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THE HADRONIC HIGGS DECAY AND THE CONNECTION TO THE DECOUPLING RELATIONS IN QCD*

Matthias Steinhauser

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 Munich, Germany

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The decoupling relations for the strong coupling constant, α_s, and the light quark masses are considered in the framework of perturbative QCD. A low-energy theorem is derived which connects the decoupling constants to the coefficient functions of the effective Lagrangian responsible for the decay of a Higgs boson in the intermediate-mass range. The evaluation of the imaginary part of the correlators formed by the corresponding operators completes the calculation of the decay of the Higgs boson into hadrons.

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1. Introduction

The Higgs boson is the only not yet detected particle of the Standard model. From the direct search at LEP up to now only a lower limit of $M_H \gtrsim 90$ GeV could be determined. Precision measurements in combination with higher order calculations could set indirect limits of $M_H = 66^{+74}_{-39}$ GeV with an upper bound of 215 GeV at 95% C.L. [1]. In this paper we will restrict ourselves to a Higgs boson in the intermediate-mass range which means that $M_H \lesssim 2M_W$.

It is well known that the Appelquist-Carazzone decoupling theorem [2] does not in general apply to quantities that do not represent physical observables. This means that, *i.e.*, heavy quarks with mass m_h do not automatically decouple even if we have $m_h^2 \gg \mu^2$ where μ is the energy scale under consideration. The standard way out is to use the language of effective theory and do the decoupling "by hand". In Sec. 2 this will be demonstrated in the case of QCD. Afterwards, in Sec. 3, an effective Lagrangian for the

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interaction of an intermediate-mass Higgs boson to quarks and gluons is considered and the connection to the decoupling relation is established. Finally we present numerical values for the decay of the Higgs boson into quarks up to order α_s^3 and into gluons up to order α_s^4.

2. Decoupling constants

In this section the decoupling constants are computed. They relate the strong coupling α_s and, respectively, the light quark masses in the full theory to the corresponding quantities in the effective theory where the heavy quark with mass m_h is integrated out. To set up the notation let us introduce the renormalization constants and write down the relations between the bare and the renormalized quantities:

$$\begin{aligned} g_s^0 &= \mu^\varepsilon Z_g g_s\,, \quad m_q^0 \;=\; Z_m m_q\,, \quad \xi^0 - 1 \;=\; Z_3(\xi - 1)\,, \\ \psi_q^0 &= \sqrt{Z_2}\psi_q\,, \; G_\mu^{0,a} \;=\; \sqrt{Z_3}G_\mu^a\,, \quad c^{0,a} \;=\; \sqrt{\tilde{Z}_3}c^a\,, \end{aligned} \tag{1}$$

where $g_s = \sqrt{4\pi\alpha_s}$ is the QCD gauge coupling, μ is the renormalization scale, $D = 4 - 2\varepsilon$ is the dimensionality of space time, ψ_q is a quark field with mass m_q, G_μ^a is the gluon field, and c^a is the Faddeev-Popov-ghost field. For simplicity, we do not display the colour indices of the quark fields. The gauge parameter, ξ, is defined through the gluon propagator in lowest order,

$$\frac{i}{q^2 + i\epsilon}\left(-g^{\mu\nu} + \xi\frac{q^\mu q^\nu}{q^2}\right).$$

The index '0' marks bare quantities.

The bare decoupling constants are defined as follows:

$$\begin{aligned} g_s^{0\prime} &= \zeta_g^0 g_s^0\,, \quad m_q^{0\prime} = \zeta_m^0 m_q^0\,, \quad \xi^{0\prime} - 1 = \zeta_3^0(\xi^0 - 1)\,, \\ \psi_q^{0\prime} &= \sqrt{\zeta_2^0}\psi_q^0\,, \quad G_\mu^{0\prime,a} = \sqrt{\zeta_3^0}G_\mu^{0,a}\,, \quad c^{0\prime,a} = \sqrt{\tilde{\zeta}_3^0}c^{0,a}\,, \end{aligned} \tag{2}$$

where the prime refers to the effective theory.

