

# ISOLATED AND NON-ISOLATED PHOTON RATES AT LEP\*

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We present the results of the calculation of the ‘photon’ + 1 jet rate at  $\mathcal{O}(\alpha\alpha_s)$  for LEP energies. By comparing these results with the data from the ALEPH Collaboration we make a next-to-leading order determination of the quark-to-photon fragmentation function  $D_{q\rightarrow\gamma}(z, \mu_F)$  at  $\mathcal{O}(\alpha\alpha_s)$ . The predictions obtained using this fragmentation function for the *isolated* rate, defined as the ‘photon’ + 1 jet rate for  $z > 0.95$ , are found in good agreement with the ALEPH data. The next-to-leading order corrections are moderate demonstrating the perturbative stability of this particular isolated photon definition. We have also computed the inclusive photon energy distribution and found good agreement with the OPAL data.

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## 1. Introduction

Photons produced in hadronic collisions arise essentially from two different sources: the *direct* production of a photon off a primary parton or through the *fragmentation* of a hadronic jet into a single photon carrying a large fraction of the jet energy. The former gives rise to perturbatively calculable short-distance contributions whereas the latter is primarily a long distance process which cannot be calculated within perturbative QCD. It is described by the process-independent parton-to-photon fragmentation function [1] which must be determined from experimental data. Its evolution with the factorization scale  $\mu_F$  can however be determined by perturbative methods.

The ALEPH Collaboration at CERN has measured the quark-to-photon fragmentation function  $D_{q\rightarrow\gamma}$  [2] from an analysis of two jet events in which one of the jets contains a photon carrying a large fraction ( $z > 0.7$ ) of the

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jet energy. These ‘photon’ +1 jet events are defined by the application of the Durham jet clustering algorithm [3] to both the hadronic and electromagnetic clusters. In this democratic approach, the photon is called *isolated* if it carries a large fraction, typically 95%, of the jet energy and said to be non-isolated otherwise. A comparison between this measured rate and the calculated rate up to  $\mathcal{O}(\alpha)$  [4] using the same clustering approach to define the photon yielded a first determination [2] of the quark-to-photon fragmentation function accurate at this order. Furthermore, the insertion of this measured function into the  $\mathcal{O}(\alpha)$  ‘isolated’ rate, defined as the ‘photon’ +1 jet rate for  $z > 0.95$ , yielded a good agreement with the ALEPH data.

In this fixed order framework, the distribution of electromagnetic energy within the photon jet of photon + 1 jet events, for a single quark of charge  $e_q$ , at  $\mathcal{O}(\alpha)$  can be written [4],

$$\frac{1}{\sigma_0} \frac{d\sigma}{dz} = 2D_{q \rightarrow \gamma}(z, \mu_F) + \frac{\alpha e_q^2}{\pi} P_{q\gamma}^{(0)}(z) \log \left( \frac{s_{\text{cut}}}{\mu_F^2} \right) + R_\Delta \delta(1-z) + \dots, \quad (1.1)$$

where  $\dots$  represents terms which are well behaved as  $z \rightarrow 1$ . In the Durham jet algorithm and at large  $z$ ,  $s_{\text{cut}} \sim sz(1-z)^2/(1+z) \sim p_T^2$  [4] where  $p_T$  is the transverse momentum of the photon with respect to the cluster. The non-perturbative fragmentation function is an exact solution at  $\mathcal{O}(\alpha)$  of the evolution equation in the factorization scale  $\mu_F$ ,

$$D_{q \rightarrow \gamma}(z, \mu_F) = \frac{\alpha e_q^2}{2\pi} P_{q\gamma}^{(0)}(z) \log \left( \frac{\mu_F^2}{\mu_0^2} \right) + D_{q \rightarrow \gamma}(z, \mu_0). \quad (1.2)$$

In this equation, all unknown long-distance effects are related to the behaviour of  $D_{q \rightarrow \gamma}(z, \mu_0)$ , the initial value of this fragmentation function which has been fitted to the data at some initial scale  $\mu_0$  in [2]. As  $D_{q \rightarrow \gamma}(z, \mu_F)$  is exact, this solution does not take the commonly implemented [5] resummations of  $\log(\mu_F^2)$  into account and when used to evaluate the photon +1 jet rate at  $\mathcal{O}(\alpha)$  yields a factorization scale independent prediction for the cross section.

