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LOW-ENERGY GRAVITINO INTERACTIONS*

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Some recent results on the effective interactions of a light gravitino with ordinary particles are reviewed. In particular, the low-energy behaviour of electron-positron and photon-photon annihilation into gravitinos are carefully discussed, and a new "low-energy theorem" is established in the electron-positron case. These results are applied to derive model independent bounds on the supersymmetry breaking scale and to organize the search for a superlight gravitino at high-energy colliders.

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One of the main motivations for supersymmetry as a relevant symmetry at low-energies is the solution of the gauge hierarchy problem. As a consequence, supersymmetric extensions of the Standard Model (SM) may possess a domain of validity that extends up to energies much larger than the electroweak scale, possibly near the Planck scale M_P. The electroweak scale is kept stable and close to Δm, the mass splitting between ordinary particles and superpartners, which fixes $\Delta m \sim 1$ TeV. This requirement, however, leaves largely undetermined the supersymmetry-breaking scale $\sqrt{F}$, or, equivalently, the gravitino mass $m_{3/2} = F/(\sqrt{3}M_P)$. The ratio $\Delta m^2/F$, proportional to the goldstino coupling to matter, parametrizes the strength of the interactions transmitting supersymmetry breaking to the observable sector and may span a wide range.

In general, the low-energy theory provides a sensible description of the underlying fundamental theory up to energies close to $F/\Delta m$, where perturbative unitarity breaks down and new quanta/interactions are required. Thus to be able to stretch the low-energy description up to the highest available scale, M_P, while taking full advantage of the stable gauge hierarchy,

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we should demand that supersymmetry breaking takes place at the scale $\sqrt{F} \sim 10^{10}$ GeV. In this case, $m_{3/2} \sim \Delta m$ and the goldstino coupling is of gravitational strength.

On the other hand, the extrapolation of the known properties of electroweak and strong interactions over 16 orders of magnitude might represent a too naive scenario. The absence of a theory of supersymmetry breaking makes it impossible to formulate a definite prediction for $\sqrt{F}$. The flavour problem, even more serious in the supersymmetric case, might require the presence of new physical thresholds that cannot be very far from the electroweak scale. A limiting case is represented by $\sqrt{F}$ close to the electroweak scale, when the low-energy theory has an energy cut-off not much larger than the electroweak scale itself. In such a theory the gravitino becomes superlight, with a mass of about 10^{-5} eV and the goldstino coupling to matter is of $O(1)$.

There is no doubt that if the energy range where the low-energy theory correctly applies is so limited, we lose a part of the initial advantage of supersymmetry: that of naturally extending the validity of the effective theory beyond the TeV scale. Nevertheless here we will take a strictly phenomenological point of view and, waiting for new theoretical or experimental informations on $\sqrt{F}$, for the time being we will treat the supersymmetry breaking scale as a free parameter. In particular, in this note we will review some properties of a superlight gravitino and we will investigate to what extent this possibility is excluded or favoured by the existing data.

If the gravitino is superlight, then one expects a phenomenology which is substantially different from the one characterizing the Minimal Supersymmetric Standard Model (MSSM). In this case, only the $\pm 3/2$ gravitino helicity states can be safely omitted from the low-energy effective theory, when gravitational interactions are neglected. The $\pm 1/2$ helicity states, essentially described by the goldstino field, should instead be accounted for at low energy, because of their non-negligible coupling to matter. The lightest supersymmetric particle is the gravitino and peculiar experimental signatures can arise from the decay of the next-to-lightest supersymmetric particle into its ordinary partner plus a gravitino [1].

Moreover, even when all supersymmetric particles of the MSSM are above the production threshold, interesting signals could come from those processes where only ordinary particles and gravitinos occur. As soon as the typical energy of the process is larger than $m_{3/2}$, a condition always fulfilled in the applications discussed below, one can approximate the physical amplitudes by replacing external gravitinos with goldstinos, as specified by the equivalence theorem [2]. If the masses of the ordinary particles involved are negligible with respect to the energy of interest, these processes are controlled by just one dimensionful parameter, the supersymmetry-breaking scale $\sqrt{F}$, entering the amplitudes in the combination $(\widetilde{G}/\sqrt{2}F)$, $\widetilde{G}$ denoting the goldstino wave function.

