Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 10

THE RARE DECAY $B \to X_s\gamma$ * **

A. Kwiatkowski

Physik Department, Technische Universität München
D-95747 Garching b. München, FRG

(Received July 17, 1998)

The next-to-leading-order (NLO) analysis of the decay $B \to X_s\gamma$ is reviewed in this talk. The theoretical framework and the various ingredients for a theoretical prediction of the branching ratio $\mathrm{Br}(B \to X_s\gamma)$ are briefly outlined. The numerical analysis focuses on an estimate of the theoretical uncertainties. It is pointed out that the theoretical error is presently dominated by parametric uncertainties that may be reduced in the future. In view of oncoming measurements at future B factories this underlines the importance of the decay $B \to X_s\gamma$ for the search for new physics.

PACS numbers: 13.90.+i

1. Introduction

The inclusive rare decay $B \to X_s\gamma$ plays an outstanding role in present day phenomenology. Being forbidden at tree level in the Standard Model, this flavour changing neutral current process may only proceed as a loop induced transition. Such rare processes attracted much interest in the last decade.

Due to the occurrence of the top quark as a virtual particle inside the loop and the resulting dependence on the Cabibbo–Kobayashi–Maskawa (CKM) matrix element $|V_{ts}|$, the $B \to X_s\gamma$ decay provides a handle to constrain this CKM parameter of the Standard Model that is hardly accessible by other processes.

Moreover, the process $B \to X_s\gamma$ may be able to open the window to new physics when looking beyond the Standard Model. The rare decays $B \to X_s\gamma$ leave room in their loops for potential nonstandard particles of

* Presented at the DESY Zeuthen Workshop on Elementary Particle Theory "Loops and Legs in Gauge Theories", Rheinsberg, Germany, April 19-24, 1998.

** Work supported by the German Bundesministerium für Bildung und Forschung under contract 06 TM 874 and DFG Project Li 519/2-2.

extended theories (like charged Higgs bosons or supersymmetric particles). Those contributions would be of the same order in the gauge coupling as the Standard Model contributions themselves and thus be unsuppressed. Sizeable deviation of the branching ratio $\mathrm{Br}(B \to X_s\gamma)$ from the Standard Model prediction could therefore lead to an indirect observation of new physics effects.

The above agenda is challenging both from the experimental and theoretical point of view as sufficient accuracy is required on both sides for a test of the theory at the quantum level.

Experimental measurements were performed by CLEO in the energy range of the $\Upsilon(4S)$ resonance, where they find a branching ratio [1]

$$\mathrm{Br}(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \tag{1}$$

Here the first error is statistical and the second one is systematic. A recent preliminary update of this result is quoted [2]

$$\mathrm{Br}(B \to X_s\gamma) = (2.50 \pm 0.47 \pm 0.39) \times 10^{-4}\,. \tag{2}$$

ALEPH has investigated the decays of b-hadrons produced at LEP on the Z_0 peak. Their finding is [3]

$$\mathrm{Br}(H_b \to X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}. \tag{3}$$

The experimental situation will improve further with the oncoming operation of an upgraded CLEO detector and the future B-factories at SLAC and KEK. The experimental error may be expected to drop below the 10% level.

On the theoretical side, during the last five years considerable progress has been made for an accurate prediction of the branching ratio $\mathrm{Br}(B \to X_s\gamma)$. The combined effort of several groups to calculate the large QCD corrections to this decay, including the next-to-leading-order (NLO) corrections, was finally completed by 1998. This enterprise was mainly motivated by large renormalization scale uncertainties of $\pm 25\%$ [4,5] which could only be reduced by extending the calculations beyond the leading order.

The numerical analysis shows that a theoretical uncertainty below 10% is already achieved at present. More important, only a part of these uncertainties is due to unknown higher order corrections while the remaining ones arise from the uncertain values of the input parameters. Once the latter are known with better precision, a theoretical uncertainty for $\mathrm{Br}(B \to X_s\gamma)$ of 5% may be within reach.

Continued work is devoted to estimate non-perturbative corrections to the $B \to X_s\gamma$ decay with higher precision [6–9] as well. It appears that these corrections amount only to a few percent and constitute a rather small theoretical uncertainty.

