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CALCULATION OF TWO-LOOP RADIATIVE CORRECTIONS TO $b \to c$ DECAY AT ZERO RECOIL*

J. Franzkowski

Institut für Physik, Johannes Gutenberg-Universität
Staudinger Weg 7, D-55099 Mainz, Germany

and J.B. Tausk

Fakultät für Physik, Albert-Ludwigs-Universität
Hermann-Herder-Straße 3, D-79104 Freiburg, Germany

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We review some aspects of our calculation of two-loop QCD corrections to $b \to c$ decay, which confirmed the results of Czarnecki and Melnikov.

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1.

Amplitudes for weak decays of mesons containing a b quark, such as *e.g.* $\overline{B^0} = \bar{d}b$, into charmed mesons such as $D^{*+} = \bar{d}c$, are proportional to the Kobayashi-Maskawa matrix element V_{cb}. The decay rate is also affected by strong interactions. One strategy for determining $|V_{cb}|$ involves measuring exclusive semileptonic decays $B \to D^* l\nu$ at the special kinematical point where the B and D^* mesons have equal velocities. The reason for choosing this zero recoil point is that non-perturbative effects, which cannot be calculated from first principles, are at least suppressed by a factor of $\Lambda^2_{\rm QCD}/m_c^2$ at this point. The zero recoil condition also entails some technical simplifications which make a complete analytical calculation of the perturbative QCD corrections to the underlying hard process $b \to cl\nu$ possible up to two loops. That is the subject of this paper. Our complete results have already been presented in [1] and will not be repeated here. Instead, we will focus on some aspects of the calculation itself.

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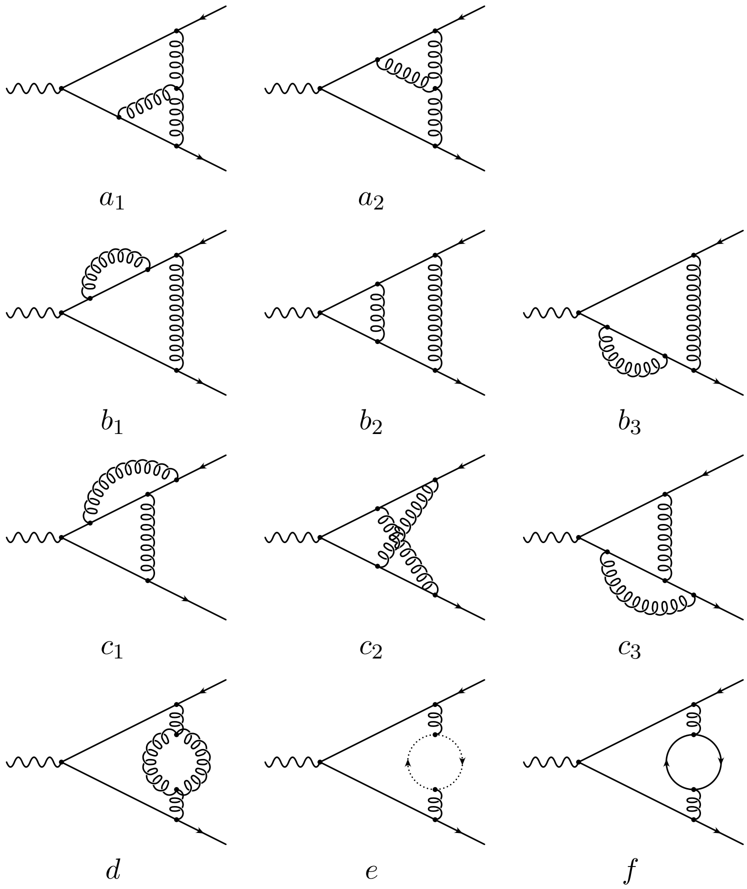

Fig. 1. Irreducible Feynman diagrams contributing to $b \to cW$ at order α_s^2. The dotted line in diagram e represents a Faddeev-Popov ghost. The fermion in the loop in diagram f can be either a light quark, a b or a c quark. Note the symmetry when the b and c quarks are interchanged: $a_1 \leftrightarrow a_2$, $b_1 \leftrightarrow b_3$, $b_2 \leftrightarrow b_2$, *etc.*

For a general review on semileptonic B decays, see [2]. Experimental results can be found in [3,4], and theoretical issues connected with the heavy quark expansion are discussed in [5]. The one-loop corrections to $b \to c$ decay at zero recoil were calculated in [6]. The two-loop corrections were first obtained as a Taylor series expansion in $(m_b - m_c)/m_b$ [7], and subsequently in a closed analytical form [8].

2.

