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INDIRECT CP VIOLATION IN THE B_d-SYSTEM*

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Recently a rephasing-invariant definition of the CP-mixing parameter for Indirect CP Violation has been introduced. This is made possible by the explicit use of the CP operator in the analysis. The problem is the determination of the CP operator for a CP violating scenario. We discuss it and provide a definite solution.

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1. Introduction

In 1964, Christenson, Cronin, Fitch and Turlay [1] discovered that the long-lived neutral kaon K_L also decays to $\pi^+\pi^-$ with a branching ratio of $\sim 2\times 10^{-3}$. This discovery established CP-violation and the fact that K_L is not identical to the CP-eigenstate with CP-eigenvalue equal to -1.

Similarly, the short-lived neutral kaon K_S is not identical to the CP-eigenstate with CP-eigenvalue equal to $+1$. CP-violation was confirmed later by the decay $K_L \to \pi^0\pi^0$ [2] and by the charge asymmetry [3] in the K_{l3} decays $K_L \to \pi^\pm l^\mp \nu_l$. In particular, this semileptonic asymmetry measures whether CP-violation is present in the physical eigenstates of the Meson Mass Matrix, referred to as Indirect CP Violation. The present value of the world average [4] of the charge asymmetry gives a CP-violation in the Mixing $\mathrm{Re}\,\varepsilon_K = (1.63\pm 0.06)\times 10^{-3}$.

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For the non-leptonic $K_{S,L} \to 2\pi$ decays, the experimentally observable quantities are the ratios

$$\eta_{\pm} = \frac{\langle \pi^+\pi^- | K_L \rangle}{\langle \pi^+\pi^- | K_S \rangle} \quad , \quad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L \rangle}{\langle \pi^0\pi^0 | K_S \rangle} \tag{1}$$

which can be rewritten in terms of CP-violating indirect ε and direct ε' parameters as

$$\eta_{\pm} \simeq \varepsilon + \varepsilon' \quad , \quad \eta_{00} \simeq \varepsilon - 2\varepsilon' \tag{2}$$

when the $\Delta I = 1/2$ rule is used.

The ratio $\frac{\varepsilon'}{\varepsilon}$ can be determined by the "method of ratio of ratios" when comparing the $\pi^0\pi^0$ and $\pi^+\pi^-$ decay channels

$$\mathrm{Re}(\frac{\varepsilon'}{\varepsilon}) \simeq \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\} . \tag{3}$$

Present results at CERN [5] and FermiLab [6] give conflicting results at the level of 10^{-3}. New experiments with better sensitivity at CERN, FermiLab and the dedicated Φ-factory at Frascati [7] will push the precision to reach sensitivities better than 10^{-4}.

Recently, the first direct observation of a difference in the decay rates between particles and antiparticles has been accomplished by the CP-LEAR experiment [8]. They make use of flavour-tagging either K^0 or $\bar{K}^0$ at $t = 0$, and study their time evolution in the decays to 2π. One concludes that, in the $K^0 - \bar{K}^0$ system, Indirect CP Violation governed by ε_K plays the most prominent role.

The main question is whether the origin of CP-violation can be explained within the Standard Model or whether it needs physics beyond the Standard Model. In particular, is CP-violation to be described by charged current flavour mixing of quarks? We believe that the $B^0 - \bar{B}^0$ system is going to play a fundamental role in this respect, and its experimental study will be the task of all big facilities in the world: B-factories, Cornell, HERA-B, B-TeV, LHC.

2. Orthodoxy for indirect CP-violation in the B_d-system

Flavour Number is not conserved by weak interactions, so in 2^{nd} order B^0 and $\bar{B}^0$ mix. The existence of decay channels leads to a non-Hermitian Mixing Matrix

$$H = M - \frac{i}{2}\Gamma \tag{4}$$

in the $B^0, \bar{B}^0$ basis. The physical eigenstates of mass and lifetime diagonalize H as

$$\begin{aligned} |B_1\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \\ |B_2\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle \end{aligned} \tag{5}$$

with the amplitude

$$\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma} \quad ; \quad M_{12} - \frac{i}{2}\Gamma_{12} \equiv \langle B^0|H|\bar{B}^0\rangle \tag{6}$$

obtained in the Weisskopf–Wigner approximation.

