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SYMMETRIES IN LOW ENERGY PION PHYSICS AND THE πNN COUPLING CONSTANT*

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We use a new value for the s-wave matrix element at threshold for the $pp \to d\pi^+$ reaction to determine the isoscalar πN scattering length. This is done by invoking symmetries and three ratios. Two of the ratios are measured and the third one is obtained from a new calculation. This leads to a new value for the πNN coupling constant $f^2/4\pi = 0.0760 \pm 0.0011$.

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The s-wave matrix element at threshold α_0 for the reaction $pp \to d\pi^+$ can be related to the πN isovector scattering length b_1 through a chain of symmetries and ratios given in Fig. 1. The cross section $\sigma(pp \to d\pi^+)$ can be transformed into $\sigma(\pi^+ d \to 2p)$ by making use of time reversal invariance (TRI). From this one has $\sigma(\pi^- d \to 2n)$ by applying charge symmetry (CSY). The cross section is converted into a rate $w(\pi^- d \to 2n) \propto \lim_{k\to 0} \sigma(\pi^- d \to 2n)/k$ by extrapolation to zero energy EZE). Then one has to apply three ratios: $S = w(\pi^- d \to 2n)/w(\pi^- d \to 2n\gamma)$, $T = w(\pi^- d \to 2n\gamma)/\ w(\pi^- p \to n\gamma)$, and the Panofsky ratio $P = w(\pi^- p \to n\gamma)/w(\pi^- p \to \pi^0 n)$. Then one can go back to a cross section via $w(\pi^- p \to \pi^0 n) \propto \lim_{k\to 0} \sigma(\pi^- p \to \pi^0 n)$. Finally, isospin symmetry yields from this cross section the one for elastic π scattering on the proton. This cross section is determined by the isovector scattering length.

Recently new cross section data for the $pp \to d\pi^+$ reaction close to threshold were published by [1] and [2] making a reliable extraction of the s-wave partial cross section possible. Older data are dominated by the p-wave cross sections due to Δ excitation. The data in the threshold region were fitted by different models as is discussed in [3]. Since one can not a priori distinguish between the different models the arithmetic mean is taken for the amplitudes. The fit labelled with E is closest to these mean values.

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$$
\begin{tikzcd}
 & \sigma(np \to \pi^0 d) \arrow[d, leftrightarrow, "\text{ISY}"] & \\
 & \sigma(pp \to \pi^+ d) \arrow[d, leftrightarrow, "\text{TRI}"] & \\
 & \sigma(\pi^+ d \to pp) \arrow[d, leftrightarrow, "\text{CSY}"] & \\
 & \sigma(\pi^- d \to nn) \arrow[d, leftrightarrow, "\text{EZE}"] & \\
 & w(\pi^- d \to nn) \arrow[d, leftrightarrow, "\text{S}"] & \\
 & w(\pi^- d \to nn\gamma) \arrow[d, leftrightarrow, "\text{T}"] & \\
 & w(\pi^- p \to n\gamma) \arrow[dl, leftrightarrow, "\text{EZE}"'] \arrow[dr, leftrightarrow, "\text{P}"] & \\
\sigma(\pi^- p \to n\gamma) \arrow[d, leftrightarrow, "\text{TRI}"'] & & w(\pi^- p \to \pi^0 n) \arrow[d, leftrightarrow, "\text{EZE}"] \\
\sigma(\gamma n \to \pi^- p) \arrow[d, leftrightarrow, "\text{R}"'] & & \sigma(\pi^- p \to \pi^0 n) \arrow[d, leftrightarrow, "\text{ISY}"] \\
\sigma(\gamma p \to \pi^+ n) & & \sigma(\pi^\pm p \to \pi^\pm p)
\end{tikzcd}
$$

Fig. 1. Chain of reactions coupled by symmetries and ratios R, S, T and P .

In Fig. 2 it is compared with the data. Also shown are separately the s-wave and the p-wave contributions to the total cross sections. These can be compared with calculations which became recently available [5] and show a remarkable degree of agreement for the p-wave. The s-wave part of the calculation overestimates the data slightly. Also shown is the prediction from a recent phase shift analysis [4]. Again there is excellent agreement in the p-wave, the s-wave is below the fit E.

