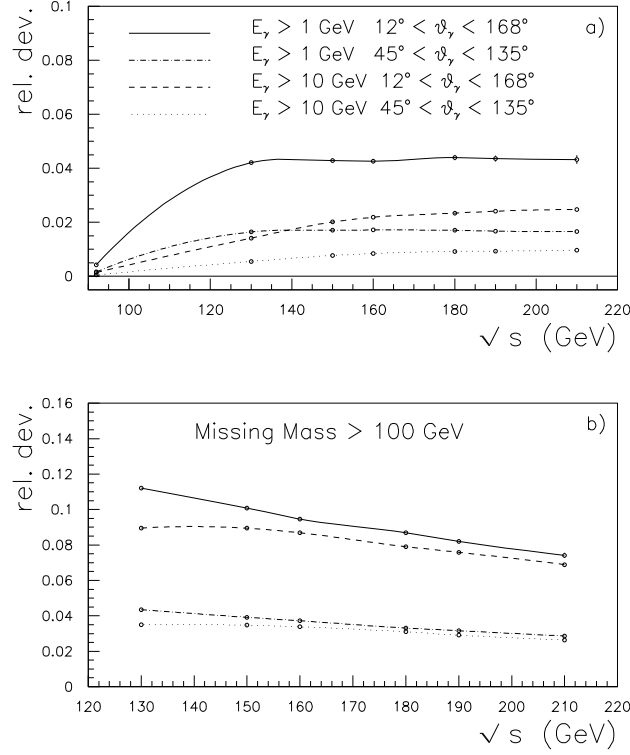


Fig. 3. Contribution of the “overlapping effects” (see the text for definition) to the cross section for the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma(\gamma)$ . In Fig. 3a the four lines correspond to the cuts on the photon energy and angle specified in the plot; (b) is the same as (a), with an additional cut on the missing mass of the event.

#### 4. Conclusions

The search for new physics in single- and multi-photon final states with large missing energy at LEP requires the best knowledge of the SM irreducible background given by the processes  $e^+e^- \rightarrow \nu\bar{\nu}n\gamma$ . To this end, the tree-level matrix elements for the SM processes with neutrino pairs and up to three photons in the final state have been calculated without any approximation. The exact treatment of the lowest-order transition amplitudes has been seen to be actually necessary in view of an expected precision at the 1% level. At this accuracy level, also a careful treatment of the (large) effect of the higher-order corrections introduced by ISR is unavoidable. Indeed, it has been shown that the usual implementation of ISR via collinear SFs,

which is a good approximation at LEP1 energies and with the usual selection criteria, at LEP2 can lead to a significant overestimate of the physical cross section and should be carefully considered in a sensible experimental analysis. The remaining uncertainty in the present study is left to the yet unknown exact  $O(\alpha)$  electroweak corrections to the process  $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ . Such a complete calculation should be actually desirable to reach a theoretical error not exceeding the 1% level.

As a result of the present study, an improved version of the event generator NUNUGPV is by now available. The program can be used for a full analysis of single- and multi-photon events with missing energy at LEP2 and beyond.