At zero recoil, the amplitude for $b \to c$ is given by $\bar{u}(c)\gamma^{\mu}\,(\eta_V - \eta_A\gamma_5)\,u(b)$, with form factors $\eta_{V,A}$ that are normalized to 1 at tree level. The relevant two-loop Feynman diagrams are shown in figure 1. Their contributions to $\eta_{V,A}$ are combinations of integrals that can all be written as

$$\iint d^d k\, d^d l\, \frac{1}{P_1^{\nu_1} P_2^{\nu_2} \ldots P_9^{\nu_9}}\,, \tag{1}$$

where $d = 4 - 2\varepsilon$ regularizes both ultraviolet and infrared divergences, the ν_i are integer powers, and the propagator denominators P_i are defined by

$$\begin{array}{lll} P_1 = (l+k)(2p_1 + l + k)\,, & P_3 = l(2p_1 + l)\,, & P_5 = k(2p_1 + k)\,, \\ P_2 = (l+k)(2p_2 + l + k)\,, & P_4 = l(2p_2 + l)\,, & P_6 = k(2p_2 + k)\,, \\ P_7 = k^2\,, & P_8 = l^2\,, & P_9 = (l+k)^2\,. \end{array} \tag{2}$$

The four-momenta of the external b and c quarks, p_1 and p_2, are proportional to each other and satisfy $p_1^2 = m_1^2$, $p_2^2 = m_2^2$ and $p_1 p_2 = m_1 m_2$. Therefore, the integrals (1) only depend on the two variables m_1 and m_2. Sometimes (whenever $\nu_2 = \nu_4 = \nu_6 = 0$ or $\nu_1 = \nu_3 = \nu_5 = 0$), they only depend on one of the masses. In such one-scale cases, the result is a power of the mass, which follows from dimensional arguments, times a coefficient that can be calculated recursively using integration by parts [10].

The recurrence relations for the two-scale cases are more complicated. In our calculation, we used them as checks, and also to reduce the number of scalar integrals (1) needed, without solving the full system of equations. A solution to the recurrence relations for the special case where $m_1 = 2m_2$ was applied in [9]. In any case, a few scalar integrals have to be calculated by other means in order to start off the recursion. The results (expanded in ε) can be expressed in terms of polylogarithms. One can almost predict which polylogarithms appear by considering where the scalar integrals (1) have singularities. In this respect, there is a difference between graphs that have a cut going across three massive quark lines, like the graphs denoted by N in [10], and those that do not, like the M-graphs in [10]. We find the following solutions to the Landau equations:

	$m_1 = m_2$	$m_1 = 0$	$m_2 = 0$	$m_1 + m_2 = 0$
M-like graphs	x	x	x	
N-like graphs	x	x	x	x

At $m_1 = m_2$, poles in the integrand of (1) coincide without trapping the integration contour. Therefore, there is no singularity at this point on the physical sheet, but we do find singularities at $m_1 = m_2$ when we consider the analytical continuation to other Riemann sheets. Similarly, the singularity at $m_1 + m_2 = 0$ can only be reached by analytic continuation and is not relevant for physical, positive masses m_1 and m_2.

The table below shows all the (poly)logarithms (up to the level of trilogarithms) of m_1 and m_2 that do not have any singularities except at points corresponding to the solutions of the Landau equations shown above. The ones in the lower part of the table are singular at $m_1+m_2 = 0$ and therefore we only expect them to appear in N-like graphs:

$\log\left(\frac{m_1}{m_2}\right)$	$\mathrm{Li}_2\left(\frac{m_1-m_2}{m_1}\right)$		$\mathrm{Li}_3\left(\frac{m_1-m_2}{m_1}\right)$	$\mathrm{Li}_3\left(\frac{m_2-m_1}{m_2}\right)$
$\log\left(\frac{2m_1}{m_1+m_2}\right)$	$\mathrm{Li}_2\left(\frac{m_1-m_2}{m_1+m_2}\right)$	$\mathrm{Li}_2\left(\frac{m_2-m_1}{m_1+m_2}\right)$	$\mathrm{Li}_3\left(\frac{m_1-m_2}{m_1+m_2}\right)$	$\mathrm{Li}_3\left(\frac{m_2-m_1}{m_1+m_2}\right)$
			$\mathrm{Li}_3\left(\frac{m_1^2-m_2^2}{m_1^2}\right)$	$\mathrm{Li}_3\left(\frac{m_2^2-m_1^2}{m_2^2}\right)$
			$\mathrm{Li}_3\left(\frac{m_1-m_2}{2m_1}\right)$	$\mathrm{Li}_3\left(\frac{m_2-m_1}{2m_2}\right)$

This table is complete in the sense that any other such function one might consider (say, *e.g.*, an $S_{1,2}$) can be written as a linear combination of functions and products of functions in the table. So we expect that all the scalar integrals we need can be expressed in terms of these functions, and this turns out to be true.[1]

Often, a convenient way of calculating the scalar integrals is by using differential equations in m_1 and m_2. This has two advantages. Firstly, it enables us to avoid having to deal with polylogarithms of horrible, unnecessarily complicated arguments in the intermediate steps of the calculation. Secondly, it provides a nice way of extracting infrared divergences. Things can be arranged in such a way that all infrared divergences are expressed in terms of one-scale integrals. An example of how this works for a six-propagator, N-like integral is explained in detail in [1].