In the conventional approach, a resummation of the logarithms of the factorization scale is performed to all orders in  $\alpha_s$  [5] and the solution of the evolution equation for  $D_{q \rightarrow \gamma}$  is proportional to  $\log(\mu_F^2/\mu_0^2)$ . For  $z < 1$ , we find that  $\mu_F^2 \sim s_{\text{cut}}$  and  $\mu_F^2 \gg \mu_0^2$ . The ‘direct’ contribution in Eq. (1.1) is therefore suppressed relative to the fragmentation contribution. The conventional assignment of a power of  $1/\alpha_s$  to the fragmentation function is in this case clearly motivated, this contribution is indeed more significant. However, as  $z \rightarrow 1$ , we see that the transverse size of the photon jet cluster decreases such that  $s_{\text{cut}} \rightarrow 0$ . The hierarchy  $s_{\text{cut}} \sim \mu_F^2$  and  $\mu_F^2 \gg \mu_0^2$  is no longer preserved and both contributions in Eq. (1.1) are important. Large

logarithms of  $(1 - z)$  become the most dominant contributions. Being primarily interested in the high  $z$  region, in [4] it was chosen not to impose the conventional prejudice and resum the logarithms of  $\mu_F$  *a priori* but to work within a fixed order framework, to isolate the relevant large logarithms. A detailed comparison of the evaluation of the photon production cross section in the conventional and fixed order formalisms can be found in [6].

We have performed the calculation of the  $\mathcal{O}(\alpha_s)$  corrections to the ‘photon’ + 1 jet rate using the same democratic procedure to define the photon as in [2, 4]. The details of this fixed order calculation have been presented in [7]. We shall here limit ourselves to outline the main characteristics of this calculation, to summarize the results and to show how these compare with the available experimental data.

## 2. The calculation of the ‘photon’ + 1 jet rate at $\mathcal{O}(\alpha\alpha_s)$

The ‘photon’ + 1 jet rate in  $e^+e^-$  annihilation at  $\mathcal{O}(\alpha\alpha_s)$  receives contributions from five parton-level subprocesses displayed in Fig. 1. Although the ‘photon’ + 1 jet cross section is finite at  $\mathcal{O}(\alpha\alpha_s)$ , all these contributions contain divergences (when the photon and/or the gluon are collinear with one of the quarks, when the gluon is soft or since the bare quark-to-photon fragmentation function contains infinite counter terms). All these divergences have to be isolated and cancelled analytically before the ‘photon’ + 1 jet cross section can be evaluated numerically.

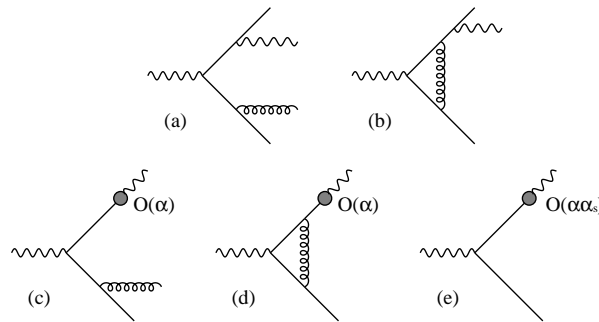


Fig. 1. Parton level subprocesses contributing to the photon + 1 jet rate at  $\mathcal{O}(\alpha\alpha_s)$ .

Within each singular region which we have defined using a theoretical criterion  $s_{\min}$ , the matrix elements are approximated and the unresolved variables analytically integrated. The evaluation of the singular contributions associated with the process  $\gamma^* \rightarrow q\bar{q}g\gamma$  is of particular interest as it contains various ingredients which could directly be applied to the calculation of jet observables at next-to-next-to-leading order. Indeed, besides the contributions arising when one final state gluon is collinear or soft, there

are also contributions where *two* of the final state partons are theoretically unresolved. The three different double unresolved contributions which occur in this calculation are: the *triple collinear* contributions, arising when the photon and the gluon are simultaneously collinear to one of the quarks, the *soft/collinear* contributions arising when the photon is collinear to one of the quarks while the gluon is soft and the *double single collinear* contributions, resulting when the photon is collinear to one of the quarks while the gluon is collinear to the other. A detailed derivation of each of these singular real contributions and of the singular contributions arising in the processes depicted in Fig. 1(b)–(d) has been presented in [7].