In a first step formulae for the bare decoupling constants are derived. With the help of Eqs (1) they are then transformed into renormalized and thus finite ones. As an example let us consider ζ_2^0 which can be derived from the propagator of a massless quark:

$$\begin{aligned} &-\frac{1}{\not p\left[1 + \Sigma_V^0(p^2)\right]} = i\int dx e^{ip\cdot x}\left\langle T\psi_q^0(x)\bar{\psi}_q^0(0)\right\rangle \\ &= \frac{i}{\zeta_2^0}\int dx e^{ip\cdot x}\left\langle T\psi_q^{0\prime}(x)\bar{\psi}_q^{0\prime}(0)\right\rangle \;=\; -\frac{1}{\zeta_2^0}\,\frac{1}{\not p\left[1 + \Sigma_V^{0\prime}(p^2)\right]}. \end{aligned} \tag{3}$$

Note that $\Sigma_V^{0\prime}(p^2)$ only depends on the light degrees of freedom and ζ_2^0 itself is independent of the external momentum p. As we are interested in $m_h \to \infty$ we may nullify p [3]. Within dimensional regularization the quantity $\Sigma_V^{0\prime}(0)$ vanishes and thus we end up with tadpole integrals where the scale is determined by the mass of the heavy quark. The formula for ζ_2^0 finally reads:

$$\zeta_2^0 \; = \; 1 + \Sigma_V^{0h}(0). \tag{4}$$

In complete analogy similar formulae can be derived for the other decoupling relations:

$$\begin{aligned} \zeta_m^0 \; &= \; \frac{1 - \Sigma_S^{0h}(0)}{1 + \Sigma_V^{0h}(0)}, \quad \zeta_3^0 \; = \; 1 + \Pi_G^{0h}(0), \quad \tilde{\zeta}_3^0 \; = \; 1 + \Pi_c^{0h}(0), \\ \zeta_g^0 \; &= \; \frac{\tilde{\zeta}_1^0}{\tilde{\zeta}_3^0 \sqrt{\zeta_3^0}}, \; \tilde{\zeta}_1^0 \; = \; 1 + \Gamma_{G\bar{c}c}^{0h}(0,0). \end{aligned} \tag{5}$$

where $\Sigma_V(p^2)$ and $\Sigma_S(p^2)$ are the vector and scalar components of the light-quark self-energy, defined through $\Sigma(p) = \not{p}\Sigma_V(p^2) + m_q \Sigma_S(p^2)$, $\Pi_G(p^2)$ and $\Pi_c(p^2)$ are the gluon and ghost vacuum polarizations, respectively, and $\Gamma_{G\bar{c}c}^{0h}(p,k)$ is determined through the one-particle-irreducible part of the amputated $G\bar{c}c$ Green function.

In Fig. 1 some sample diagrams contributing to the two-, respectively, three-point functions are pictured. At this point we refrain form listing the results explicitly. They can be found in [4, 5]. Instead, we want to discuss an application of the newly available three-loop terms for ζ_g and ζ_m. One of the numerous experimental values for $\alpha_s^{(5)}(M_Z)$ results from the measurement of the hadronic τ decay. Once it is possible to extract $\alpha_s^{(3)}(M_\tau)$ at the $\mathcal{O}(\alpha_s^4)$ precision it is necessary to use the four-loop β function [6] in order to run the coupling constant from $\sqrt{s} = M_\tau$ to some other scale. It is furthermore necessary to use the three-loop matching relations every time a quark

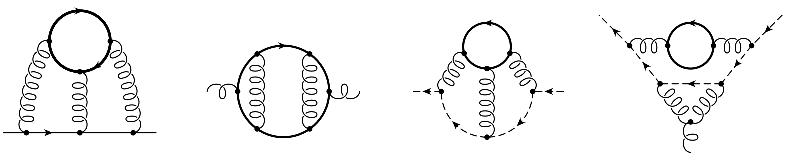

Fig. 1. Typical three-loop diagrams contributing to $\Sigma_V^{0h}(0)$, $\Sigma_S^{0h}(0)$, $\Pi_G^{0h}(0)$, $\Pi_c^{0h}(0)$, and $\Gamma_{G\bar{c}c}^{0h}(0,0)$. Solid, bold-faced, loopy, and dashed lines represent massless quarks q, heavy quarks h, gluons G, and ghosts c, respectively.