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## REFERENCES

- [1] See, for instance, G. Montagna, O. Nicrosini, F. Piccinini, Precision Physics at LEP FNT/T97/19 **hep-ph/9802302**, to appear in *La Rivista del Nuovo Cimento*.
- [2] *Physics at LEP2*, G. Altarelli, T. Sjostrand and F. Zwirner, CERN 96-01, CERN, Geneva 1996.
- [3] G. F. Giudice, M. L. Mangano, G. Ridolfi, R. Rückl *et al.*, in [2], Vol. 1, p. 463; S. Ambrosanio *et al.*, *Nucl. Phys.* **B478**, 46 (1996); *Phys. Rev.* **D56**, 1761 (1997), *Phys. Rev.* **D54**, 5395 (1996); J.L. Lopez, D.V. Nanopoulos and A. Zichichi, *Phys. Rev.* **D5**, 5813 (1997); *Phys. Rev. Lett.* **7**, 5168 (1996); A. Brignole, F. Feruglio and F. Zwirner, *Nucl. Phys.* **B516**, 13 (1998); F. Feruglio, these proceedings.
- [4] A.R. Zerwekh and R. Rosenfeld, IFT-P.027/98, **hep-ph/9805329**.
- [5] L. Trentadue *et al.*, in *Z. Phys. at LEP1*, G. Altarelli, R. Kleiss and C. Verzegnassi Eds. CERN 89-08, CERN, Geneva, 1989, Vol. 1, p. 129.
- [6] F. Boudjema, B. Mele *et al.*, in [2], Vol. 1, p. 207.
- [7] K.J.F. Gaemers, R. Gastmans and F.M. Renard, *Phys. Rev. D* **19**, 1605 (1979); G. Barbiellini, B. Richter and J.L. Siegrist, *Phys. Lett.* **B106**, 414 (1989).
- [8] O. Nicrosini, L. Trentadue, *Nucl. Phys.* **B318**, 1 (1989).
- [9] G. Montagna, O. Nicrosini, F. Piccinini, L. Trentadue, *Nucl. Phys.* **B452**, 161 (1995).
- [10] T. Sjöstrand, *Comput. Phys. Commun.* **79**, 503 (1994), CERN-TH.712/93.
- [11] S. Jadach, B.F.L. Ward, Z. Wąs, *Comput. Phys. Commun.* **79**, 503 (1994).
- [12] F.A. Berends *et al.*, *Nucl. Phys.* **B301**, 583 (1988).
- [13] R. Miquel, C. Mana, M. Martinez, *Z. Phys.* **C48**, 309 (1990).

- [14] G. Montagna, O. Nicrosini, F. Piccinini, *Comput. Phys. Commun.* **98**, 206 (1996).
- [15] Y. Kurihara, J. Fujimoto, T. Muehisa, Y. Shimizu, *Prog. Theor. Phys.* **96**, 1223 (1996); **95**, 375 (1996); J. Fujimoto, T. Muehisa, Y. Shimizu, *Prog. Theor. Phys.* **90**, 177 (1993); K. Kato, T. Muehisa, *Phys. Rev.* **D39**, 156 (1989); G. Marchesini, B.R. Webber, *Nucl. Phys.* **B238**, 1 (1984); R. Odorico, *Nucl. Phys.* **B172**, 157 (1980).
- [16] E.A. Kuraev, V.S. Fadin, *Sov. J. Nucl. Phys.* **41**, 446 (1985); G. Altarelli, G. Martinelli in *Physics at LEP*, J. Ellis, R. Peccei, Eds. CERN 86-02, CERN, Geneva 1986, Vol. 1, p. 47; O. Nicrosini, L. Trentadue *Phys. Lett.* **B196**, 551 (1987); *Z. Phys.* **C39**, 479 (1988); F.A. Berends, G. Burgers, W.L. van Neerven, *Nucl. Phys.* **B297**, 429 (1988).
- [17] S. Jadach, B.F.L. Ward, *Comput. Phys. Commun.* **56**, 351 (1990); *Phys. Rev.* **D38**, 2897 (1988); D.R. Yennie, S. Frautschi, H. Suura, *Ann. Phys. (NY)* **13**, 379 (1961).
- [18] J. Busenitz, Comparison of Single Photon Event Generators at LEP2 Energies, L3 note 2172 (1997), and private communication.
- [19] S. Mrenna, hep-ph/970541.
- [20] S. Ambrosanio *et al.*, *Phys. Rev.* **D54**, 5395 (1996).
- [21] T. Ishikawa *et al.*, KEK Report 92-19, 1993; H. Tanaka, T. Kaneko Y. Shimizu *Comput. Phys. Commun.* **64**, 149 (1991); H. Tanaka, *Comput. Phys. Commun.* **58**, 153 (1990).
- [22] E. Boos *et al.*, hep-ph/9503280; E. Boos *et al.*, *Int. J. Mod. Phys.* **C5**, 615 (1994).
- [23] P. Bain, R. Pain, HEP'97 #528, paper submitted to the HEP'97 Conference, Jerusalem, 1997.
- [24] M. Caffo, E. Remiddi, *Helv. Phys. Acta* **5**, 339 (1982); G. Passarino, *Phys. Rev.* **D28**, 2867 (1983); *Nucl. Phys.* **B237**, 249 (1984).
- [25] F. Caravaglios, M. Moretti, *Phys. Lett.* **B358**, 332 (1995).
- [26] O. Nicrosini, L. Trentadue, *Phys. Lett.* **B231**, 487 (1989).
- [27] G. Montagna, M. Moretti, O. Nicrosini, F. Piccinini, in preparation.