DIRECT DETERMINATIONS OF THE πNN COUPLING CONSTANTS *

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A novel extrapolation method has been used to deduce directly the charged πNN coupling constant from backward np differential scattering cross sections. The extracted value, $g_c^2 = 14.52(0.26)$ is higher than the indirectly deduced values obtained in nucleon-nucleon energy-dependent partial-wave analyses. Our preliminary direct value from a reanalysis of the GMO sum-rule points to an intermediate value of g_c^2 about 13.97(30).

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1. Introduction

The πNN coupling is a basic quantity both in nuclear physics and in particle physics. In nuclei it sets the scale of the interaction together with the pion mass and it the fundamental constant. In particle physics it is of great importance for one of the most important tests of chiral symmetry [1]. Its experimental error is the main obstacle in the accurate discussion of the corrections to the Goldberger–Treiman relation as predicted from chiral symmetry breaking. With the latest value for the axial coupling constant, $g_A = 1.266(4)$ [2], this relation would lead to $g_{GT}^2(q^2 = 0) = 13.16(16)$, if it were exact, which is not expected. The uncertainty, here of about $\pm 1\%$, comes from the experimental error in g_A and f_π. The value above is takes into account the remeasured value for the neutron life time: until a short time ago the old neutron lifetime gave $g_{GT}^2(q^2 = 0) = 12.81 \pm 0.12$. The

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characteristic correction to the relation can be estimated from the effect of the form factor to be of order $2m_\pi^2/\Lambda^2$, where Λ is a monopole formfactor of order 800 MeV/c. This corresponds to a +6% correction or δg^2 about +0.8 which gives a coupling constant of about 14. From the precision of this relation we note that an accuracy of the πNN coupling constant of about 1% is desirable.

The present situation is the following. In the 1980's, the πNN coupling constant was believed to be well known. In particular, Koch and Pietarinen [3] determined a value of the charged pion coupling constant, $g_c^2 = 14.28(18)$, from $\pi^\pm p$ scattering data. All this was put in question in the early 1990's when the Nijmegen group [5–7] determined smaller values on the basis of energy-dependent partial-wave analyses (PWA) of nucleon–nucleon (NN) scattering data. They obtained $g_0^2 = 13.47(11)$ and $g_c^2 = 13.58(5)$. Similar values with g^2 about $= 13.7$ have also been found by the Virginia Tech group [8–10] from analysis of both $\pi^\pm N$ and NN data. Using a very similar method in the πp sector Timmermans also finds similar values [12]. A larger value with a larger quoted error has been found by our group using the methods which will be discussed below. The situation is summarized in the Table I.

TABLE I

Some important determinations of the pion–nucleon coupling constant

Source	Year	System	$g^2_{\pi NN}$
Karlsruhe-Helsinki [3]	1980	πp	14.28(18)
Kroll[4]	1980	pp	14.52(40)
Nijmegen [7]	1993	pp	13.58(5)
VPI [9]	1994	pp	13.7
Nijmegen [11]	1997	pp, np	13.48(5)
Timmermans [12]	1997	πp	13.39(14)
VPI [10]	1994	GMO πp	13.75(15)
Uppsala [13]	1995	np charge exchange	14.62(30)
Uppsala[15]	1998	np charge exchange	14.52(26)

It has become quite clear that the determinations in the 1980's were too optimistic about the systematic errors originating in experimental uncertainties. The present global partial wave analyses based on the compounded data of many experiments treated as statistically independent, but with a selection of data, yield both from NN data and πN data a coupling constant of order 13.5 with a precision of 1/2 to 1% and in fact quoted to a precision of even 1/3% in some cases. The goal of determining the πNN coupling to 1% precision appears to have been achieved. Why do we bother to attempt a determination that cannot match the accuracy of better than 1%? The reason is methodological. The high precision has been achieved by removing obviously deviant data and some more in addition and then analyzing the rest purely statistically. The resulting value is very indirect and based on a large number of experiments, each with inevitable systematic errors. There is no way known to us to determine a true systematic error in this procedure. It may be that the value which is obtained is the correct one, but the uncertainty is unknown. This is why the usual procedure in any branch of physics is to determine fundamental constants directly from highly controlled, dedicated and improvable experiments using transparent methods of analysis subject to constructive criticism. The precision will be less, at least initially, but the approach is then controllable. There should be no disagreement in principle on the desirability of the direct approach.