We now make use of the new s-wave pion production matrix element at threshold [3]: $\alpha_0 = 0.230 \pm 0.019$ (mb). The ratios S and P are often measured and we make use of the newest values. The ratio T can be taken from the individually calculated rates as given by [6]. However, if one is only interested in the ratio this can be calculated in an quasi free model (see [3]) yielding $T = 0.78 \pm 0.04$. With these ingredients one gets $b_1 = -(87.3 \pm 4.4) 10^{-3}/m_\pi$. This value is included in Fig. 3. In this figure we compare this value with results from other measurements or analyses. Here we have restricted ourselves to processes with a real charged pion only. It may well be that for neutral pions a different πNN coupling constant exists. Recently Gibbs *et al.* [7] analyzed π-nucleon scattering in terms of a potential in the Klein–Gordon equation. Kovash *et al.* [8] extrapolated their measurements of the radiative pion capture $\pi^- p \to \gamma n$ to threshold and making then also use of the chain in Fig. 1. Elastic pion scattering was analyzed in terms of phase shift analysis with fixed-t dispersion relation in Ref.'s [9–12].

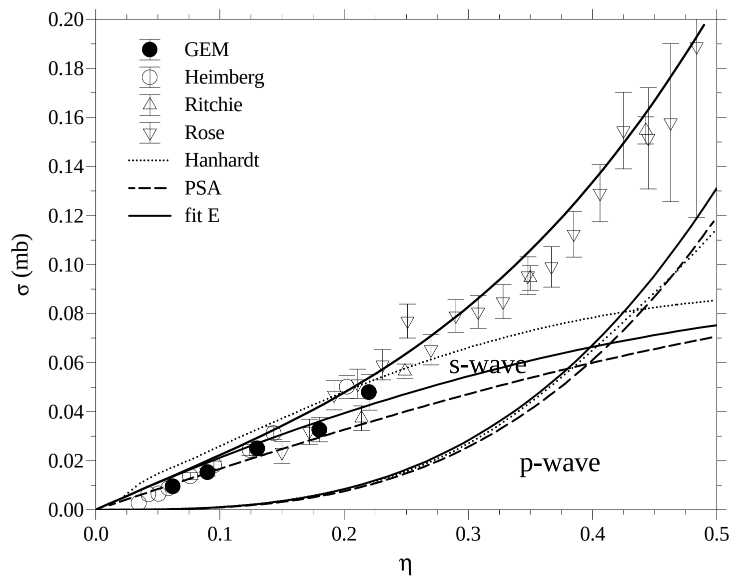

Fig. 2. The data for the $pp \to d\pi^+$ reaction in the threshold region. Also shown is fit E from Ref. [3] as solid curve as well as its s-wave and p-wave contributions. The results from phase shift analysis [4] labelled as PSA are shown as long dashed curves, a calculation [5] labelled as Hanhardt as dotted curves.

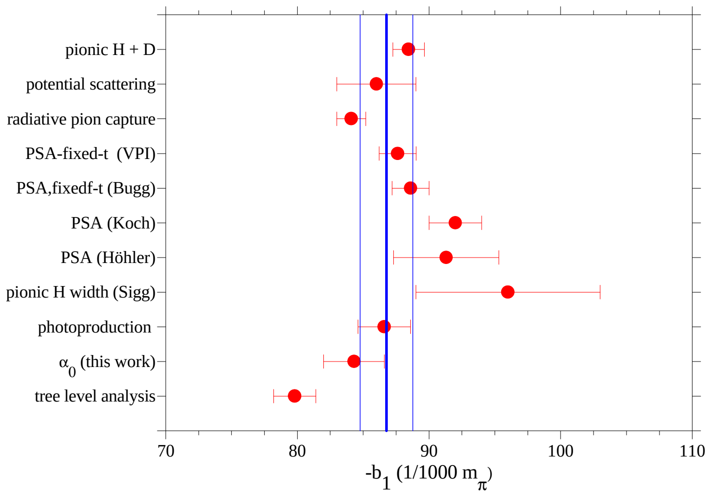

Fig. 3. The isovector πN scattering length for different reactions with a real pion.