Combining all unresolved contributions present in the processes shown in Fig. 1(a)–(d) yields a result that still contains single and double poles in  $\epsilon$ . These pole terms are however proportional to the universal next-to-leading order splitting function  $P_{q\gamma}^{(1)}$  [8] and a convolution of two lowest order splitting functions,  $(P_{qq}^{(0)} \otimes P_{q\gamma}^{(0)})$ . Hence, they can be factorized into the next-to-leading order counterterm of the bare quark-to-photon fragmentation function [9] present in the contribution depicted in Fig. 1(e), yielding a finite and factorization scale ( $\mu_F$ ) dependent result [7].

We have then chosen to evaluate the remaining finite contributions numerically using the *hybrid subtraction* method, a generalization of the *phase space slicing* procedure [10, 11]. This latter procedure turns out to be inappropriate when more than one particle is potentially unresolved. Indeed, in our calculation we found areas in the four parton phase space which belong simultaneously to two different single collinear regions. Those areas cannot be treated correctly within the phase space slicing procedure. Within the *hybrid subtraction* method developed in [12], a parton resolution criterion  $s_{\min}$  is used to separate the phase space into different resolved and unresolved regions, but, rather than assuming that the approximated matrix elements are exact in the singular regions, the difference between the full matrix element and its approximation is evaluated numerically in all unresolved regions. The non-singular contributions are calculated using Monte Carlo methods like within the phase space slicing scheme.

The numerical program finally evaluating the ‘photon’ + 1 jet rate at  $\mathcal{O}(\alpha_s)$  contains four separate contributions. Each of them depends logarithmically (in fact as  $\log^3(y_{\min})$ ) on the theoretical resolution parameter  $y_{\min} = s_{\min}/Q^2$ . The physical ‘photon’ + 1 jet cross section, which is the sum of all four contributions, *must* of course be independent of the choice of  $y_{\min}$ . In Fig. 2, we see that the cross section approaches (within numerical errors) a constant value provided that  $y_{\min}$  is chosen small enough, indicating that a complete cancellation of all powers of  $\log(y_{\min})$  takes place. This provides a strong check on the correctness of our results and on the consistency of our approach.

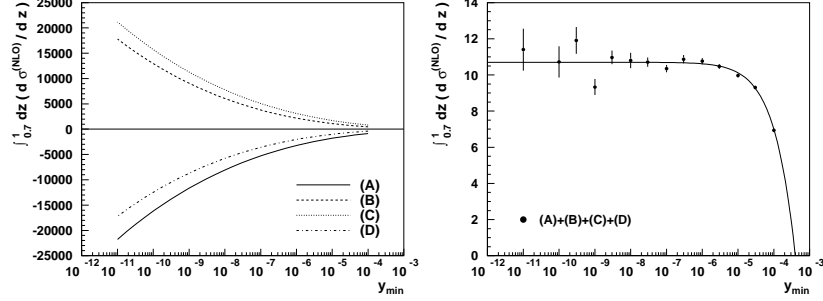


Fig. 2.  $\mathcal{O}(\alpha\alpha_s)$  individual contributions (left) and sum of all  $\mathcal{O}(\alpha\alpha_s)$  contributions (right) to the photon + 1 jet rate for a single quark of charge  $e_q$  such that  $\alpha e_q^2 = 2\pi$ ,  $\alpha_s(N^2 - 1)/2N = 2\pi$  using the Durham jet algorithm with  $y_{\text{cut}} = 0.1$ , and integrated for  $z > 0.7$ .