There are not many possibilities to determine the pion-nucleon coupling constant directly. In the πN sector the best prospect is the Goldberger-Miyazawa-Oehme (GMO) sum-rule, while in the NN sector we favor the np charge exchange reaction in the backward direction. It is in this spirit we have developed a method of analyzing accurate single energy experiments of backward np charge exchange to be described below and have collaborated closely with a dedicated experiment in this area [13, 15].

Our work has given rise to an animated debate with the Nijmegen group. In their earlier analysis [7] the pion-nucleon coupling constant appeared rather insensitive to such backward *np* cross section within the approach used by the Nijmegen group. In our work [13, 15], we demonstrate the opposite using explicit analysis of extensive sets of 'pseudodata' built from models in common use. The experimental normalization of the cross section is crucial and this has been a well known problem in the past. Most energy-dependent PWA's and in particular the Nijmegen one have therefore chosen to let the normalization of data float more or less freely. The sensitivity to the *np* cross section is then lost, and the coupling constant depends diffusely on many observables. There is strong evidence that precision data of the backward *np* cross section is one of the best places in the NN sector to determine the charged coupling constant directly. We now describe the extrapolation procedure and its results then give preliminary, independent results on the coupling constant from our ongoing re evaluation of GMO sum rule.

2. Neutron–proton charge exchange data analysis

It is very striking that the *np* unpolarized charge exchange cross sections in a very large range of energies from about 100 MeV to several GeV, have similar shape and normalization (in the laboratory system). These data contain essentially the same physical information as far as the extrapolation to the pion pole is concerned. Here we shall concentrate our analysis to new precise data at 162 MeV [15] consisting of an extension from $\theta_{\rm CM} = 72°$ to $120°$ of our previous backward measurement [13]. This allows to improve the absolute normalization to about ±2%. A study of the present *np* data base [18], shows that there are two main families with respect to the angular shape. The first one is dominated by the Bonner *et al.* data [19], which have a flattish angular distribution at backward angles. The second one, which includes our measurements and the Hürster *et al.* [20] data, have a steeper angular shape. The total c.m. cross sections can be defined in terms of the five amplitudes a, b, c, d, e as

$$\begin{aligned}\frac{{\rm d}\sigma}{{\rm d}\Omega}(q^2) &= \frac{1}{2}(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2)\\ &= \frac{1}{2}\left[\frac{1}{2}(|a+c|^2+|a-c|^2)+\frac{1}{2}(|b+d|^2+|b-d|^2)+|e|^2\right],\end{aligned}$$

where q^2 is the squared momentum transfer from the neutron to the proton.

In order to understand the qualitative contributions of pion exchange, we have chosen the regularized pion Born amplitudes of Ref. [21] with the r-space δ-function subtracted. This ensures a non-zero cross section at 180°. The different components for this Born pion terms, and for the more realistic Paris potential, are then displayed in Figs 1(a) and 1(b), respectively. The combination $|b-d|^2$, which contains the entire pion pole term, is for the Paris potential remarkably close to that of the Born term, particularly at small

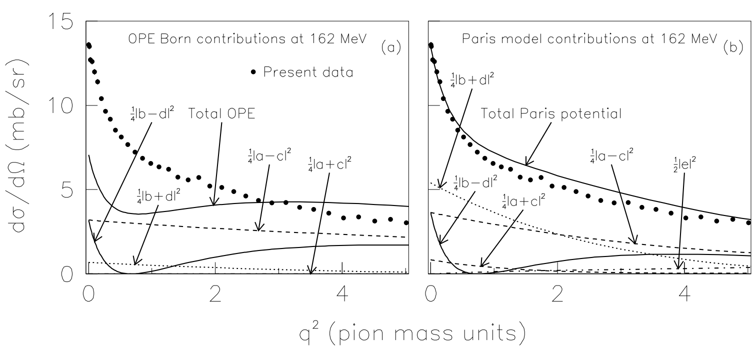

Fig. 1. Contributions to the *np* cross section at 162 MeV of combinations of the amplitudes a, b, c, d, e of Eq. 1. (a) for the regularized pion Born terms (b) for the Paris potential model.