Here, we will sketch the procedure for the M-like integral

$$I_1(m_1, m_2) = \iint d^dk\, d^dl\, \frac{1}{P_1P_2P_3P_4P_7P_8} \tag{3}$$

shown in figure 2. It has infrared divergences coming from two regions, $l \to 0$, and $k, l \to 0$, which manifest themselves as a $1/\varepsilon^2$ pole in dimensional regularization. First, using the identity $m_1P_4 - m_2P_3 = (m_1 - m_2)P_8$ (this follows from the fact that p_1 and p_2 are proportional to each other), I_1 is

[1] In fact, $\mathrm{Li}_3\left(\frac{m_1-m_2}{m_1+m_2}\right)$ and $\mathrm{Li}_3\left(\frac{m_2-m_1}{m_1+m_2}\right)$ do not appear, though it does not follow from the simple arguments we have given here.

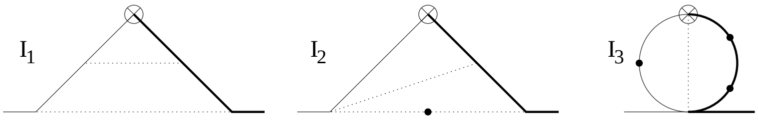

Fig. 2. The scalar integrals $I_1 \dots I_3$. The momentum p_1 enters from the left, $p_2 - p_1$ enters at the vertex marked $\otimes$, p_2 leaves at the right. The thin (thick) lines symbolize quark propagators with mass m_1 (m_2). The dotted lines are massless propagators. The heavy dots mean the corresponding propagator is squared (or cubed).

split into two partial fractions. One is the integral I_2 shown in figure 2 and the other its mirror image. Then, differentiating three times, we get

$$\frac{\partial}{\partial m_2} m_2^2 \frac{\partial}{\partial m_2} m_2 \frac{\partial}{\partial m_1} (m_1 - m_2) I_2 = 2 m_2 I_3 \,. \tag{4}$$

I_3 is a very simple, convergent integral:

$$I_3 = \frac{\pi^4}{2 m_1 m_2^2 (m_1 - m_2)^2} \left\{ m_1 - m_2 - m_1 \log\left(\frac{m_1}{m_2}\right) \right\} . \tag{5}$$

In order to find I_2, we must integrate the right hand side of (5) three times from some suitable initial point with respect to the masses. These integrations are easy to program because they all have a similar structure. If the equal mass point $m_1 = m_2$ is taken as the initial point, then the two infrared divergent integration constants required are one-scale integrals.

3.

Many cancellations occur when the diagrams of fig. 1 are combined. After renormalization, the infrared divergences that are present in individual diagrams all disappear. This is related to that fact that the cross section for gluon Bremsstrahlung vanishes at zero recoil, so that there cannot be any cancellations of infrared divergences between real and virtual graphs. Other cancellations can be understood from the symmetry of the process: provided the renormalization is performed in a symmetric way, $\eta_{V,A}(m_1, m_2) = \eta_{V,A}(m_2, m_1)$. The trilogarithms $\mathrm{Li}_3\left(\frac{m_1 - m_2}{2m_1}\right)$ and $\mathrm{Li}_3\left(\frac{m_2 - m_1}{2m_2}\right)$, which are present in the contributions of diagrams c_1, c_2 and c_3, cancel out in the sum $c_1 + c_2 + c_3$. As a consequence of all this, the final expressions for $\eta_{V,A}$ are relatively short, involving only the following functions,

$$\ell \;=\; \log\left(\frac{1+u}{1-u}\right) , \tag{6}$$

$$\mathcal{L}_1 = \mathrm{Li}_2(u) - \mathrm{Li}_2(-u)\,, \tag{7}$$

$$\mathcal{L}_2 = \mathrm{Li}_2\left(\frac{2u}{u+1}\right) + \frac{1}{4}\ell^2\,, \tag{8}$$

$$\mathcal{L}_3 = \mathrm{Li}_3\left(\frac{2u}{u+1}\right) + \mathrm{Li}_3\left(\frac{2u}{u-1}\right) + \frac{2}{3}\ell\,\mathcal{L}_2\,, \tag{9}$$

$$\begin{aligned}\mathcal{L}_4 = {} & \mathrm{Li}_3\left(\frac{4u}{(u+1)^2}\right) + \mathrm{Li}_3\left(\frac{-4u}{(u-1)^2}\right) + \frac{16}{3}\ell\,\mathcal{L}_2 \\ & -2\zeta(2)\ \left(\log(1+u) + \log(1-u)\right) - \frac{8}{3}\ell\,\mathcal{L}_1\,,\end{aligned} \tag{10}$$

with coefficients that are rational functions of $u = (m_1 - m_2)/(m_1 + m_2)$. Note that ℓ, $\mathcal{L}_1$ and $\mathcal{L}_2$ are odd functions of u, while $\mathcal{L}_3$ and $\mathcal{L}_4$ are even. Clearly, it is possible to expand $\eta_{V,A}$ in a Taylor series in u^2 to any order. The series converges for $|u^2| < 1$. If one substitutes the actual numerical values of m_b and m_c, $u^2 \approx 0.29$, the convergence is quite good, and only a few terms are needed to get a reasonable precision. Another option is to use ℓ^2/π^2 as the expansion parameter.

We have checked that our results are equivalent to the formulae presented in [8], and we agree with their numerical conclusions.

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