### 3. Results

A comparison between the measured ‘photon’ + 1 jet rate [2] and our calculation yielded a first determination of the quark-to-photon fragmentation function accurate up to  $\mathcal{O}(\alpha\alpha_s)$  [13]. This function, which parameterizes the perturbatively incalculable long-distance effects, has to satisfy a perturbative evolution equation in the factorization scale  $\mu_F$ . Indeed, the next-to-leading order fragmentation function can be expressed as an *exact* solution of the evolution equation up to  $\mathcal{O}(\alpha\alpha_s)$  [7],

$$\begin{aligned}
 D(z, \mu_F) = & \frac{\alpha e_q^2}{2\pi} P_{q\gamma}^{(0)}(z) \log\left(\frac{\mu_F^2}{\mu_0^2}\right) + \frac{\alpha e_q^2}{2\pi} \frac{\alpha_s}{2\pi} \left(\frac{N^2 - 1}{2N}\right) P_{q\gamma}^{(1)}(z) \log\left(\frac{\mu_F^2}{\mu_0^2}\right) \\
 & + \frac{\alpha_s}{2\pi} \left(\frac{N^2 - 1}{2N}\right) \log\left(\frac{\mu_F^2}{\mu_0^2}\right) P_{qq}^{(0)}(z) \otimes \frac{\alpha e_q^2}{2\pi} \frac{1}{2} P_{q\gamma}^{(0)}(z) \log\left(\frac{\mu_F^2}{\mu_0^2}\right) \\
 & + \frac{\alpha_s}{2\pi} \left(\frac{N^2 - 1}{2N}\right) \log\left(\frac{\mu_F^2}{\mu_0^2}\right) P_{qq}^{(0)}(z) \otimes D(z, \mu_0) + D(z, \mu_0).
 \end{aligned} \tag{3.1}$$

The initial function  $D(z, \mu_0)$  has been fitted to the ALEPH 1 jet data [13] for  $\frac{1}{\sigma_0} \frac{d\sigma}{dz}$ , for the jet resolution parameter  $y_{\text{cut}} = 0.06$  and  $\alpha_s(M_z^2) = 0.124$  to yield <sup>1</sup>,

$$D^{\text{NLO}}(z, \mu_0) = \frac{\alpha e_q^2}{2\pi} \left( -P_{q\gamma}^{(0)}(z) \log(1 - z)^2 + 20.8(1 - z) - 11.07 \right), \tag{3.2}$$

<sup>1</sup> Note that the logarithmic term proportional to  $P_{q\gamma}^{(0)}(z)$  is introduced to ensure that the predicted  $z$  distribution is well behaved as  $z \rightarrow 1$  [4].

where  $\mu_0 = 0.64$  GeV. The next-to-leading order ( $\overline{\text{MS}}$ ) quark-to-photon fragmentation function (for a quark of unit charge) at a factorization scale  $\mu_F = M_Z$  were shown in [13] and compared with the lowest order fragmentation function obtained in [2]. A large difference between the leading and next-to-leading order quark-to-photon fragmentation functions was observed only for  $z$  close to 1, indicating the presence of large  $\log(1 - z)$ .

Moreover, a comparison between the ALEPH data and the results of the  $\mathcal{O}(\alpha\alpha_s)$  calculation using the fitted next-to-leading order fragmentation function for different values of  $y_{\text{cut}}$  can be found in [7, 13]. The next-to-leading order corrections were found to be moderate for all values of  $y_{\text{cut}}$  demonstrating the perturbative stability of our fixed order approach. To test the generality of our results, we have considered two further applications: the ‘isolated’ photon rate and the inclusive photon distribution which we shall now briefly present.

Using the results of the calculation of the photon + 1 jet rate at  $\mathcal{O}(\alpha\alpha_s)$  and the fitted quark-to-photon fragmentation function, we have determined the *isolated* rate defined as the photon + 1 jet rate for  $z > 0.95$  in the democratic approach. The result of this calculation compared with data from ALEPH [2] and the leading order calculation [4] is shown in Fig. 3. It can clearly be seen that inclusion of the next-to-leading order corrections improves the agreement between data and theory over the whole range of  $y_{\text{cut}}$ . It is also apparent that the next-to-leading order corrections to the isolated photon + 1 jet rate obtained in this democratic clustering approach are of reasonable size indicating a good perturbative stability of this *isolated* photon definition.

