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DIAGRAMMATIC CALCULATIONS OF THE MSSM NEUTRAL HIGGS MASSES UP TO 2-LOOP*

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Diagrammatic two-loop results are presented for the leading QCD corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the Minimal Supersymmetric Standard Model (MSSM). The results are valid for arbitrary values of the parameters of the Higgs and scalar top sector of the MSSM. Their impact on a precise prediction for the mass of the lightest Higgs boson is briefly discussed.

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1. Introduction

Tests of supersymmetric extensions of the Standard Model of electroweak and strong interactions, in particular tests of the MSSM, are a central theme for present and future particle physics. While for the predicted superpartners of the SM particles present-day accelerators can cover only a very limited part of the MSSM parameter space, the Higgs sector of the model provides the opportunity for a stringent direct test based on the prediction of a light neutral Higgs boson. At tree level its mass m_h is constrained from above by the Z boson mass. Large one-loop corrections [1, 2], however, shift the upper bound to about 150 GeV. Beyond one-loop order renormalization group methods have been applied in order to include higher-order leading and next-to-leading logarithmic contributions [3–5]. A diagrammatic calculation is available for the dominant two-loop contributions in the limiting case of vanishing $\tilde{t}$-mixing and infinitely large M_A and $\tan\beta$ [6]. These results indicate that the two-loop corrections considerably reduce the predicted value of m_h. A precise prediction for m_h in terms of the relevant

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SUSY parameters is crucial in order to determine the discovery and exclusion potential of LEP2 and the upgraded Tevatron, and also for physics at future colliders where high-precision measurements of m_h are feasible.

Diagrammatic calculations at the two-loop level which take into account virtual particle effects without restrictions of their masses and mixing are therefore very desirable. We have performed a Feynman diagrammatic calculation of the leading two-loop QCD corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM [7]. The results are valid for arbitrary values of the parameters of the Higgs and scalar top sector of the MSSM.

2. Calculational frame

The Higgs sector of the MSSM contains two doublets $H_i = \begin{pmatrix} H_i^2 \\ H_i^1 \end{pmatrix}$, with the components

$$H_1 = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}. \quad (1)$$

The tree level Higgs potential can be written as follows:

$$\begin{aligned} V &= m_1^2 H_1^2 + m_2^2 H_2^2 + \varepsilon_{ij}(m_{12}^2 H_1^i H_2^j + \text{h.c.}) \\ &\quad + \frac{g^2 + g'^2}{8}(H_1^2 - H_2^2)^2 + \frac{g^2}{4}(H_1 H_2)^2 . \end{aligned} \quad (2)$$

Diagonalization of the mass matrices for the $\mathcal{CP}$-even and the $\mathcal{CP}$-odd scalars, following from the potential (2), leads to three physical particles: two $\mathcal{CP}$-even Higgs bosons H^0, h^0 and one $\mathcal{CP}$-odd Higgs boson A^0. The tree-level masses of h^0, H^0 follow from the coefficients $m_{\phi_1}^2$, $m_{\phi_2}^2, m_{\phi_1\phi_2}^2$ of the quadratic terms of (2) in the $\phi_{1,2}$ basis. They are determined by the values of two input parameters, conventionally chosen as $\tan\beta = v_2/v_1$ and $M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$, where M_A is the mass of the $\mathcal{CP}$-odd A boson, and by the Z boson mass M_Z.