This class of processes includes $\gamma\gamma \to \widetilde{G}\widetilde{G}$, $e^+e^- \to \widetilde{G}\widetilde{G}$, which may influence primordial nucleosynthesis, stellar cooling and supernovae explosion [3–5]. Of direct interest for LEP2 and for the future linear colliders is the reaction $e^+e^- \to \widetilde{G}\widetilde{G}\gamma$. Partonic reactions such as $q\bar{q} \to \widetilde{G}\widetilde{G}\gamma$, $q\bar{q} \to \widetilde{G}\widetilde{G}g$ and $qg \to \widetilde{G}\widetilde{G}q$ can be indirectly probed at the Tevatron collider or in future hadron facilities. In the absence of experimental signals, one can use these processes to set absolute limits on the gravitino mass. At variance with other bounds on $m_{3/2}$ discussed in the literature [6], these limits have the advantage of not depending on detailed assumptions about the spectrum of supersymmetric particles. Finally, the study of these processes can reveal unexpected features of the low-energy theory, which were overlooked in the standard approach to goldstino low-energy interactions.

The natural tools to analyse the above processes are the so-called low-energy theorems [7]. According to these, the low-energy amplitude for the scattering of a goldstino on a given target is controlled by the energy–momentum tensor $T_{\mu\nu}$ of the target. To evaluate the physical amplitudes, it is more practical to make use of an effective Lagrangian, containing the goldstino field and the matter fields involved in the reactions, and providing a non-linear realization of the supersymmetry algebra [8]. For instance, in the non-linear construction of [9], the goldstino field $\widetilde{G}$ and the generic matter field φ are incorporated into the following superfields:

$$\varLambda_\alpha \equiv \exp(\theta Q + \overline{\theta Q})\,\widetilde{G}_\alpha = \frac{\widetilde{G}_\alpha}{\sqrt{2}F} + \theta_\alpha + \frac{i}{\sqrt{2}F}(\widetilde{G}\sigma^\mu\overline{\theta} - \theta\sigma^\mu\overline{\widetilde{G}})\partial_\mu\frac{\widetilde{G}_\alpha}{\sqrt{2}F} + \dots\,, \tag{1}$$

$$\varPhi \equiv \exp(\theta Q + \overline{\theta Q})\,\varphi = \varphi + \frac{i}{\sqrt{2}F}(\widetilde{G}\sigma^\mu\overline{\theta} - \theta\sigma^\mu\overline{\widetilde{G}})\partial_\mu\varphi + \dots\,. \tag{2}$$

The goldstino–matter system is described by the supersymmetric Lagrangian:

$$\int d^2\theta d^2\bar{\theta}\varLambda^2\bar{\varLambda}^2\left[2F^2 + \mathcal{L}(\varPhi, \partial\varPhi)\right]\,, \tag{3}$$

where $\mathcal{L}(\varphi, \partial\varphi)$ is the ordinary Lagrangian for the matter system. This non-linear realization automatically reproduces the results of the low-energy theorems, in particular the expected goldstino coupling to the energy–momentum tensor $T_{\mu\nu}$ associated to φ.

An alternative approach consists in constructing a low-energy Lagrangian, starting from a general supersymmetric theory defined, up to terms with more than two derivatives, in terms of a Kähler potential, a superpotential and a set of gauge kinetic functions. The effective theory can be obtained by integrating out, in the low-energy limit, the heavy superpartners [3].