2. $B \to X_s\gamma$ in the next-to-leading-logarithmic approximation

The $B \to X_s\gamma$ decay is conveniently considered on the parton level in the spectator model

$$\frac{\mathrm{Br}(B \to X_s\gamma)}{\mathrm{Br}(B \to X_c e\bar{\nu}_e)} \simeq \frac{\mathrm{Br}(b \to s\gamma)}{\mathrm{Br}(b \to ce\bar{\nu}_e)} = R_{\mathrm{quark}}\,. \tag{4}$$

Corrections to this approximation are small of order $\mathcal{O}(1/m_{\mathrm{b}}^2)$ and will be discussed later. The normalization with the semileptonic branching ratio is usually employed for a cancellation of the CKM factors and the overall bottom mass dependence.

Perturbative calculations are associated with large logarithms $\alpha_s^n(\mu_b)\ln^m(\mu_W/\mu_b)$ $(m \le n)$, due to the large difference between the scale $\mu_W \simeq M_{\mathrm{W}}, m_{\mathrm{t}}$ of the heavy virtual particles and the scale $\mu_b \simeq m_{\mathrm{b}}$ of the hadronic decay under consideration.

The resummation of these logarithms to all orders in leading $(n = m)$ or next-to-leading $(n = m + 1)$ approximation is achieved in the framework of a low energy effective theory where all heavy particles like the W boson and the top quark are integrated out. The relevant effective Hamiltonian reads

$$\mathcal{H}^{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i\,. \tag{5}$$

Here Q_1, Q_2 denote the commonly used current-current operators and Q_3, $\ldots$, Q_6 the QCD penguin operators. For the explicit formulae that we choose not to display in this paper, the reader is referred to the reviews [11–13].

Although the matrix elements $\langle \mathcal{H}^{\mathrm{eff}} \rangle$ are renormalization scale invariant, the Wilson coefficients $C_i(\mu)$ and the matrix elements $\langle Q_i(\mu) \rangle$ show a separate μ-dependence, reflecting the factorization of short-distance and long-distance physics. The scale dependence of the coefficient functions is governed by the renormalization group equation

$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j \hat{\gamma}_{ji}(\alpha_s) C_j(\mu)\,, \tag{6}$$

where $\hat{\gamma}(\alpha_s)$ is the 8×8 anomalous dimension matrix. The solution of (6) is given by

$$C_i(\mu_b) = \sum_j \hat{U}_{ij}(\mu_b, \mu_W) C_j(\mu_W) \tag{7}$$

with the evolution matrix $\hat{U}(\mu_b, \mu_W)$ and the initial conditions $C_i(\mu_W)$.

The complete analysis at NLO therefore consists of the following steps and requires several ingredients.

First, the effective theory is defined by fixing the coefficient functions $C_i(\mu_W)$ through matching of the full and the effective theory at a matching scale $\mu_W = \mathcal{O}(M_{\rm W})$. The NLO $\mathcal{O}(\alpha_s)$ corrections for $C_7(\mu_W)$ and $C_8(\mu_W)$ amount to a two-loop calculation which was first performed by Adel and Yao [10] and confirmed by [14–16].

Second, the coefficient functions C_i evolve from their initial values at the high energy scale μ_W to the low energy scale μ_b via the evolution matrix $\hat{U}$ which itself is determined by the anomalous dimension. The NLO anomalous dimension matrix has been calculated in [17]– [22] of which [17]– [21] are two-loop calculations and [22] is even a three-loop calculation.

Third, the one loop matrix elements $\langle s\gamma|Q_{7,8}|b\rangle$ and the gluon bremsstrahlung $\langle s\gamma g|Q_i|b\rangle$ were calculated in [23, 24] whereas the two-loop corrections to $\langle s\gamma|Q_i|b\rangle$ were presented in [25].

The complete NLO result for the $b \to s\gamma$ decay can be cast into the form

$$R_{\rm quark} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} \frac{1}{\kappa(z, \bar{\mu}_b)} \left(\frac{\bar{m}_b(m_{\rm b})}{m_{b,\rm pole}} \right)^2 \left(|D|^2 + A \right) , \tag{8}$$

where

$$D = C_7^{(0)\rm eff}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} \left\{ C_7^{(1)\rm eff}(\mu_b) + \sum_{i=1}^{8} C_i^{(0)\rm eff}(\mu_b) \left[r_i + \frac{\gamma_{i7}^{(0)\rm eff}}{2} \ln \frac{m_b^2}{\mu_b^2} \right] \right\} . \tag{9}$$

We do not display all quantities in Eqs. (8), (9) in their full form, but refer the reader in our discussion to various places in the literature.