q^2. The term $|a+c|^2$ is very small in both cases and the more important $|a-c|^2$ terms are again very similar. The simple structure of the term which contains the pion pole gives considerable confidence that the extrapolation can be achieved realistically.

3. Extrapolation to the pion pole

The basic step to extrapolate to the pion pole is to construct a smooth physical function, the Chew function [22],

$$y(x) = \frac{sx^2}{m_\pi^4 g_R^4}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(x) = \sum_{\mathrm{i=0}}^{\mathrm{n-1}} a_i x^i. \tag{1}$$

Here s is the square of the total energy and $x = q^2 + m_\pi^2$. At the pion pole $x = 0$, the Chew function gives $y(0) \equiv a_0 \equiv g^4/g_R^4$, g being the pseudoscalar coupling constant related to the pseudovector coupling by $f = (m_{\pi^+}/2M_p)g$. The quantity g_R^2 is a reference scale for the coupling chosen for convenience. The model-independent extrapolation requires accurate data with absolute normalization of the differential cross section. If the differential cross section is incorrectly normalized by a factor N, the extrapolation determines $\sqrt{N}g^2$.

The Difference Method, which we introduced to obtain a substantial improvement [13], is based on the Chew function, but it recognizes that a major part in the cross section behaviour is described by models with exactly known values for the coupling constant. It applies the Chew method to the *difference* between the function $y(x)$ obtained from a model and from the experimental data, *i.e.*,

$$y_M(x) - y_{\mathrm{exp}}(x) = \sum_{\mathrm{i=0}}^{\mathrm{n-1}} d_i x^i \tag{2}$$

with g_R of Eq. 1 replaced by the model value g_M. At the pole $y_M(0) - y_{\mathrm{exp}}(0) \equiv d_0 \equiv (g_M^4 - g^4)/g_M^4$. This should diminish systematic extrapolation errors and remove a substantial part of the irrelevant information at large momentum transfers.

In our work we have explicitly shown, using 'pseudodata' generated from models in common use including the Nijmegen potential [17], that we can reproduce the input coupling constants of the models to a precision less than 1%. We have grouped the data into a "reduced range", $0 < q^2 < 4\ m_\pi^2$ with 31 data points and a "full range", $0 < q^2 < 10.1\ m_\pi^2$ with 54 data points. The reduced range is the range of the data available for the analysis in our previous work [13]. This allows to check the sensitivity and stability of the extrapolation to a particular cut in momentum transfer and to verify that it is the small q^2 region that carries most of the pion pole information.

The Difference Method requires only a few terms in the polynomial expansion in favorable cases, and this gives a small, statistical extrapolation error. The similarity between the angular distributions from models and the experimental data is exploited, particularly for large q^2. This incorporates substantial additional physical information without introducing any model dependence. We apply the method using three comparison models: the Nijmegen potential [17], the Nijmegen energy-dependent PWA NI93 [23] as well as the Virginia SM95 energy-dependent PWA [24, 25]. The result, for this last case, is shown in Fig. 2 for the reduced and full ranges of data. In all cases the extrapolation to the pole can be made easily and already a visual extrapolation gives a good result. The polynomial fits cause no problem as long as the data are not overparametrized. The resulting $g^2 = 14.52 \pm 0.26$ is consistent with our previous finding [13].