• $\gamma\gamma \to \widetilde{G}\widetilde{G}$

When applied to the process $\gamma\gamma \to \widetilde{G}\widetilde{G}$, the two procedures yield the same result. The only independent, non-vanishing, helicity amplitude for the process is:

$$a(1,-1,\tfrac{1}{2},-\tfrac{1}{2}) = 8\sin\theta\cos^2\frac{\theta}{2}\,\frac{E^4}{F^2}\,, \tag{4}$$

where $(1,\,-1)$ and $(1/2,\,-1/2)$ are the helicities of the incoming and outgoing particles, respectively; E and θ are the goldstino energy and scattering angle in the centre-of-mass frame. The total cross section is

$$\sigma_{\gamma\gamma} = s^3/(640\pi F^4)\quad . \tag{5}$$

• $e^+e^- \to \widetilde{G}\widetilde{G}$

When considering $e^+e^- \to \widetilde{G}\widetilde{G}$, in the limit of a massless electron, one has to face an unexpected result [10]. On the one hand, by integrating out the heavy selectron fields, one finds the following helicity amplitude:

$$a(\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}) = \frac{4(1+\cos\theta)^2E^4}{F^2}\,, \tag{6}$$

all other non-vanishing amplitudes being related to this one. On the other hand, by using the non-linear realization of [9], one obtains:

$$a(\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2}) = \frac{4\sin^2\theta E^4}{F^2}\,. \tag{7}$$

The amplitudes of Eqs. (6) and (7) scale in the same way with the energy, but have a different angular dependence. We should conclude that the low-energy theorems do not apply to the case of a massless fermion. A particularly disturbing aspect is that the non-linear realization of Eq. (3) is supposed to provide the most general parametrization of the amplitude in question. In the case at hand, the Lagrangian of Eq. (3) reads:

$$\mathcal{L}_e = \int d^2\theta d^2\bar\theta \Lambda^2\bar\Lambda^2\left[2F^2 + iE\sigma^\mu\partial_\mu\bar{E} + iE^c\sigma^\mu\partial_\mu\bar{E}^c\right]\,, \tag{8}$$

where E and E^c are the superfields associated to the two Weyl spinors e and e^c describing the electron, according to Eq. (2). The solution to this puzzle [10] is provided by the existence of an independent supersymmetric invariant (unique for a single Majorana matter fermion):

$$\delta\mathcal{L}_e = \int d^2\theta d^2\bar\theta(\Lambda E\bar\Lambda\bar{E} + \Lambda E^c\bar\Lambda\bar{E}^c)\,. \tag{9}$$

The amplitudes of Eq. (6) are reproduced, up to an overall sign, by the combination $\mathcal{L}_e - 8\ \delta\mathcal{L}_e$.

On the other hand, there is no reason to prefer either the result of Eq. (6) or that of Eq. (7). The process $e^+e^- \to \widetilde{G}\widetilde{G}$ does not have a universal low-energy behaviour. In the framework of non-linear realizations, this freedom can be described by the invariant Lagrangian $\mathcal{L}_e + 2a\ \delta\mathcal{L}_e$, where a is a free parameter of the low-energy theory. This gives rise to the total cross section:

$$\sigma_{e^+e^-} = \frac{s^3}{7680\pi F^4}(8 + 10a + 5a^2)\,. \tag{10}$$

This cross-section has a minimum at $a = -1$.

• Astrophysical bounds

The success of primordial nucleosynthesis limits the effective number of degrees of freedom that remained coupled to the thermal bath of the early universe down to temperatures of O(MeV). This limit requires to conveniently deplete the annihilation processes $\widetilde{G}\widetilde{G} \to \gamma\gamma$ and $\widetilde{G}\widetilde{G} \to e^+e^-$, that control the goldstino decoupling temperature. These considerations lead [11] to a typical lower bound on $m_{3/2}$ of $O(10^{-7})$ eV. The opposite reactions $\gamma\gamma \to \widetilde{G}\widetilde{G}$ and $e^+e^- \to \widetilde{G}\widetilde{G}$ might influence the stellar evolution. The most stringent bound comes from the requirement that the energy loss of the supernova SN1987A remains within the range predicted by the theory and verified experimentally through the detection of the emitted neutrinos. One obtains an excluded window for the gravitino mass: 10^{-9} eV $< m_{3/2} < 10^{-7}$ eV. These bounds are quite interesting, but they are not competitive with those coming from the high-energy colliders. They can become more significant only if the spin 0 partners of the goldstino are so light that the cross sections in Eqs. (5) and (10) gets modified [5, 11].

