# TWO-LOOP LARGE HIGGS MASS CONTRIBUTION TO VECTOR BOSON ANOMALOUS QUARTIC COUPLINGS\*

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The calculation of the two-loop corrections to the electroweak gauge boson quartic couplings, growing quadratically with the Higgs boson mass, is reviewed. The potential of the CERN Large Hadron Collider and  $e^+e^$ linear collider to study such anomalous interactions is discussed.

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### 1. Introduction

The remarkable precision of the electroweak experimental data [1, 2] makes it possible to test the predictions of the Standard Model ( $\mathcal{SM}$ ) at the quantum loop level. After the successful prediction of the top-quark mass from the  $m_t^2$  one-loop electroweak radiative corrections and the actual observation of the top quark signal at the Tevatron, the mechanism of the spontaneous electroweak symmetry breaking, connected to the existence of the Higgs boson in the  $\mathcal{SM}$ , remains the last untested property of the  $\mathcal{SM}$ . Electroweak observables are influenced also by the presence of the Higgs boson, but contrary to the  $m_t^2$  dependence at the one-loop level they depend only logarithmically on the Higgs boson mass. From the high-precision data at LEP, SLC and the Tevatron an upper limit of  $m_H < 430$  GeV has been derived at the 95% confidence level [1,2]. This bound is not very sharp however. In a conservative conclusion the experimental limit may be interpreted in the  $\mathcal{SM}$  as an indication for a scale  $m_H \leq \mathcal{O}(1)$  TeV.

If the Higgs boson is really heavy the study of its indirect effects at the quantum loop level at energies much smaller than  $m_H$  will be one of

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the most important goals for the future experiments. Starting from the two-loop level radiative corrections exhibit power growth for  $m_H \gg M_W$ . Leading  $m_H^2$  two-loop corrections to the  $\rho$ -parameter [3, 5] and to vector boson masses [4] were calculated more than ten years ago. These two-loop large Higgs mass calculations were later extended to the case of the triple vector boson couplings [6]. Power counting shows, that only vertex functions with maximally four vector boson external legs can have two-loop large Higgs mass corrections proportional to  $m_H^2$ , while for five and higher point vertex functions no power growth of the two-loop corrections with the Higgs mass is possible. In this talk we review the results of our recent paper [7], where these calculations were completed and the analytical expressions have been obtained for the two-loop  $m_H^2$  corrections to quartic electroweak gauge boson couplings in the SM in the limit  $m_H \gg M_W$  at low energy  $E \ll m_H$ .

# 2. The calculation

In order to calculate the four vector boson vertex function contribution to the low energy effective action  $\Gamma_{\text{eff}}$  one has to take into account both oneparticle irreducible (OPI) four-vertex graphs and one particle Higgs reducible graphs with four external vector particles, as shown in Fig. 1.

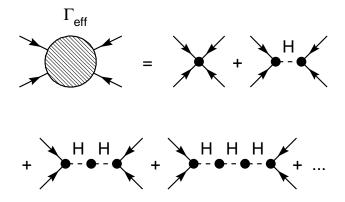


Fig. 1. One particle irreducible and Higgs reducible graphs contributing to low energy quartic vector boson vertex. Bold blobs denote the one particle irreducible four-, three- and two-point vertex functions.

The two-loop topologies and one-loop topologies with counterterm insertions contributing to OPI four-, three-, and two-point vertex functions are shown in Fig. 2. The numbers in parentheses show the total number of corresponding topologies, the external lines are assumed to be topologically different.

Two-Loop Large Higgs Mass Contribution to Vector

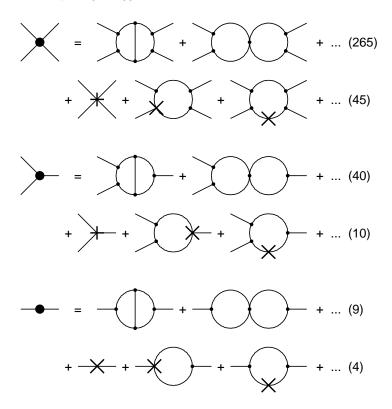


Fig. 2. One-particle irreducible two- and one-loop topologies.