The Standard Model $\Delta B = 2$ transition is given by the box diagram[1] in Fig. 1.

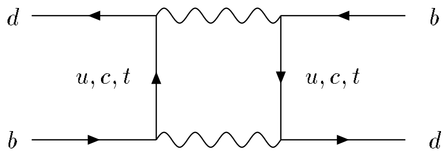

Fig. 1. The box diagram leading to the Standard Model $\Delta B = 2$ transition.

It generates matrix elements with $|\Gamma_{12}| \ll |M_{12}|$ and almost an alignment of the complex values of Γ_{12} and M_{12}. As a consequence, to a good approximation [9], the flavour mixing amplitude q/p is just a pure phase!

The parameter q/p is phase-convention-dependent on the definition of the CP-transformed states and thus its phase is not, by itself, observable. The best prospects use then the strategy of the interplay between Mixing and Decay [10].

The non-observability of the flavour mixing phase is made apparent in the CP-violating rate asymmetry, from a flavour tag, in the semileptonic decay $B^0 \to l\nu_l X$:

$$a_{SL} \equiv \frac{N(l^+l^+) - N(l^-l^-)}{N(l^+l^+) + N(l^-l^-)} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2}\,. \tag{7}$$

To generate $|q/p| \neq 1$, one would need both $\Delta\Gamma_B \neq 0$ and a misalignment such that $\mathrm{Im}(M_{12}^*\Gamma_{12}) \neq 0$. Some prospects could appear for physics beyond the Standard Model [11].

[1] This is my reference to the title of the Workshop

3. Phase-convention-independent CP mixing

The question is whether Indirect CP Violation and $|q/p| \neq 1$ are equivalent. We propose to establish the concept of Indirect CP Violation by means of CP Mixing in the physical states. We define the ε-parameter [12] as

$$\left.\begin{aligned} |B_1\rangle &= \tfrac{1}{\sqrt{1+|\varepsilon|^2}}(|B_+\rangle + \varepsilon|B_-\rangle) \\ |B_2\rangle &= \tfrac{1}{\sqrt{1+|\varepsilon|^2}}(|B_-\rangle + \varepsilon|B_+\rangle) \end{aligned}\right\} \quad |B_\pm\rangle \equiv \frac{1}{\sqrt{2}}(I \pm \mathrm{CP})|B^0\rangle\,, \tag{8}$$

where $|B_\pm\rangle$ are the CP eigenstates. For a given CP-operator, this ε is phase-convention-independent. This is seen explicitly by the relation

$$\frac{1-\varepsilon}{1+\varepsilon} = \frac{q}{p}\mathrm{CP}_{12}\,, \tag{9}$$

and the result (6): The phase convention for $B^0, \bar{B}^0$ states is irrelevant. ε involves, on the other hand, the three operators M, Γ and CP. The matrix element $\mathrm{CP}_{12} \equiv \langle B^0|\mathrm{CP}|\bar{B}^0\rangle$ plays the role of a reference phase. In the CP-conserving limit it is given by the flavour mixing amplitude:

$$\left(\frac{q}{p}\right)_{\mathrm{CP}} = -\mathrm{CP}_{12}^*\,. \tag{10}$$

With three directions in the complex plane, those of $M_{12}, \Gamma_{12}, \mathrm{CP}_{12}$, we have two <u>relative</u> phases that can become observable: one is well known

$$\frac{2\mathrm{Re}(\varepsilon)}{1+|\varepsilon|^2} = \frac{1-|q/p|^2}{1+|q/p|^2}\,, \tag{11}$$

which involves the relative phase between M_{12} and Γ_{12}. For the B_d-system, with $\Delta\Gamma \sim 0$, one has $\mathrm{Re}(\varepsilon) \sim 0$. In this limit,

$$\frac{\mathrm{Im}(\varepsilon)}{1+|\varepsilon|^2} \simeq \frac{\mathrm{Im}(M_{12}^*\mathrm{CP}_{12})}{\Delta m}\,, \tag{12}$$

and we observe that the (second) relative phase between M_{12} and CP_{12} gives $\mathrm{Im}(\varepsilon)$. Is this a quantum-mechanical observable?