• $e^+e^- \to \widetilde{G}\widetilde{G}\gamma$

Also the low-energy limit of the process $e^+e^- \to \widetilde{G}\widetilde{G}\gamma$ is not universal. For nearly collinear photons, the differential cross section is dominated by the initial state radiation:

$$\frac{d\sigma}{dx_\gamma \cos\theta_\gamma} = \sigma_{e^+e^-}[(1-x_\gamma)s] \cdot \frac{\alpha_{em}}{\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma \sin\theta_\gamma} + \ldots\,, \tag{11}$$

where x_γ is the fraction of the beam energy carried away by the photon, θ_γ is the photon scattering angle in the center-of-mass frame and dots stand for terms that are non-singular for $\theta_\gamma \to 0$. There is a dependence on the unknown parameter a, via the total cross-section for $e^+e^- \to \widetilde{G}\widetilde{G}$. General expressions for the differential cross-section evaluated according to the two different approaches outlined above can be found in Refs. [12, 13]. The photon energy and angular distributions are not universal, as for the case of the goldstino angular distribution in $e^+e^- \to \widetilde{G}\widetilde{G}$. In both cases the total cross-section, with appropriate cuts on the photon energy and scattering angle, scales as $\alpha_{em} s^3/F^4$. To the purpose of deriving a bound on $\sqrt{F}$ the collinear approximation of Eq. (11) is sufficient. Due to the quite strong power dependence of the cross-section on $\sqrt{F}$, further corrections to the leading term explicitly displayed in Eq. (11) have a small impact on the bound.

From the non-observation of single-photon events above the SM background at LEP2, one can derive a lower bound on the gravitino mass close to 10^{-5} eV, corresponding to $\sqrt{F} \sim 200$ GeV [14].

• **Hadron colliders**

As long as the approximation of heavy superparticles remains justified, the energy dependence of the cross-sections for goldstino production indicates that the most significant bound is expected to come from existing and future hadron colliders, i.e. the facilities with the highest available energy. Indeed, also the partonic processes $q\bar{q} \to \widetilde{G}\widetilde{G}\gamma$, $q\bar{q} \to \widetilde{G}\widetilde{G}g$, $qg \to \widetilde{G}\widetilde{G}q$, and $gg \to \widetilde{G}\widetilde{G}g$ have cross-sections scaling as s^3/F^4. The first reaction leads to a final state with a single photon plus missing transverse energy. This channel has been analysed by the D0 collaboration at the Tevatron Collider [15]. In a data sample corresponding to about 13 pb^{-1}, with a photon pseudorapidity η_γ in the range $|\eta_\gamma| < 2.5$, D0 found no events with a photon energy above 70 GeV. This translates in $\sqrt{F} > 245$ GeV ($m_{3/2} > 1.4 \cdot 10^{-5}$ eV), the best direct limit on F up to now [16]. A considerable improvement is expected from the channel with a single jet plus missing transverse energy, originating from the other partonic processes listed above. No experimental analysis exists at the moment, but we can estimate [16] the expected sensitivity by comparing the signal to the SM background, dominated by the associated Z plus jet production with the subsequent decay $Z \to \nu\bar{\nu}$. It is interesting to note that, as a result of the interplay between the energy dependence of the partonic cross-sections and the fractional momentum dependence of the parton densities, the signal is much less steep than the background in the jet transverse energy variable. A suitable cut on the jet transverse energy can optimize the significance of the signal. Already at the Tevatron Collider, with an energy of 1.8 TeV and an integrated luminosity of 100 pb^{-1}, a sensitivity to $\sqrt{F}$ up to 340 GeV ($m_{3/2} > 2.7 \cdot 10^{-5}$ eV) is expected. The upgraded Tevatron Collider, with $\sqrt{s} = 2$ TeV and an integrated luminosity of 2 fb^{-1} might reach $\sqrt{F} > 450$ GeV ($m_{3/2} > 5 \cdot 10^{-5}$ eV). Finally, the LHC, with $\sqrt{s} = 14$ TeV and an integrated luminosity of 10 fb^{-1} will be sensitive to a supersymmetry breaking scale beyond 2 TeV ($m_{3/2} > 10^{-3}$ eV). The LHC saturates the region of applicability of the low-energy approximation, since the superpartners are expected to show up at the TeV scale. In summary: hadron colliders represent the best opportunity either to discover a superlight gravitino or to set a direct bound on $\sqrt{F}$.