Photoproduction data were similarly analysed by Hanstein *et al.* [13]. The isovector scattering length is then again deduced by applying the corresponding chain in Fig. 1. This quantity was also measured via the width of the pionic hydrogen atom [15].

It is interesting to note that the value obtained from pionic hydrogen width is not in agreement with the one from a combined analysis of the shifts of pionic hydrogen and pionic deuterium measured by the same group [15,16]. The origin may that for the pionic deuterium a scaling of the Thomas and Landau theory was necessary [17]. In order to demonstrate this problem we have reanalyzed the data in terms of a theory by Weinberg including three body forces in the πNN-system [18]. Besides the two-body interaction Weinberg calculated three-body interactions based on CHPT-Lagrangians. His final result is

$$a_{\pi d} = \frac{1 + m_\pi/m_N}{1 + m_\pi/m_d}(a_{\pi p} + a_{\pi n}) + \sum_{r=a}^{e} a_r \ , \tag{1}$$

where the first and second term refer to the two- and the three-body contributions, respectively. For the most important graph the result is $a_a = (-26 \pm 1) * 10^{-3}/m_\pi$. This is in agreement with the double scattering term given in Ref. [17]. The remaining terms in the sum give $\sum_{r=b}^{e} a_r = (-0.5 \pm 0.5) * 10^{-3}/m_\pi$. The error we quote is an educated guess. From the experimental value we find by using $(a_{\pi p} + a_{\pi n}) = 2b_0$ a value $b_0 = (0.047 \pm 0.73) * 10^{-3}/m_\pi$. This is compatible with the soft-pion result $b_0 = 0$ but even a small negative value remains possible. On the other hand one can relate b_0 and b_1 via

$$b_0 = a(\pi^- p \to \pi^- p) + b_1. \tag{2}$$

With the experimental result of Sigg *et al.* [15] $a(\pi p \to \pi p) = (88.5 \pm 0.9) * 10^{-3}/m_\pi$ we obtain $b_1 = (-88.6 \pm 1.1) * 10^{-3}/m_\pi$. This is the number with by far the smallest error. We note that similar results for the combined analysis of the pionic hydrogen and pionic deuterium data have been obtained in the framework of a multiple scattering model in Ref. [19], $b_0 \in \{-1.3, 0.6\} * 10^{-3}/m_\pi$ and $-b_1 \in \{87.7, 89.6\} * 10^{-3}/m_\pi$. Beane *et al.* [20] have recently recalculated the three body amplitudes in terms of chiral perturbation theory up to third order in momentum. They applied different nucleon nucleon interactions to deduce the deuteron wave function yielding different results. The arithmetic mean of all these numbers yields $a_a = (-19.6 \pm 0.4) * 10^{-3}/m_\pi$ and $\sum_{r=b}^{e} a_r = (-0.72 \pm 0.11) * 10^{-3}/m_\pi$. Then the isoscalar scattering length becomes larger but changes its sign to $b_0 = (-2.6 \pm 0.6) * 10^{-3}/m_\pi$ and hence $b_1 = (-91.1 \pm 1.1) * 10^{-3}/m_\pi$. This is just out of the error bar larger than the Weinberg result. New measurements under way at PSI will reduce the experimental errors making this type of analysis more precise. For the moment we apply only the Weinberg result. Also shown in Fig. 3 is the result from charged pion scattering analysis in terms of a tree model [21]. His results are $b_0 = (2.8 \pm 1.6) * 10^{-3}/m_\pi$ and $b_1 = (-79.8 \pm 1.6) * 10^{-3}/m_\pi$. Here again b_0 is positive but larger than our result in term of Weinberg's theory. The isovector scattering length is surprisingly small. We now calculate the mean of all the values compiled in Fig. 3. Entries which do not overlap within error bars are excluded. These are in a first step the results of Matsinos and Koch. The latter result is based on rather old and uncertain data. In the second step the uncertain shift of pionic hydrogen is excluded. The final result is

$$b_1 = (-86.7 \pm 2.0) * 10^{-3}/m_\pi . \tag{3}$$

This can be converted into a new value of the πNN coupling constant by making use of the GMO [14] sum rule: $f^2/4\pi = 0.0760 \pm 0.0011$ or $g^2/4\pi = 13.7 \pm 0.2$ which is smaller than the previously accepted value $g^2/4\pi = 14.3$ [22].