The tree-level mass predictions are affected by large corrections at one-loop order through terms proportional to $G_F m_t^4$ [1]. These dominant one-loop contributions can be obtained by evaluating the contribution of the t-$\tilde{t}$-sector to the $\phi_{1,2}$ self-energies at zero external momentum from the Yukawa part of the theory (neglecting the gauge couplings). Accordingly, the one-loop corrected Higgs masses are derived by diagonalizing the mass matrix, given in the ϕ_1-ϕ_2 basis as

$$M_{\text{Higgs}}^2 = \begin{pmatrix} m_{\phi_1}^2 - \hat{\Sigma}_{\phi_1}(0) & m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) \\ m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) & m_{\phi_2}^2 - \hat{\Sigma}_{\phi_2}(0) \end{pmatrix}. \quad (3)$$

Therein, the $\hat{\Sigma}$ denote the Yukawa contributions of the t-$\tilde{t}$-sector to the renormalized one-loop $\phi_{1,2}$ self-energies, *i.e.* including the counterterms fixed by A^0 mass renormalization and by tadpole renormalization for vanishing renormalized tadpoles. In this approximation one obtains the compact expressions (third reference of [1])

$$\begin{aligned} M^2_{H,h} &= \frac{M_A^2 + M_Z^2 + \varepsilon_t + \sigma_t}{2} \\ &\pm \left[\frac{(M_A^2 + M_Z^2)^2 + (\varepsilon_t - \sigma_t)^2}{4} - M_A^2 M_Z^2 \cos^2 2\beta \right. \\ &+ \left. \frac{(\varepsilon_t - \sigma_t)\cos 2\beta}{2}(M_A^2 - M_Z^2) - \lambda_t \sin 2\beta (M_A^2 + M_Z^2) + \lambda_t^2 \right]^{1/2} \end{aligned} \quad (4)$$

with

$$\begin{aligned} \varepsilon_t &= \frac{N_C G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \left[\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{A_t(A_t - \mu\cot\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right. \\ &\left. + \frac{A_t^2(A_t - \mu\cot\beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} \left(1 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}} \right) \right] \\ \lambda_t &= \frac{N_C G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \left[\frac{\mu(A_t - \mu\cot\beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right. \\ &\left. + \frac{2\mu A_t(A_t - \mu\cot\beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} \left(1 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}} \right) \right] \\ \sigma_t &= \frac{N_C G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \frac{\mu^2(A_t - \mu\cot\beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} \left[1 - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}} \right] . \end{aligned} \quad (5)$$

These formulae contain the masses $m_{\tilde{t}_{1,2}}$ of the top squarks, the Higgs mass parameter μ of the superpotential, and the non-diagonal entry A_t in the stop mass matrix. By comparison with the full one-loop result [2] it has been shown that these contributions indeed contain the bulk of the one-loop corrections. Typical differences to the full one-loop result are of the order of 5 GeV.

In order to derive the leading two-loop contributions to the masses of the neutral $\mathcal{CP}$-even Higgs bosons we have evaluated the QCD corrections to Eq. (3), which because of the large value of the strong coupling constant are expected to be the most sizable ones (see also Ref. [6]). This requires the evaluation of the renormalized $\phi_{1,2}$ self-energies at the two-loop level.

The renormalization is performed in the on-shell scheme. The counterterms in the Higgs sector are derived from the Higgs potential (2), analogously to the 1-loop calculation. The renormalization conditions for the tadpole counterterms are chosen in such a way that they cancel the tadpole contributions in one- and two-loop order. The renormalization in the t-$\tilde{t}$-sector is performed in the same way as in Ref. [8]. For the present calculation the one-loop counterterms δm_t, $\delta m_{\tilde{t}_1}$, $\delta m_{\tilde{t}_2}$ for the top-quark and scalar top-quark masses and $\delta\theta_{\tilde{t}}$ for the mixing angle contribute, which enter via the subloop renormalization. The appearance of the $\tilde{t}$ mixing angle $\theta_{\tilde{t}}$ reflects the fact that the current eigenstates, $\tilde{t}_{\rm L}$ and $\tilde{t}_{\rm R}$, mix to give the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$. Since the non-diagonal entry in the scalar quark mass matrix is proportional to the quark mass the mixing is particularly important in the case of the third generation scalar quarks. The mixing angle counterterm $\delta\theta_{\tilde{t}}$ is chosen in such a way that there is no transition between $\tilde{t}_1$ and $\tilde{t}_2$ when $\tilde{t}_1$ is on-shell. The numerical result, however, is insensitive to the choice of the renormalization point. Counterterms for μ and $\tan\beta$ do not appear in $\mathcal{O}(\alpha_s)$.