• **Indirect limits**

If the goldstino is superlight, it can also modify low-energy observables via loop contribution evaluated in the effective theory. For instance, the 1-loop contribution of the goldstino to $a_\mu \equiv (g-2)_\mu/2$ is given by [17]

$$\delta a_\mu = \frac{1}{48\pi^2} \frac{m_\mu^2 m_{\tilde{\mu}}^2}{F^2} \,. \tag{12}$$

By asking that $|\delta a_\mu|$ does not exceed the present experimental precision, $\sim 10^{-8}$, for a smuon mass $m_{\tilde{\mu}} = 100$ GeV, we obtain $\sqrt{F} > 70$ GeV ($m_{3/2} > 10^{-6}$ eV).

Similarly, the 1-loop contribution of the goldstino sector to the parameter ϵ_3 goes like

$$\delta\epsilon_3 \sim \frac{M_1 M_2 m_Z^2}{16\pi^2 F^2} \tag{13}$$

and, for gaugino masses $M_1 = M_2 = 100$ GeV, remains within the current experimental accuracy, $\sim 10^{-3}$, provided that $\sqrt{F} > 150$ GeV ($m_{3/2} > 5 \cdot 10^{-6}$ eV).

In the previous examples one should also account for the contribution coming from local counterterms that are present in the effective theory with unknown coefficients. Thus the bounds quoted rely on the assumption that the loop contribution well represents the size of the overall effect in the effective theory: loop plus counterterm. Nevertheless these bounds are quite interesting and their peculiar feature is that they become stronger for heavier superparticles.

• **Naturalness bounds**

The superlight gravitino scenario might also lead to direct modifications of the four-fermion interactions among ordinary quarks and leptons. The effective theory contains four-fermion operators involving four goldstinos or two goldstinos and two ordinary fermions. They are controlled by the coefficient $\Delta m^2/F^2$. This is the reason why perturbative unitarity gets violated at a critical energy of $O(F/\Delta m)$. One may wonder if the same physics that gives rise to these interactions may also produce four-fermion operators involving only ordinary matter fermions. Indeed, in the simplest models, the natural coefficient for such operators is again $\Delta m^2/F^2$ [18]. It is possible to adjust the theory in such a way that this particular set of operators is absent either at the classical level or in the 1-loop approximation. Nevertheless, the low-energy effective theory does not seem to acquire any additional symmetry in this limit, at least in the simplest cases examined. For squark masses of O(200 GeV), the present limits on four-quark operators from the Tevatron Collider lead to a lower bound on $\sqrt{F}$ close to 400 GeV.

Conclusions

The supersymmetry breaking scale $\sqrt{F}$ is a fundamental parameter of any supersymmetric extension of the Standard Model. If we regard $\sqrt{F}$ as a free parameter, with no theoretical prejudice about the possible preferred values, we can presently tolerate a supersymmetry breaking scale as small as 300–400 GeV, without any conflict with the present experimental data. The corresponding gravitino would be superlight and could produce visible signals at the upgraded Tevatron Collider and at the LHC. The most promising channels to detect a superlight gravitino at hadron colliders are missing transverse energy plus either a single photon or a single jet. The non-observation of events of this type over the SM backgrounds at the LHC would allow to raise the lower bound on $\sqrt{F}$ up to about 2 TeV.

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