Here we present only the simplest terms of the two-loop low-energy effective action  $\Gamma_{\text{eff}}$  to order  $m_H^2$ , which contribute to the ZZZZ anomalous quartic vertex, which is not present in the SM at the tree-level:

$$\frac{\delta\Gamma_{\text{eff}}}{\delta Z_{\mu_1}(k_1)\delta Z_{\mu_2}(k_2)\delta Z_{\mu_3}(k_3)\delta Z_{\mu_4}(k_4)} = \frac{e^6}{(16\pi^2)^2 s_W^6 c_W^4} \frac{m_H^2}{M_W^2} \left(g^{\mu_1\mu_2}g^{\mu_3\mu_4} + g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right) \times \left( +\frac{337}{64} - \frac{39}{16}\pi Cl + \frac{105}{64}\pi\sqrt{3} - \frac{557}{576}\pi^2 - 2Cl\sqrt{3} + \frac{63}{16}\zeta(3) \right). \quad (1)$$

As is well known [3–6] the two-loop  $m_H^2$  corrections to fermion scattering processes and triple vector boson couplings are very small, in spite of the  $m_H^2/M_W^2$  enhancements, not only because of the small two-loop factor  $g^4/(16\pi^2)^2$ , but also because the dimensionless coefficients themselves are of the order of  $10^{-1}-10^{-2}$ , *i.e.* quite small. In this respect the  $W^+W^-W^+W^-$ ,  $W^+W^-ZZ$  and ZZZZ quartic couplings represent a drastic contrast to the

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other vertices. The dimensionless coefficients in (1) are about 2, *i.e.* about 20 times larger, than the largest dimensionless coefficients for fermionic and triple vector boson couplings! As was mentioned previously, these particular vertices are distinguished, due to a contribution from two-loop Higgs self energy insertion in the Higgs-reducible graphs. These vertices receive a contribution from the  $\zeta(3)$  and  $\pi Cl$  terms, which originate only from the two-loop Higgs mass counterterm [7] as a term proportional to a linear combination  $21\zeta(3) - 13\pi Cl$ . In a sense these couplings could be considered "genuine" quartic couplings, which are the most sensitive to the details of the mechanism of the electroweak symmetry breaking.

## 3. Numerical results

The possibilities to probe the quartic vector boson couplings through the WW-, ZZ-fusion reactions

$$pp \to VVX$$
, (2)

$$ee \to VVff$$
, (3)

at the CERN Large Hadron Collider (LHC) or the electron-positron linear collider are under intense study. Here  $V = \gamma$ , Z or  $W^{\pm}$  and f = e or  $\nu_e$ .

In order to demonstrate the potential importance of large Higgs mass corrections at high energies, we present in Fig. 3 the energy dependence of the Born and corrected cross section of vector boson scattering integrated over scattering angles in the region  $30^{\circ} < \theta < 150^{\circ}$  for  $\sqrt{s_{VV}}$  up to 1 TeV for the very heavy Higgs boson mass of 1.5 TeV. The existence of a physical Higgs particle with such large mass seems to be excluded due to triviality bounds (see [2] and references therein). We can consider however such a value of  $m_H$  as an effective ultraviolet cut-off in the theory without visible scalar Higgs particle. We see that the growth with energy of the longitudinal vector boson scattering cross sections, which is the experimental indication of the existence of heavy Higgs sector and/or strong interactions among longitudinal  $W_L$ ,  $Z_L$  bosons, is strongly modified by the two-loop  $m_H^2$  corrections. At high energy the cross sections of neutral channel reactions are diminished, and those of charged channel reactions are enhanced. The large value of the two-loop correction is not only due to  $m_H^2/M_W^2$  enhancement factor, but also due to violations of unitarity cancellations in the presence of anomalous quartic couplings. E.g. at 500 GeV the correction to longitudinal WW scattering for  $m_H = 1.5$  TeV is

$$\delta \sim \frac{e^4}{(16\pi^2)^2 s_W^4} \frac{m_H^2}{M_W^2} \times \frac{s}{M_W^2} \sim 0.25\% \times 40 \sim 10\%, \tag{4}$$

which gives the correct order of magnitude for corrections in Fig. 3.

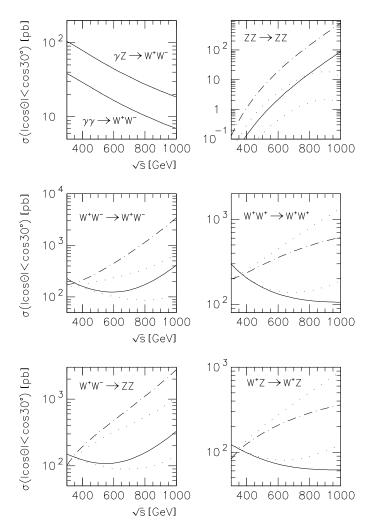


Fig. 3. Energy dependence of the Born cross sections for unpolarized UUUU (solid lines) and longitudinal LLLL (dash-dotted lines) vector boson scattering reactions. Dotted lines show corresponding corrected cross sections. Higgs mass is taken to be 1.5 TeV.

