Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 11

MESON–MESON AND MESON–BARYON INTERACTIONS IN A CHIRAL NON-PERTURBATIVE APPROACH*

E. Oset, J.A. Oller

Departamento de Física Teórica
and IFIC, Centro Mixto Universidad de Valencia-CSIC
46100 Burjassot (Valencia), Spain

J.R. Pelaez

Stanford Linear Accelerator Center, Stanford University
Stanford, California 94309, USA

and A. Ramos

Departament d'Estructura i Constituents de la Materia, Universitat de Barcelona
Diagonal 647, 08028 Barcelona, Spain

(Received June 22, 1998)

A qualitative account of the meson-meson and meson-baryon interactions using chiral Lagrangians and the inverse amplitude method in coupled channels is done. The method, imposing exact unitarity, proves to be a very useful tool to extend the information contained in the chiral Lagrangians at energies beyond the realm of applicability of chiral perturbation theory.

PACS numbers: 12.39.Fe, 13.75.Jz, 13.75.Lb

1. Introduction

The meson-meson interaction has been the key problem to test Chiral Perturbation Theory (χPT), which has proved rather successful at low energies [1, 2]. The underlying idea is that an expansion in powers of the meson momenta converges at sufficiently low energy, which in practice is $\sqrt{s} \leq 500$ MeV. However, the convergence at higher energies becomes progressively

* Presented at the MESON '98 and Conference on the Structure of Meson, Baryon and Nuclei, Cracow, Poland, May 26–June 2, 1998.

worse. Even more, one of the peculiar features of the meson-meson interaction is the presence of resonances like the f_0, a_0 in the scalar sector and the ρ, K^* or the ϕ in the vector channels. These resonances will show up in the T matrix as poles that cannot be obtained using standard χPT. Nevertheless, the constraints imposed by chiral symmetry breaking are rather powerful and not restricted to the region where χPT is meant to converge [3].

Two independent approaches of non perturbative character have extended the use of chiral Lagrangians to higher energies and have been rather successful, reproducing important features of the meson-meson interaction including several resonances. One of them [4,5], based upon the Inverse Amplitude Method (IAM), first suggested in [6], makes use of χPT amplitudes at $\mathcal{O}(p^4)$. Elastic unitarity is imposed and thus no mixture of channels is allowed. Then, the coefficients of the $\mathcal{O}(p^4)$ Lagrangian are fitted to the data. The absence of coupled channels has obvious limitations, but in channels predominantly elastic the IAM is successful and able to generate dynamically the ρ, K^* and σ resonances, and to reproduce $\pi\pi$ scattering in the $(I, J) = (0, 0)$, (1,1), (2,0) partial waves, as well as in the (3/2,0),(1/2,1) and (1/2,0) channels of πK scattering. The results are very successful up to 1 GeV in all these channels except the (0,0), where it only yields good results up to 700 MeV. The limitations of this single channel approach become evident, for instance, in the $f_0(980)$ and $a_0(980)$ resonances ($J = 0$ and $I = 0$ and 1, respectively) which do not appear.

The second approach dealt with the $J = 0$ sector [7]. The input consists of the $\mathcal{O}(p^2)$ Lagrangian, which is used as the source of a potential between mesons. This potential enters in a set of coupled channel Bethe-Salpeter (BS) equations, which leads to the scattering matrix. The method imposes unitarity in coupled channels; hence it yields inelasticities when inelastic channels open up. Amazingly, the approach has only one free parameter, which is a cut-off that regularizes the loop integrals of the BS equation. Such a method proves rather successful since phase shifts and inelasticities are reproduced accurately up to 1200 MeV. The $f_0(980)$ and $a_0(980)$ resonances appear as poles of the T matrix for $I = 0$ and 1, respectively, and their widths and partial decay widths are very well reproduced. In addition, one finds a pole when $I = 0$ at $\sqrt{s} \simeq 500$ MeV with a width of around 400 MeV, corresponding to the σ meson, which was also found with similar properties with the IAM [5].

In this talk we will report on the method proposed in [8] with applications to the meson-meson interaction and K^-p interaction. It consists of a generalization to coupled channels of the inverse amplitude method and unifies the two methods discussed above.

2. Unitary amplitude in coupled channels

We denote by T_{IJ} the partial wave amplitude with isospin I and angular momentum J. For each value of I and J one has a definite channel with several meson-meson states coupled to each other. In Table I, we have listed these states for the $J = 0, 1$ channels.

TABLE I

Physical states used in the different I, J channels

	I = 0	I = 1/2	I = 1	I = 3/2	I = 2
J = 0	$\pi\ \pi$ $K\ \bar{K}$	$\bar{K}\ \pi$ $K\ \eta$	$\pi\ \eta$ $K\ \bar{K}$	$K\pi$	$\pi\pi$
J = 1	$K\bar{K}$	$\bar{K}\ \pi$ $K\ \eta$	$\pi\ \pi$ $K\ \bar{K}$		

Hence, throughout the present work, T_{IJ} will be either a 2×2 symmetric matrix when two states couple, or just a number when there is only one state. In what follows we omit the I, J labels and use a matrix formalism, which will be valid for the general case of $n\times n$ matrices corresponding to n coupled states.