Due to the normalization with the semileptonic decay a phase space factor $f(z)$ with $z = m_{c,\rm pole}^2 / m_{b,\rm pole}^2$ and QCD corrections [26] $\kappa(z, \bar{\mu}_b)$ are introduced. An approximation formula for κ can be found in [27] and an exact analytic formula in [28]. The scale $\bar{\mu}_b$ of the semileptonic decay is in general different from the low energy scale μ_b of the $b \to s\gamma$ decay as was pointed out in [29].

The amplitude D is composed of the Wilson coefficient of Q_7

$$C_7^{\rm eff}(\mu_b) = C_7^{(0)\rm eff}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_7^{(1)\rm eff}(\mu_b) \tag{10}$$

containing the information of the three-loop anomalous dimension [22]. The index "eff" indicates that a regularization scheme independent combination of coefficient functions as introduced in [5] is considered. Because of their top dependence the coefficient functions also depend on the scale $\mu_t = \mathcal{O}(m_{\rm t})$ at which the top mass $\overline{m}_{\rm t}(\mu_t^2)$ is defined. As was discussed in [29], the scales μ_t and μ_W do not have to be equal. Furthermore D contains the contributions of the virtual corrections [25] to the matrix elements $\langle s\gamma|Q_i|b\rangle$ represented by the last two terms in Eq. (9).

Finally the term A in Eq. (8) originates from the bremsstrahlung corrections and the necessary virtual corrections needed for the cancellation of the infrared divergences. These have been calculated in [23, 24] and are also considered in [22, 25] (see also [30]) in the context of the full analysis. As in [22] we have chosen in the numerical analysis a lower cut on the photon energy of $(1 - E_\gamma/E_\gamma^{\rm max}) < \delta = 0.99$.

In order to pass from the partonic picture to the hadronic decay rates and to obtain the final result for the B-meson decay rate, non-perturbative contributions should be taken into account as well. According to Heavy Quark Effective Theory calculations they are included as power corrections [22] in the numerical analysis

$$\mathrm{Br}(B \to X_s\gamma) = \mathrm{Br}(B \to X_c e\bar{\nu}_e) \cdot R_{\rm quark}\left(1 - \frac{\delta_{\rm sl}^{\rm NP}}{m_{\rm b}^2} + \frac{\delta_{\rm rad}^{\rm NP}}{m_{\rm b}^2}\right), \qquad (11)$$

where $\delta_{\rm sl}^{\rm NP}$ and $\delta_{\rm rad}^{\rm NP}$ parametrize non-perturbative corrections to the semileptonic and radiative B-meson decay rates respectively. However, due to partial cancellations [6] the modifications turn out to be small and sum up to around 1%.

In addition we have included in our numerical analysis a 3% enhancement [9] from $1/m_{\rm c}^2$ corrections [8].

3. Numerical analysis

Adding the NLO corrections to the LO result an explicit cancellation of large logarithms can be observed [13, 29] leading to the considerable reduction of the renormalization scale dependence. To be precise, the following scales are present in this process.

- The low energy scale $\mu_b = \mathcal{O}(m_b)$ at which the hadronic decay takes place. At this scale the Wilson coefficients and the matrix elements of the operators for the $B \to X_s\gamma$ decay are evaluated. Notice that the semileptonic normalization process $B \to X_c e\bar{\nu}_e$ is also associated with a low energy scale $\bar{\mu}_b = \mathcal{O}(m_b)$. Since both scales are completely independent one has in general $\mu_b \neq \bar{\mu}_b$ [29].

- The high energy scale $\mu_W = \mathcal{O}(M_{\rm W})$ at which the full theory is matched with the effective theory.

- The scale $\mu_t = \mathcal{O}(m_t)$ at which the running top quark mass is defined.