Of course at center-of-mass energy of 1 TeV  $s_{VV}$  is not very much smaller than  $m_H^2$ , which is the condition under which our low-energy effective action was calculated. Nevertheless, we think that the qualitative trend, namely the fact that the account of large Higgs mass corrections at high energy can change the value of the cross section by a large factor of 2 - 4, is important for all considerations of the signal from strong scattering of longitudinal vector boson at TeV energy. G. Jikia

In fact, using the results of a thorough phenomenological analysis of the effects of anomalous quartic couplings in pp and  $e^{\pm}e^{-}$  collisions [9,10] we can estimate the potential of TeV colliders in investigating the effects of enhanced  $m_{H}^{2}$  two-loop corrections more quantitatively. Anomalous quartic couplings are defined in Refs. [9,10] through the following effective electroweak chiral Lagrangians:

$$\mathcal{L}_{4} = g^{4} \alpha_{4} \left[ \frac{1}{2} [(W^{+}W^{-})^{2} + (W^{+2})(W^{-2})] + \frac{1}{c_{W}^{2}} (W^{+}Z)(W^{-}Z) + \frac{1}{4c_{W}^{4}} Z^{4} \right],$$
  
$$\mathcal{L}_{5} = g^{4} \alpha_{5} \left[ (W^{+}W^{-})^{2} + \frac{1}{c_{W}^{2}} (W^{+}W^{-})Z^{2} + \frac{1}{4c_{W}^{4}} Z^{4} \right],$$
 (5)

where  $g = e/s_W$ . These operators introduce all possible quartic couplings among the weak gauge bosons, that are compatible with custodial  $SU(2)_c$ symmetry [8]. Although our complete effective action given in [7] does not obey this symmetry and as a consequence can not be described by the combination of operators (5), the dominating terms which originate from two-loop Higgs self energy insertions in the Higgs reducible graphs have exactly the structure of Lagrangian (5). Using our expression (1) and analogous expressions for WWWW and WWZZ vertices we can calculate the coupling constant  $\alpha_5$ :

$$\alpha_5 \approx -\frac{g^2}{(16\pi^2)^2} \frac{m_H^2}{M_W^2}.$$
 (6)

In our approach the constant  $\alpha_4$  should be about an order of magnitude smaller.

The potential of the LHC and TeV  $e^{\pm}e^{-}$  linear collider to study anomalous quartic vector boson interactions was carefully analyzed in recent papers [9, 10]. The limit on the anomalous quartic coupling  $\alpha_5$  which will be accessible at LHC is [10]:

$$-7.2 \le \alpha_5 \le 13. \tag{7}$$

An integrated luminosity of  $100 f b^{-1}$  was assumed. The invariant mass of the vector boson pair was required to be in the range  $0.5 < M_{VV} < 1.25$  TeV. As one could expect from Fig. 3, the lower bound on  $\alpha_5$  is determined by the limits from same sign  $W^{\pm}W^{\pm}$ -pair production, because for negative  $\alpha_5$  (6) the correction in this channel is positive.

Study of the reaction (3) at the linear collider running at 1.6 TeV energy will allow one to improve the LHC limits by a factor of five [9]. Indeed, the 90% bound on  $\alpha_5$ , obtained by combining the  $e^+e^- \rightarrow \nu_e \bar{\nu}_e W^+W^-$  and  $e^+e^- \rightarrow \nu_e \bar{\nu}_e ZZ$  channels is

$$|\alpha_5| \le 1.5 \times 10^{-3} \tag{8}$$

for integrated luminosity of 500 fb<sup>-1</sup>. These limits were obtained under the assumption that only the anomalous parameter  $\alpha_5$  is non-vanishing.

For a Higgs mass of 1.5 TeV the value of  $\alpha_5$  from Eq. (6) is approximately  $-6 \times 10^{-3}$  (and  $-2 \times 10^{-3}$  for  $m_H = 900$  GeV), which is four times larger than the experimental limit achievable at the linear collider. This comparison is a very good indication that if a heavy Higgs scenario of the electroweak symmetry breaking is realized in nature its indirect quantum effects could probably be measured.

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