In Ref. [12] we have discussed an interference experiment between the CP-eigenstates $|B_\pm\rangle$ obtained from the time evolution of a CP-tag. The corresponding CP asymmetries to common leptonic final states l^+ and l^- are able to separate out $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\varepsilon)$. If $B_+(t)$ denotes the time-evolved state from a CP eigenstate B_+ prepared at $t=0$, one has

$$\begin{aligned} A_+^{\mathrm{CP}}(t) &= \frac{\Gamma[B_+(t)\to l^+] - \Gamma[B_+(t)\to l^-]}{\Gamma[B_+(t)\to l^+] + \Gamma[B_+(t)\to l^-]} \\ &= \frac{2\mathrm{Re}(\varepsilon)}{1+|\varepsilon|^2}\left[1 - e^{\frac{\Delta\Gamma}{2}t}\cos(\Delta m\, t)\right] - \frac{2\mathrm{Im}(\varepsilon)}{1+|\varepsilon|^2}e^{\frac{\Delta\Gamma}{2}t}\sin(\Delta m\, t)\,. \end{aligned} \tag{13}$$

We conclude that $\mathrm{Im}(\varepsilon)$ is a physical quantity and observable iff the CP-transformation is well defined. Is the CP-operator determined? In the case of an invariant theory, a symmetry transformation is well defined. Otherwise the corresponding symmetry operator is undetermined.

4. The CP-conserving direction

It is possible to have a well defined CP operator even for a non-invariant theory. This is the situation when the structure of the Lagrangian allows the separation of a CP-conserving part which includes flavour mixing, from a different interaction responsible for the CP non-invariance. In this type of models, as in superweak interaction [13], the invariant part of the Lagrangian determines the action of the symmetry operation on the fields.

The most interesting case is that of the Standard Model: a theory where flavour mixing and CP-violation cannot be separated in that way. Therefore, there is no phase choice for the CP transformed fields which leaves the Lagrangian invariant. Different choices of phases, and thus of CP operator, will yield different observables: our ε-parameter would not be unique. We are going to show, however, that the use of the quark mixing hierarchy (empirically well established) leads to a unique separation of the weak Lagrangian into a CP-conserving and a CP-violating part.

The CP operation is defined by the invariance of strong and electromagnetic interactions. When the mass matrices for the up M and down M' sectors are considered, the corresponding electroweak quark fields have CP transformations which include unitary matrices Φ and Φ', respectively, in family space. The invariance condition on the Lagrangian determines [14] Φ, Φ' up to diagonal unitary phases $e^{2i\theta}, e^{2i\theta'}$ in terms of the diagonalizing matrices U, U' for the quark fields. These diagonal phases are the (arbitrary) CP-phases of the physical quark fields. If the charged current Lagrangian was absent, there would be no cross-talk between up and down quarks: the arbitrariness would have no physical effect and we could consider the CP operator to be determined.

On the contrary, the existence of a charged current Lagrangian induces both

— Flavour Mixing through $V = UU'^{\dagger}$

— CP-violation through $B = \Phi\Phi'^{\dagger}$

The arbitrariness of CP-phases is now relevant to induce different B-matrices. Is it possible to choose θ, θ' such that $B = I$? If the answer is positive, we have a theory with Flavour Mixing $V \neq I$, but CP invariance $B = I$: a CP-conserving Standard Model. CP-violation would have to be understood from a Superweak-type Model. The necessary and sufficient conditions for CP invariance to be satisfied [15] by the mass matrices M, M' and the corresponding phase fixing were discussed [16] some time ago.