To estimate the remaining sensitivity of the branching ratio on the scales, we have varied them in the ranges

$$2.5\ \text{GeV} \le \mu_b, \bar{\mu}_b \le 10\ \text{GeV}$$
$$40\ \text{GeV} \le \mu_W \le 160\ \text{GeV} \qquad 80\ \text{GeV} \le \mu_t \le 320\,\text{GeV} \tag{12}$$

and find

$$\Delta \text{Br}(B \to X_s\gamma) = \left\{ \begin{array}{lll} \text{LO} & \text{NLO} & \\ \pm 13\% & \pm 1.1\% & (\mu_W) \\ \pm\ \ 3\% & \pm 0.4\% & (\mu_t) \\ \pm 22\% & \pm 4.3\% & (\mu_b)\,. \end{array} \right. \tag{13}$$

The huge reduction of the scale uncertainties in the NLO result compared to the LO is striking. Combined with the uncertainty due to the variation of the semileptonic scale $\bar{\mu}_b$ of $\Delta\text{Br}(B \to X_s\gamma) = \pm 1.7\%$ one arrives at the total scale uncertainty

$$\Delta \text{Br}(B \to X_s\gamma) = \pm 4.8\% \quad \text{(scale)}. \tag{14}$$

This number is a genuine theoretical uncertainty that is due to the truncated perturbation series. It estimates the magnitude of unknown higher order corrections and can only be reduced by going beyond the NLO approximation. It should be distinguished from the parametric uncertainties that are introduced through the errors of the various input parameters. In Table I the relative importance of various uncertainties are compared.

TABLE I

Uncertainties in $\text{Br}(B \to X_s\gamma)$ due to various sources

Scales	$\alpha_s(M_Z)$	$m_{t,\text{pole}}$	$m_{c,\text{pole}}/m_{b,\text{pole}}$
$\pm 4.8\%$	$\pm 2.9\%$	$\pm 1.7\%$	$\pm 5.4\%$
$m_{b,\text{pole}}$	α_{em}	CKM angles	$B \to X_c e \bar{\nu}_e$
$\pm 0.7\%$	$\pm 1.8\%$	$\pm 2.0\%$	$\pm 3.8\%$

One finds that at present the parametric uncertainties dominate the theoretical error. This is reflected in the final result

$$\begin{aligned} \text{Br}^{\text{QCD}}(B \to X_s\gamma) &= (3.60 \pm 0.17\ \text{(scale)}\ \pm 0.28\ \text{(par)}) \times 10^{-4} \\ &= (3.60 \pm 0.33) \times 10^{-4}. \end{aligned} \tag{15}$$

In a very recent study [31] also dominant electroweak corrections to the $B \to X_s\gamma$ decay rate were investigated. The authors considered contributions from fermion loops to the gauge boson propagators as well as photonic loop corrections to $b \to s\gamma$ and $b \to ce\bar{\nu}_e$. They also advocated the electromagnetic coupling α_{em} to be renormalized at $q^2 = 0$ and arrive at a branching ratio

$$\mathrm{Br}^{\mathrm{QCD+EW}}(B{\to}X_s\gamma) = (3.28 \pm 0.30) \times 10^{-4}\,. \qquad (16)$$

Within their errors the theoretical prediction and the experimental results are compatible. Further improvements of the values on both sides may pave the way to new physics. Recent calculations of contributions of extended theories [16, 32–34] have shown that already at present the $B \to X_s\gamma$ decay plays a keyrole in restricting the parameter space of theories beyond the Standard Model. Future developments will be exciting to watch.

I would like to thank A.J. Buras and N. Pott for their collaboration on the results presented in this talk.

REFERENCES

[1] M.S. Alam *et al.* (CLEO), *Phys. Rev. Lett.* **74**, 2885 (1995).

[2] S. Glenn (CLEO), talk presented at the Meeting of the American Physics Society, Columbus, Ohio, 18-21 March 1998.

[3] R. Barate *et al.* (ALEPH), CERN-EP/98-044.

[4] A. Ali, C. Greub, *Z. Phys.* **C60**, 433 (1993).

[5] A.J. Buras, M. Misiak, M. Münz, S. Pokorski, *Nucl. Phys.* **B424**, 374 (1994).