We now proceed and use these new values to study the breaking of the Goldberger–Treiman relation

$$g(1 - \Delta) = \frac{g_A * M}{f_\pi} \tag{4}$$

with g_A the Gamow–Teller coupling in neutron decay and f_π the pion decay constant and $1 - \Delta$ the breaking term. With $g_A = 1.266 \pm 0.004$ and $f_\pi = 92.42 \pm 0.26$ MeV we find $\Delta = 2.2 \pm 0.8\%$. This quantity is a measure of chiral symmetry breaking due to finite quark masses. Following the idea of Timmermans [23] to express the breaking factor by a Gaussian form factor interpolating between the pion pole and $t = 0$, *i.e.*, $1 - \Delta = \exp\left(-\frac{m_\pi^2}{2\Lambda^2}\right)$, we get $\Lambda = 662 \pm 26$ MeV.

REFERENCES

[1] M. Drochner *et al.*, *Phys. Rev. Lett.* **77**, 454 (1996).

[2] P. Heimberg *et al.*, *Phys. Rev. Lett.* **77**, 1012 (1996).

[3] H. Machner, *Nucl. Phys.* **A 633**, 341 (1998).

[4] Ch.H. Oh, R.A. Arndt, I.I. Strakowsky, R.L. Workman, *Phys. Rev.* **C56**, 635 (1997) and SAID interactive program.

[5] Ch. Hanhart, Ph. D. thesis, Jülich 1997 and private communications to H. M.

[6] W.R. Gibbs *et al.*, *Phys. Rev.* **C16**, 322 (1977) and ibid. 327.

[7] W.R. Gibbs, Li Ai, W.B. Kaufmann, *Phys. Rev.* **C57**, 784 (1998).

[8] M.A. Kovash *et al.*, πN *Newsletter* **12**, 51 (1997).

[9] G. Höhler, *Pion–Nucleon Scattering, Landoldt-Börnstein,* Vol. I/9b2, ed. H. Schopper, Springer (1983).

[10] R. Koch, *Nucl. Phys.* **A448**, 707 (1986).

[11] F.G. Markopoulou–Kalamara, D.V. Bugg, *Phys. Lett.* **B318** (1993) 565.

[12] R.A. Arndt, I.I. Strakovsky, R.L. Workman, M. Pavan, *Phys. Rev.* **C52**, 2120 (1995).

[13] O. Hanstein, D. Drechsel, L. Tiator, *Phys. Lett.* **B399**, 13 (1997).

[14] M.L. Goldberger, H. Miyazawa, R. Oehme, *Phys. Rev.* **99**, 986 (1955).

[15] D. Sigg *et al.*, *Phys. Rev. Lett.* **75**, 3245 (1995); *Nucl. Phys.* **A609**, 269 (1996); **A617**, 526(E) (1997).

[16] D. Chattelard *et al.*, *Phys. Rev. Lett.* **74**, 4157 (1995); *Nucl. Phys.* **A625**, 855 (1997).

[17] A.W. Thomas, R.H. Landau, *Phys. Rep.* **C58**, 122 (1980).

[18] S. Weinberg, *Phys. Lett.* **B295**, 114 (1992).

[19] V.V. Baru, A.E. Kudryavtev, πN *Newsletter* **12**, 64 (1997).

[20] S.R. Beane *et al.*, *Phys. Rev.* **C57**, 424 (1998).

[21] E. Matsinos, *Phys. Rev.* **C56**, 3014 (1997).

[22] T. Ericson, W. Weise, *Pions and Nuclei*, Clarendon Press, Oxford 1988.

[23] R.G.E. Timmermans, πN *Newsletter* **11**, 7 (1995).