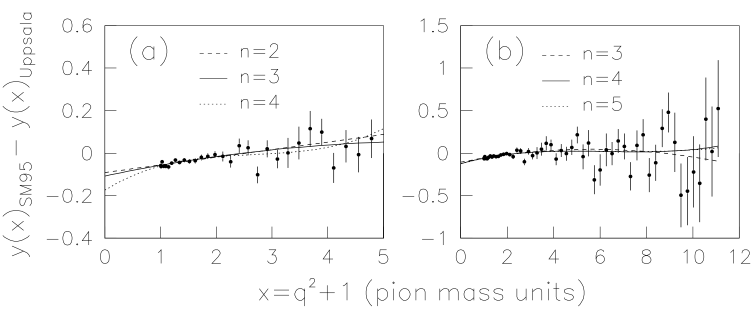

Fig. 2. Extrapolations of the Chew function $y(q^2)$ to the pion pole at 162 MeV with the Difference Method using PWA SM95 as comparison model, different order of polynomials and different intervals in q^2. The left panel uses the reduced range $0 < q^2 < 4\ m_\pi^2$; the right panel uses the full range $0 < q^2 < 10.1\ m_\pi^2$.

Subsequent to our first publication [13] Arndt *et al.* [25] subjected a major part of the np charge exchange cross section data to an analysis using the Difference Method at energies from 0.1 to 1 GeV. They found an average value 13.75 using SM95 as comparison model. Their individual results show a considerable scatter of approximately $\pm 10\%$. This appears to come from the quality of the data. In particular, the deduced g^2 shows systematic trends with energy as can be seen in Fig. 3 for the Bonner data leading to an increase of g^2 with energy. Note that for energies above 400 MeV the slope of the data at large angle is as steep as that of the Uppsala data.

There is a spread of 7 % for the value of the πNN coupling constant between our determination and the one PWA value. If one or the other is adopted has important consequences on our present understanding of QCD. Here we have shown that using the most accurate extrapolation method, the Difference Method, on high precision np differential cross section measure-

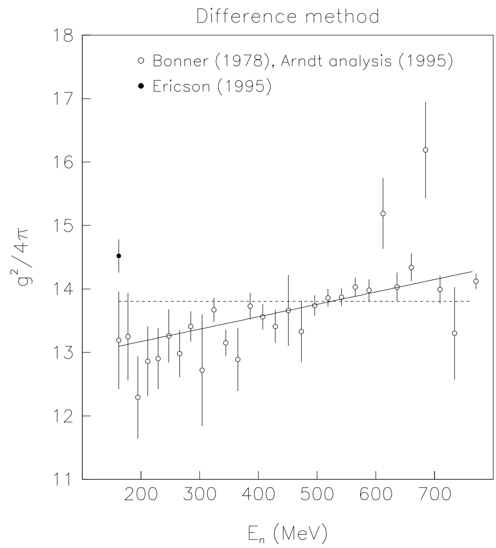

Fig. 3. g^2 values obtained [25] with the Difference method applied to the Bonner data [19] using the PWA SM95 as comparison model. The dotted line is the average g^2 when applied to many data from .1 to 1 GeV, the solid line a linear fit to the Bonner's g^2. The systematic variation with energy indicates systematic uncertainties in these experiments.

ments at 162 MeV in the angular range 72°–180° one can obtain a precise value of this coupling, namely $\sqrt{N}g^2 = 14.52 \pm 0.13$ with a systematic error of about ± 0.15 and a normalization uncertainty of ± 0.17. We have no difficulty in reproducing the input coupling constants of models using equivalent pseudo-data. The practical usefulness of the method, its precision and its relative insensitivity to systematics appear to be in hand without serious problems. The data were normalized using the total *np* cross section, which is one of the most accurately known cross sections in nuclear physics, together with a novel approach, in which the differential cross section measurement was considered as a simultaneous measurement of a fraction of the total cross section. In the angular region 150°–180°, our data are steeper than those of the large data set of Bonner *et al.* [19] below about 400 MeV. This steeper behaviour, leading to a high value of g^2, should be confirmed and a dedicated *np* charge exchange precision experiment at 200 MeV with a tagged neutron, to allow an absolute measurement, is going to be performed at IUCF.