[6] A.F. Falk, M. Luke, M. Savage, *Phys. Rev.* **D49**, 3367 (1994).

[7] G. Eilam, A. Ioannissian, R.R. Mendel, P. Singer, *Phys. Rev.* **D53**, 3629 (1996); J.M. Soares, *Phys. Rev.* **D53**, 241 (1996); N.G. Deshpande, X.-G. He, J. Trampetic, *Phys. Lett.* **B367**, 362 (1996).

[8] M.B. Voloshin, *Phys. Lett.* **B397**, 275 (1997); A. Khodjamirian, R. Rückl, G. Stoll, D. Wyler, *Phys. Lett.* **B402**, 167 (1997); Z. Ligeti, L. Randall, M.B. Wise, *Phys. Lett.* **B402**, 178 (1997); A.K. Grant, A.G. Morgan, S. Nussinov, R.D. Peccei, *Phys. Rev.* **D56**, 3151 (1997).

[9] G. Buchalla, G. Isidori, S.J. Rey, *Nucl. Phys.* **B511**, 594 (1998).

[10] K. Adel, Y.P. Yao, *Mod. Phys. Lett.* **A8**, 1679 (1993); *Phys. Rev.* **D49**, 4945 (1994).

[11] G. Buchalla, A.J. Buras, M. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).

[12] A.J. Buras, R. Fleischer, hep-ph/9704376, in: Heavy Flavours II, World Scientific, Singapore 1997, eds. A.J. Buras, M. Lindner.

[13] A.J. Buras, hep-ph/9806471.

[14] C. Greub, T. Hurth, *Phys. Rev.* **D56**, 2934 (1997).

[15] A.J. Buras, A. Kwiatkowski, N. Pott, *Nucl. Phys.* **B517**, 353 (1998), hep-ph/9710336.

[16] M. Ciuchini, G. Degrassi, P. Gambino, G.F. Giudice, hep-ph/9710335.

[17] G. Altarelli, G. Curci, G. Martinelli, S. Petrarca, *Nucl. Phys.* **B187**, 461 (1981).

[18] A.J. Buras, P.H. Weisz, *Nucl. Phys.* **B333**, 66 (1990).

[19] A.J. Buras, M. Jamin, M.E. Lautenbacher, P.H. Weisz, *Nucl. Phys.* **B370**, 69 (1992); *Nucl. Phys.* **B375**, 501 (1992) (addendum); *Nucl. Phys.* **B400**, 37 (1993).

[20] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, *Phys. Lett.* **B301**, 263 (1993); *Nucl. Phys.* **B415**, 403 (1994).

[21] M. Misiak, M. Münz, *Phys. Lett.* **B344**, 308 (1995).

[22] K.G Chetyrkin, M. Misiak, M. Münz, *Phys. Lett.* **B400**, 206 (1997), erratum *Phys. Lett.* **B425**, 414 (1998).

[23] A. Ali, C. Greub, *Z. Phys.* **C49**, 431 (1991); *Phys. Lett.* **B259**, 182 (1991); *Phys. Lett.* **B361**, 146 (1995).

[24] N. Pott, *Phys. Rev.* **D54**, 938 (1996).

[25] C. Greub, T. Hurth, D. Wyler, *Phys. Lett.* **B380**, 385 (1996); *Phys. Rev.* **D54**, 3350 (1996); C. Greub, T. Hurth, hep-ph/9608449.

[26] N. Cabibbo, L. Maiani, *Phys. Lett.* **B79**, 109 (1978).

[27] C.S. Kim, A.D. Martin, *Phys. Lett.* **B225**, 186 (1989).

[28] Y. Nir, *Phys. Lett.* **B221**, 184 (1989).

[29] A.J. Buras, A. Kwiatkowski, N. Pott, *Phys. Lett.* **B414**, 157 (1997).

[30] M. Neubert, hep-ph/9803368.

[31] A. Czarnecki, W.J. Marciano, hep-ph/9804252.

[32] P. Ciafaloni, A. Romanino, A. Strumia, hep-ph/9710312.

[33] F.M. Borzumati, C. Greub, hep-ph/9802391.

[34] M. Ciuchini, G. Degrassi, P. Gambino, G.F. Giudice, hep-ph/9806308.