The calculations, strongly supported by computer-algebra tools [9], are performed in the dimensional reduction scheme [10], which preserves the relevant SUSY relations. They result in analytical expressions for the two-loop $\phi_{1,2}$ self-energies in terms of the SUSY parameters $\tan\beta$, M_A, μ, $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$, and the gluino mass $m_{\tilde{g}}$. Inserting the one-loop and two-loop $\phi_{1,2}$ self-energies into Eq. (3), the predictions for the masses of the neutral h^0, H^0 bosons follow from the diagonalization of the two-loop mass matrix.

3. Results and discussion

For the numerical illustration we choose two typical values for $\tan\beta$ which are favored by SUSY-GUT scenarios [11]: $\tan\beta\ = 1.6$ for the SU(5) and $\tan\beta\ = 40$ for the SO(10) scenario. The scalar top masses and the mixing angle follow from the parameters $M_{\tilde{t}_{\rm L}}$, $M_{\tilde{t}_{\rm R}}$ and $M_t^{\rm LR}$ of the $\tilde{t}$ mass matrix, where $M_t^{\rm LR} = A_t - \mu\cot\beta$ (same conventions as in Ref. [8]). In the figures below, $m_{\tilde{q}} \equiv M_{\tilde{t}_{\rm L}} = M_{\tilde{t}_{\rm R}}$ is assumed.

Fig. 1 shows m_h as a function of $M_t^{\rm LR}/m_{\tilde{q}}$, where $m_{\tilde{q}}$ is fixed to 500 GeV. A minimum is reached for $M_t^{\rm LR} = 0$ GeV which we refer to as 'no mixing'. A maximum in the two-loop result for m_h is reached for about $M_t^{\rm LR}/m_{\tilde{q}} \approx 2$ in both the low and high $\tan\beta$ scenario. This case we refer to as 'maximal mixing'. Note that the maximum position is shifted compared to its one-loop value of about $M_t^{\rm LR}/m_{\tilde{q}} \approx 2.4$.

In Fig. 2 the two scenarios with $\tan\beta\ = 1.6$ and $\tan\beta\ = 40$ are displayed. The tree-level, the one-loop and the two-loop results for m_h are

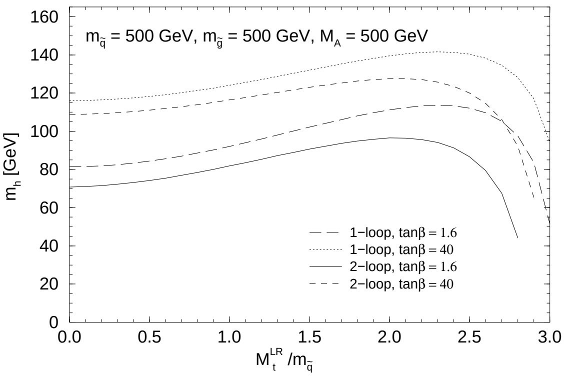

Fig. 1. One- and two-loop results for m_h as a function of $M_t^{\mathrm{LR}}/m_{\tilde{q}}$ for two values of $\tan\beta$.