threshold is crossed. Let us illustrate the procedure by means of the following example: For a given value of $\alpha_s^{(4)}(M_\tau)$ we use the N-loop evolution with four active flavours in order to get $\alpha_s^{(4)}(\mu^{(5)})$. N-loop running must be accompanied with $N-1$-loop matching which fixes $\alpha_s^{(5)}(\mu^{(5)})$. Finally again the N-loop β function is used for the computation of $\alpha_s^{(5)}(M_Z)$. In Fig. 2, $\alpha_s^{(5)}(M_Z)$ is shown for $N=1,2,3$ and 4 as a function of $\mu^{(5)}$ normalized to M_b which constitutes somehow the natural scale for the matching procedure as then the logarithms in the decoupling constants vanish. In a

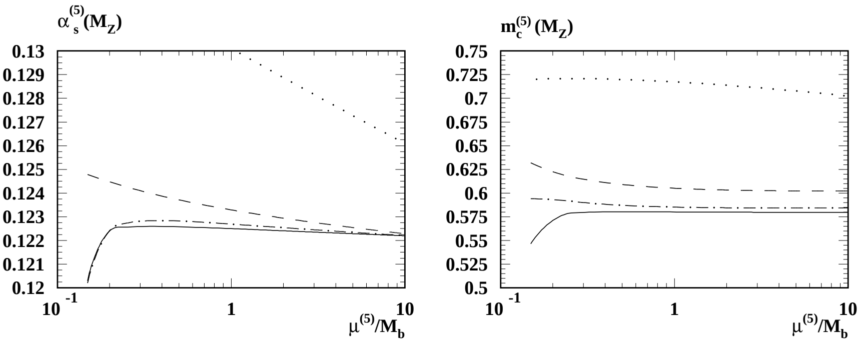

Fig. 2. $\mu^{(5)}$ dependence of $\alpha_s^{(5)}(M_Z)$ and $m_c^{(5)}(M_Z)$ calculated from $\alpha_s^{(4)}(M_\tau) = 0.36$, respectively, $\mu_c = m_c^{(4)}(\mu_c) = 1.2$ GeV. $M_b = 4.7$ GeV and for the plot on the r.h.s. $\alpha_s^{(5)}(M_Z) = 0.118$ has been chosen. The dotted, dashed, dot-dashed and solid curves correspond to one-, two-, three- and four-loop running, respectively.

similar way also the charm quark mass $m_c^{(5)}(M_Z)$ can be computed from $m_c^{(4)}(\mu_c)$ where $\mu_c = m_c^{(4)}(\mu_c)$. The result is also shown in Fig. 2. By general grounds $\alpha_s^{(5)}(M_Z)$ and $m_c^{(5)}(M_Z)$ should be independent of $\mu^{(5)}$ which is varied rather extremely by almost two orders of magnitude. The analysis in getting gradually more stable and the four-loop curves are almost flat for $\mu^{(5)} \gtrsim 1$ GeV.

3. Low-energy theorems for an intermediate-mass Higgs boson

The interaction of the Standard Model Higgs boson with light quarks and gluons is affected by the virtual presence of the top quark. As we are interested in a Higgs boson in the intermediate-mass range it is very promising to construct an effective Lagrangian where the top quark is integrated out. The starting point is the Yukawa Lagrangian of the full theory which generates in the limit $M_t \to \infty$ the effective interactions:

$$\mathcal{L} = -\frac{H^0}{v^0}\sum_{i=1}^{n_f} m_{q_i}^0 \bar{\psi}_{q_i}^0 \psi_{q_i}^0 \quad \stackrel{M_t\to\infty}{\longrightarrow} \quad \mathcal{L}_{\rm eff} = -\frac{H^0}{v^0}\sum_{i=1}^{5} C_i^0 \mathcal{O}_i'. \tag{6}$$