In view of this situation we have asked ourselves, whether a larger value is supported by other direct evidence. An obvious place to investigate is the GMO relation.

4. The GMO relation

The GMO dispersion relation[16] is the following relation between the charged πNN coupling constant, the πp scattering lengths and an integral over the difference between the total cross sections for π^+p and π^-p. Although it is often discussed in terms of isospin in the literature, isospin conservation is not assumed in this form.

$$g_c^2 = 90.0 J^- + 10.35\left(\frac{a_{\pi^-p} - a_{\pi^+p}}{2}\right). \tag{3}$$

Here the integral is given by

$$J^- = \frac{1}{4\pi^2}\int_0^\infty \frac{\sigma^T_{\pi^+p} - \sigma^T_{\pi^-p}}{\omega} dk\,. \tag{4}$$

Everything is in principle measurable to good precision. Still this expression has not been too useful in the past and even now there are a few question marks. The reason was that the scattering lengths for a long time were theoretical constructs from the analysis of scattering experiments at higher energies. Recent splendid experiments at PSI[26] now determine the π^-p and π^-d energy shifts and widths in pionic atoms to very high precision and from that the corresponding scattering lengths follow accurately. Electromagnetic corrections are under excellent control. We (B. Loiseau, T. Ericson and A.W. Thomas) have critically examined the situation with careful attention to errors. In particular, we have examined the accuracy of the constraints due to pion-deuteron data.

In order to get a robust evaluation we write the relation as

$$g_c^2 = 90.0 J^- + 10.35 a_{\pi^-p} - 10.35\left(\frac{a_{\pi^-p} + a_{\pi^+p}}{2}\right). \tag{5}$$

or, numerically,

$$g_c^2 = 4.85(22) + 9.12(9) - 10.35\left(\frac{a_{\pi^-p} + a_{\pi^+p}}{2}\right). \tag{6}$$

Here the first two terms add up to 13.97(24), while the last term is a small quantity which we can evaluate using the deuteron scattering length. Uncertainties in this term will not have a major impact on the result which is stable. We arrive at the following preliminary conclusions.

$$g_c^2(GMO) = 13.98(29?)\,.$$

The magnitude of contributions to the correction term are seen as follows

TABLE II

Magnitude of terms in $-10.35\left(\frac{a_{\pi^- p}+a_{\pi^+ p}}{2}\right)$

Source	$\delta g^2_{\pi NN}$
Impulse approximation	
$-10.35 a_{\pi^- d}$	$+1.34(11)$
double s wave scattering	-1.30
p wave single scattering	$+0.25$
s-p wave double scattering[27]	
(probably spurious)	-0.022
Dispersion correction [28]	$+0.27(9)$
Net correction (no s-p interference)	$+0.04(14)$

These results requires a correct analysis of the deuteron scattering length only to about 12–15% precision and the uncertainties do not come from the deuteron. Instead, the largest uncertainty comes from the dispersive integral, although it only represents 30% of the sum rule and the cross sections are rather accurately known to nearly 10 GeV. Present estimates suggest a 5% systematic uncertainty in the integral, mainly associated with a high energy extrapolation. The result is intermediate between the Difference Method and the PWA ones and consistent with the former.

5. Conclusion

We have two direct determinations of the coupling constant.

$g_c^2 = 14.52 \pm 0.13 \pm 0.15 \pm 0.17 = 14.52(26)$ (Difference Method)

$g_c^2 = 13.98 \pm 0.29$? (GMO relation: preliminary, error probably overestimated)

Both of these approaches can be improved. The Difference Method would profit mainly from an improved absolute normalization and a precision of about 1% appears possible. In the GMO relation the large systematic error in the dispersion integral can certainly be reduced. Although the multiple scattering description and dispersive corrections in the deuteron presently give no major limitation to the precision both are improvable. An ultimate precision of 1/2 to 1 % in the value from the GMO relation seems within reach.

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