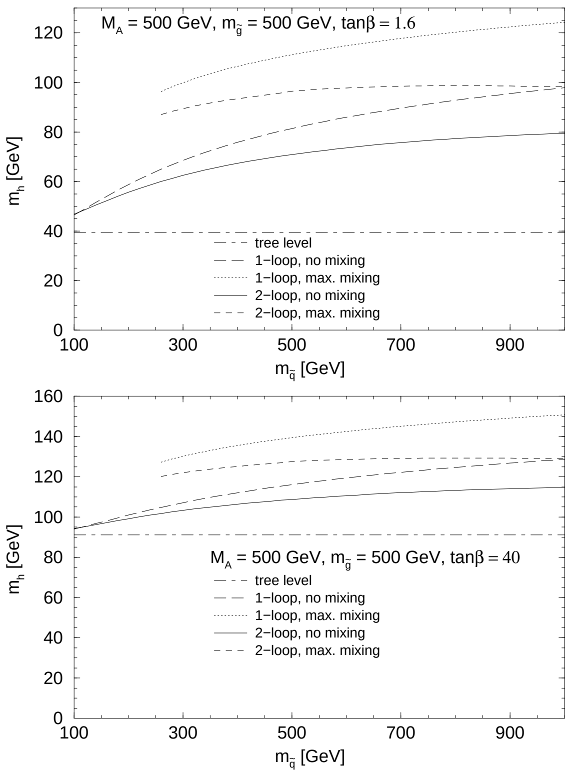

Fig. 2. The mass of the lightest Higgs boson for the two scenarios with $\tan\beta\ = 1.6$ and $\tan\beta\ = 40$. The tree-, the one- and the two-loop results for m_h are shown as a function of $m_{\tilde{q}}$ for the no-mixing and the maximal-mixing case.

shown as a function of $m_{\tilde{q}}$ for no mixing and maximal mixing (the curves for the maximal-mixing case start at higher values of $m_{\tilde{q}}$ than those for the no-mixing case, since below these values of $m_{\tilde{q}}$ the resulting $\tilde{t}$-masses are unphysical or experimentally excluded). The plots show that the one-loop on-shell result for m_h is in general considerably reduced by the inclusion of the two-loop corrections. For the low-$\tan\beta$ scenario the difference between the one-loop and two-loop result amounts to up to about 18 GeV for $m_{\tilde{q}} = 1$ TeV in the no-mixing case, and up to about 25 GeV for $m_{\tilde{q}} = 1$ TeV in the maximal-mixing case. For the high-$\tan\beta$ scenario the reduction of the one-loop result is slightly smaller than for $\tan\beta = 1.6$. The variation with $m_{\tilde{q}}$ is of the order of few GeV.

Supplementing our results for the leading $\mathcal{O}(\alpha\alpha_s)$ corrections with the leading higher-order Yukawa term of $\mathcal{O}(\alpha^2 m_t^6)$ given in Ref. [4] leads to an increase in the prediction of m_h, up to about 3 GeV. A similar shift towards higher values of m_h emerges if at the two-loop level the running top-quark mass $\overline{m_t}(m_t) = 166.5$ GeV is used instead of the pole mass $m_t = 175$ GeV, thus taking into account leading higher-order effects beyond the two-loop level. We have compared our results with the results of a renormalization group improvement of the leading one-loop contributions given in Ref. [5]. Good agreement is found for the case of vanishing $\tilde{t}$-mixing, while for larger $\tilde{t}$-mixing sizable deviations exceeding 5 GeV occur. In particular, the value of $M_t^{\mathrm{LR}}/m_{\tilde{q}}$ for which m_h becomes maximal is shifted from $M_t^{\mathrm{LR}}/m_{\tilde{q}} \approx 2.4$ in the one-loop case to $M_t^{\mathrm{LR}}/m_{\tilde{q}} \approx 2$ when our diagrammatic two-loop results are included (see Fig. 1). In the results based on renormalization group methods [3, 5], on the other hand, the maximal value of m_h is obtained for $M_t^{\mathrm{LR}}/m_{\tilde{q}} \approx 2.4$, *i.e.* at the one-loop value.

In summary, we have diagrammatically calculated the leading $\mathcal{O}(\alpha\alpha_s)$ corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM, in an on-shell renormalization scheme. No restrictions on the parameters of the Higgs and scalar top sector of the model are imposed. The two-loop correction leads to a considerable reduction of the prediction for the mass of the lightest Higgs boson compared to the one-loop value. The reduction turns out to be particularly important for low values of $\tan\beta$. Compared to the results obtained via renormalization group methods sizable deviations are observed for large mixing in the $\tilde{t}$-sector.

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