Here, v is the Higgs vacuum expectation value. The operators $\mathcal{O}_i'$ are constructed from the light degrees of freedom. It turns out that only two of them contribute to the physical process [7, 8]:

$$\mathcal{O}_1' \;=\; \left(G_{\mu\nu}^{0\prime,a}\right)^2, \quad \mathcal{O}_2' \;=\; \sum_{i=1}^{n_l} m_{q_i}^{0\prime} \bar{\psi}_{q_i}^{0\prime} \psi_{q_i}^{0\prime}, \tag{7}$$

where $G_{\mu\nu}^a$ is the colour field strength. The residual dependence on the top quark is contained in the coefficient function C_i^0. For the physical applications the renormalized Lagrangian is needed. It reads:

$$\mathcal{L}_{\rm eff}^{\rm phys} \;=\; -\frac{H}{v}\left(C_1\left[\mathcal{O}_1'\right] + C_2\left[\mathcal{O}_2'\right]\right), \tag{8}$$

with

$$\begin{aligned}
\left[\mathcal{O}_1'\right] &= \left[1 + 2\left(\frac{\alpha_s'\partial}{\partial\alpha_s'}\ln Z_g'\right)\right]\mathcal{O}_1' - 4\left(\frac{\alpha_s'\partial}{\partial\alpha_s'}\ln Z_m'\right)\mathcal{O}_2', \quad \left[\mathcal{O}_2'\right] \;=\; \mathcal{O}_2', \\
C_1 &= \frac{1}{1 + 2(\alpha_s'\partial/\partial\alpha_s')\ln Z_g'} C_1^0, \quad C_2 \;=\; \frac{4(\alpha_s'\partial/\partial\alpha_s')\ln Z_m'}{1 + 2(\alpha_s'\partial/\partial\alpha_s')\ln Z_g'} C_1^0 + C_2^0.
\end{aligned}$$

In Eq. (8) the factor H^0/v^0 has been replaced by the renormalized one as it doesn't receive QCD corrections.

The derivation of formulae to compute the coefficient functions is very similar to the case of the decoupling relations: One considers a one-particle-irreducible Green function involving next to the Higgs boson a certain number of gluons and light quarks. After performing the limit $m_h \to \infty$ the contributions form the individual operators can be identified. For the computation of the coefficient functions it is again possible to nullify the external momenta which reduces the calculation again to the evaluation of tadpole integrals. Finally one can exploit the fact that the Yukawa coupling of the Higgs boson is proportional to m_h^0 which leads to the following set of equations:

$$\begin{aligned}
\zeta_3^0(-4C_1^0 + 2C_4^0) &= -\frac{1}{2}\partial_h^0 \Pi_G^{0h}(0), \quad \zeta_2^0 C_3^0 \;=\; -\frac{1}{2}\partial_h^0 \Sigma_V^{0h}(0), \\
\zeta_m^0\zeta_2^0(C_2^0 - C_3^0) &= 1 - \Sigma_S^{0h}(0) - \frac{1}{2}\partial_h^0 \Sigma_S^{0h}(0), \quad \tilde{\zeta}_3^0(C_4^0 + C_5^0) \;=\; \frac{1}{2}\partial_h^0 \Pi_c^{0h}(0), \\
\tilde{\zeta}_1^0 C_5^0 &= \frac{1}{2}\partial_h^0 \Gamma_{G\bar{c}c}^{0h}(0,0),
\end{aligned} \tag{9}$$

where $\partial_h^0 = \left[(m_h^0)^2\partial/\partial(m_h^0)^2\right]$. On the r.h.s. of Eqs. (9) the same functions appear as in Eqs. (5). Therefore it is tempting to express the coefficient functions directly in terms of the decoupling relations. We were able to find the following two equations for C_1 and C_2:

$$C_1 \;=\; -\frac{1}{2}\partial_h \ln\zeta_g^2, \quad C_2 \;=\; 1 + 2\partial_h \ln\zeta_m. \tag{10}$$

These equations have the nice feature that according to the logarithmic derivative even the $\mathcal{O}(\alpha_s^4)$ contributions to C_1 and C_2 can be computed as it is possible to reconstruct the $\ln m_h$ terms of the four-loop decoupling relations using common renormalization group techniques. It is also possible to relate C_1 and C_2 directly to the β [6] and γ_m [9] functions. The corresponding relations can be found in [5].

4. Higher order QCD corrections to the hadronic Higgs decay

The missing pieces for the computation of the hadronic decay rate are the imaginary parts of the correlators $\langle[\mathcal{O}_i'][\mathcal{O}_j']\rangle$. Typical Feynman diagrams which have to be considered are pictured in Fig. 3. It turns out that at three-loop level it is more convenient to compute in a first step the real part and extract the imaginary part afterwards [10, 12].