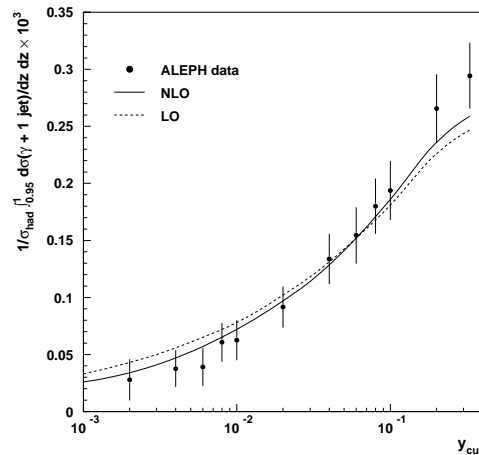


Fig. 3. The integrated photon + 1 jet rate above  $z = 0.95$  as function of  $y_{\text{cut}}$ , compared with the  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\alpha\alpha_s)$  order calculations including the appropriate quark-to-photon fragmentation functions.

The OPAL collaboration has recently measured the inclusive photon distribution for final state photons with energies between 10 and 42 GeV [14]. They have compared their results with the model estimates of [5, 15] and found reasonable agreement for  $\mu_F \sim M_Z$  in all cases. Fig. 4 shows our (scale independent) predictions for the inclusive photon energy distribution at both leading and next-to-leading order. We see good agreement with the data, even though the phase space relevant for the OPAL data far exceeds that used to determine the fragmentation functions from the ALEPH photon + 1 jet data [6].

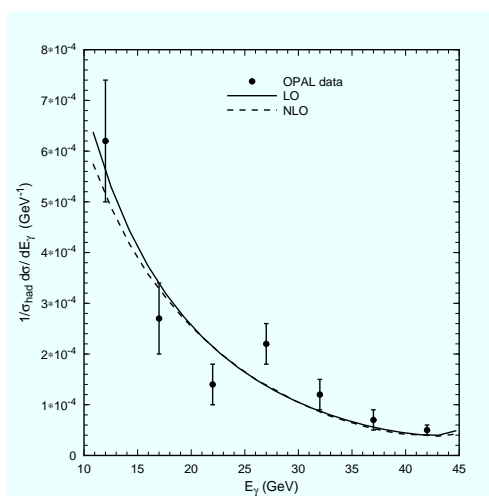


Fig. 4. The inclusive photon energy distribution normalized to the hadronic cross section as measured by the OPAL Collaboration compared with the  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\alpha\alpha_s)$  order calculations including the appropriate quark-to-photon fragmentation functions determined using the ALEPH photon + 1 jet data.

#### 4. Conclusions

In summary, we have outlined the main features of the calculation [7] of the ‘photon’ + 1 jet rate at  $\mathcal{O}(\alpha\alpha_s)$ . Although only next-to-leading order in perturbation theory, this calculation contains several ingredients appropriate to the calculation of jet observables at next-to-next-to-leading order. In particular, it requires to generalize the phase space slicing method of [10, 11] to take into account contributions where more than one theoretically unresolved particle is present in the final state. The ‘photon’ + 1 jet rate has then been evaluated for a democratic clustering algorithm with a Monte Carlo program using the hybrid subtraction method of [12]. The results of our calculation, when compared to the data [2] on the ‘photon’ + 1 jet

rate obtained by ALEPH, enabled a first determination of the process independent quark-to-photon fragmentation function at  $\mathcal{O}(\alpha\alpha_s)$  in a fixed order approach. As a first application, we have used this function to calculate the ‘isolated’ photon + 1 jet rate in a democratic clustering approach at next-to-leading order. The inclusion of the QCD corrections improves the agreement between theoretical prediction and experimental data. Moreover, it was shown that these corrections are moderate, demonstrating the perturbative stability of this particular isolated photon definition. Finally, we have computed the inclusive photon energy distribution and found good agreement with the recent OPAL data.

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