If there is no CP-phase choice for θ, θ' to get $B = I$, the theory is CP-violating. Can the theory still filter a well defined CP operator, at least in a perturbative sense? We know that, in the K-system, the CP symmetry is only slightly violated and its size [1] is of the order $O(10^{-3})$. This is understood in the Standard Model as a consequence of the need to involve the three families to generate CP-violation. Thus its effective coupling contains higher powers of the quark mixing λ than that of the CP-conserving flavour mixing $K^0\bar{K}^0$. This justifies the idea to look for a "natural" CP definition in the Standard Model based on the empirically known quark mixing hierarchy.

Take one of the sides (fixed k) of the (bd) unitarity triangle. It can be decomposed in the (complex) plane into CP-conserving and CP violating parts as

$$\begin{aligned} V^*_{kb}V_{kd}(\mathrm{CP}) &= e^{i(\theta_b-\theta_d)}\mathrm{Re}(e^{-i\theta_b}V^*_{kb}V_{kd}e^{i\theta_d})\,, \\ V^*_{kb}V_{kd}(\not{C}\not{P}) &= ie^{i(\theta_b-\theta_d)}\mathrm{Im}(e^{-i\theta_b}V^*_{kb}V_{kd}e^{i\theta_d})\,. \end{aligned} \tag{14}$$

Eq. (14) tells that $e^{i(\theta_b-\theta_d)}$ defines the CP-conserving direction associated to the (bd) triangle. It depends on the choice of CP-phases. However, the three CP-conserving directions of the three "down" triangles are not independent, due to the cyclic relation

$$e^{i(\theta_b-\theta_d)} = e^{i(\theta_b-\theta_s)}e^{i(\theta_s-\theta_d)}\,. \tag{15}$$

These CP-conserving directions are attached to the triangles, so they would rotate with them under quark rephasing. They are not physical by themselves, but the relative phases between triangle sides and them are rephasing-invariant.

According to the experimentally known hierarchy in the quark mixing, the magnitude of V matrix elements can be written in terms of a perturbative parameter λ. We can estimate the relative size of every side in the three triangles of the down sector. To order λ^3, the two triangles (bs) and (sd) collapse to a line each, thus giving CP conservation and a natural choice for the attached CP direction: the CP invariance requirement on the effective Hamiltonian fixes the corresponding CP-phases of these sectors. Due to (15) the CP-conserving direction for the (bd) system is already fixed. This is particularly attractive because the (bd) system keeps a CP violating triangle to order λ^3. One obtains [14]

$$e^{i(\theta_b-\theta_d)} = \left.\frac{V_{cd}V^*_{cb}}{|V_{cd}V^*_{cb}|}\right|_{O(\lambda^3)}\,. \tag{16}$$

Thus the CP-conserving direction matches one of the sides of the (bd) triangle to $O(\lambda^3)$.

This result controls the value of $\mathrm{Im}(\varepsilon)$ for the B_d-system as an OBSERVABLE even in the Standard Model.

To order λ^3, we have

$$\mathrm{Im}(M_{12}^* \mathrm{CP}_{12}) \propto \mathrm{Im}(V_{td}^* V_{tb} V_{cd} V_{cb}^*) \,. \tag{17}$$

Eq. (17) proves definitively the phase-convention-independence of our analysis.

5. Conclusions

These are summarized in the following two points:

1. There exists a rephasing-invariant measure of CP Mixing given by our ε-parameter. It is independent of the rephasing of $B^0 - \bar{B}^0$ states and of quark fields, *i.e.*, independent of a specific parametrization of the Mixing Matrix $V(CKM)$.

2. The ε-parameter is unique iff the CP operator is well defined. The use of the quark mixing hierarchy leads to determining CP to $O(\lambda^3)$.

As a final comment, the definite CP-conserving direction found in Eq. (16) implies that decays of B_d to final CP eigenstates which are dominated by the amplitude $V_{cd}V_{cb}^*$ constitute excellent CP tags.

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