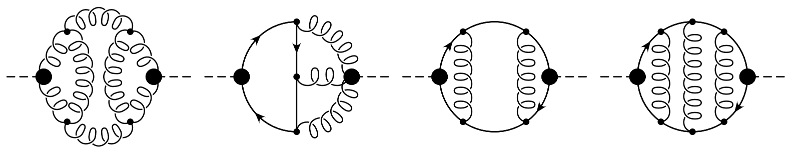

Fig. 3. Typical Feynman diagrams contributing to the correlators $\langle\mathcal{O}_1'\mathcal{O}_1'\rangle$, $\langle\mathcal{O}_1'\mathcal{O}_2'\rangle$ and $\langle\mathcal{O}_2'\mathcal{O}_2'\rangle$. In the latter case also the imaginary part of the four-loop correlator is available. Looped, solid, and dashed lines represent gluons, light quarks, and H bosons, respectively. Solid circles represent insertions of $\mathcal{O}_1'$, respectively, $\mathcal{O}_2'$.

Note, that C_1 starts at order α_s. Hence the combination $C_1^2\mathrm{Im}\langle\mathcal{O}_1'\mathcal{O}_1'\rangle$ governing the gluonic decay rate of the Higgs boson is available up to $\mathcal{O}(\alpha_s^4)$. Normalized to the Born rate it reads in numerical form ($\mu = M_H$):

$$\frac{\Gamma(H\to gg)}{\Gamma^{\mathrm{Born}}(H\to gg)} \;\approx\; 1 + 17.917\,\frac{\alpha_s^{(5)}(M_H)}{\pi} + \left(\frac{\alpha_s^{(5)}(M_H)}{\pi}\right)^2\left(156.808 - 5.708\,\ln\frac{M_t^2}{M_H^2}\right) \approx 1 + 0.66 + 0.21,$$

with $\Gamma^{\mathrm{Born}}(H \to gg) = G_F M_H^3/36\pi\sqrt{2} \times (\alpha_s^{(5)}(M_H)/\pi)^2$. In the last line $M_t = 175$ GeV and $M_H = 100$ GeV has been chosen. The analytical expressions can be found in [10]. We observe that the new $\mathcal{O}(\alpha_s^2)$ term further increases the well-known $\mathcal{O}(\alpha_s)$ enhancement [7,11] by about one third. If we assume that this trend continues to $\mathcal{O}(\alpha_s^3)$ and beyond, then Eq. (11) may already be regarded as a useful approximation to the full result. Inclusion of the new $\mathcal{O}(\alpha_s^2)$ correction leads to an increase of the Higgs-boson hadronic width by an amount of order 1%.

Concerning the decay rate into quarks we restrict ourselves to the case of bottom quarks. Inserting numerical values into the coefficient functions C_1 and C_2 and the correlators $\mathrm{Im}\langle \mathcal{O}_1' \mathcal{O}_2' \rangle$ [12] and $\mathrm{Im}\langle \mathcal{O}_2' \mathcal{O}_2' \rangle$ [12,13] leads to:

$$\begin{aligned} \Gamma(H \to b\bar{b}) = A_{b\bar{b}} \Bigg\{ & 1 + 5.667\, a_H^{(5)} + 29.147 \left(a_H^{(5)}\right)^2 + 41.758 \left(a_H^{(5)}\right)^3 \\ & + \left(a_H^{(5)}\right)^2 [3.111 - 0.667\, L_t] + \left(a_H^{(5)}\right)^3 \left[50.474 - 8.167\, L_t - 1.278\, L_t^2\right] \Bigg\}, \end{aligned} \tag{11}$$

with $A_{b\bar{b}} = 3G_F M_H m_b^2/4\pi\sqrt{2}$, $L_t = \ln M_H^2/M_t^2$ and $a_H^{(5)} = \alpha_s^{(5)}(M_H)/\pi$. In Eq. (11) electromagnetic and electroweak corrections have been neglected. Also mass correction terms and second order QCD corrections which are suppressed by the top quark mass are not displayed. One observes from Eq. (11) that the top-induced corrections at $\mathcal{O}(\alpha_s^3)$ (second line) are of the same order of magnitude than the "massless" corrections (first line).

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