Unitarity in coupled channels implies

$$\operatorname{Im} T_{if} = T_{in}\,\sigma_{nn}\,T^*_{nf}\,, \tag{1}$$

where σ is a real diagonal matrix whose elements account for the phase space of the two meson intermediate states n which are physically accessible. With our normalization σ_{nn} is given by the imaginary part of the loop integral of two meson propagators in the n state

$$\sigma_{nn}(s) = \operatorname{Im} G_{nn}(s) = -\frac{k_n}{8\pi\sqrt{s}}\theta(s-(m_{1n}+m_{2n})^2)\,,$$

$$G_{nn}(s) = i\int \frac{d^4q}{(2\pi)^4}\,\frac{1}{q^2-m_{1n}^2+i\varepsilon}\,\frac{1}{(P-q)^2-m_{2n}^2+i\varepsilon}\,, \tag{2}$$

where k_n is the on-shell CM momentum of the meson in the intermediate state n, P is the initial total four-momentum and m_{1n}, m_{2n} the masses of the two mesons in the state n.

From Eq. (1) we can extract σ and express it, in matrix form, as

$$\operatorname{Im} G = T^{-1}\cdot \operatorname{Im} T\cdot T^{*-1} = \frac{1}{2i}T^{-1}\cdot(T-T^*)\cdot T^{*-1} = \frac{1}{2i}(T^{-1*}-T^{-1}) = -\operatorname{Im} T^{-1}\,. \tag{3}$$

Hence,

$$T^{-1} = \mathrm{Re}T^{-1} - i\,\mathrm{Im}\,G; \quad T = \left[\mathrm{Re}\,T^{-1} - i\,\mathrm{Im}\,G\right]^{-1}. \tag{4}$$

This is a practical way to write the unitarity requirements of Eq. (1) which tells us that we only need to know $\mathrm{Re}\,T^{-1}$ since $\mathrm{Im}\,T^{-1}$ is given by the phase space of the intermediate physical states.

The next point is to realize that the T matrix has poles associated to resonances, which implies that the standard perturbative evaluation of χPT will necessarily fail close to these poles. As a consequence, one might try to exploit the expansion of T^{-1}, which will have zeros at the poles of T, and in principle does not present convergence problems around the poles of T. With this idea in mind let us expand T^{-1} in powers of p^2 as one would do for T using χPT:

$$T \simeq T_2 + T_4 + ...; \quad T^{-1} \simeq T_2^{-1}\cdot[1 + T_4\cdot T_2^{-1}...]^{-1} \simeq T_2^{-1}\cdot[1 - T_4\cdot T_2^{-1}...]\,. \tag{5}$$

Multiplying formaly by $T_2\,T_2^{-1}$ to the right and by $T_2^{-1}T_2$ to the left, Eq. (4) can be rewritten as

$$T = T_2\cdot[T_2\cdot\mathrm{Re}\,T^{-1}\cdot T_2 - iT_2\cdot\mathrm{Im}\,G\cdot T_2]^{-1}\cdot T_2\,. \tag{6}$$

Now, using the expansion for T^{-1} of Eq. (5) we find $T_2\cdot\mathrm{Re}\,T^{-1}\cdot T_2 \simeq T_2 - \mathrm{Re}\,T_4 + ...$, and recalling that $\mathrm{Im}\,T_4 = T_2\cdot\mathrm{Im}\,G\cdot T_2$, we finally obtain, within the $\mathcal{O}(p^4)$ approximation

$$T = T_2\cdot[T_2 - T_4]^{-1}\cdot T_2\,. \tag{7}$$

Note, as it is clear from Eq. (6), that what we are expanding is actually $T_2\cdot\mathrm{Re}\,T^{-1}\cdot T_2$ which is also convergent for low energy.

This equation is a generalization to multiple coupled channels of the IAM of Ref. [4,5]. It makes the method more general and powerful and also allows to evaluate transition cross sections as well as inelasticities.

It is now important to realize that Eq. (7) requires the complete evaluation of T_4, which is rather involved when dealing with many channels, as it is the case here. This has been done in [9] for the $K\bar{K}$ and $\pi\pi$ channels reproducing in very good agreement the experimental data up to around $\sqrt{s}\simeq$1.2 GeV for the $(I,J) = (0,0)$, (1,1) and (2,0) channels generating the σ, $f_0(980)$ and ρ resonances. Instead, we present a further approximation to Eq. (7) which turns out to be technically much simpler and rather accurate. In order to illustrate the steps leading to our final formula, let us make before another approximation. Let us assume that through a suitable cut-off we can approximate

$$\mathrm{Re}\;T_4 \simeq T_2\cdot\mathrm{Re}\;G\cdot T_2\,. \tag{8}$$

In such a case we go back to the former equations and immediately write

$$T = [1 - T_2 \cdot G]^{-1} \cdot T_2 \Longrightarrow T = T_2 + T_2 \cdot G \cdot T \,, \tag{9}$$

which is a BS equation for the T matrix, where T_2 plays the role of the potential. This is actually the approach followed in Ref. [7].

As we have already commented, the approximation of Eq. (8) leads to excellent results in the scalar channels. However, the generalization to $J \neq 0$ is not possible since basic information contained in the $\mathcal{O}(p^4)$ chiral Lagrangian is missing in Eq. (8). The obvious solution is to add a term to Eq. (8) such that

$$\text{Re } T_4 \simeq T_4^P + T_2 \cdot \text{Re } G \cdot T_2 \,, \tag{10}$$

where T_4^P is the polynomial tree level contribution coming from the $\mathcal{O}(p^4)$ Lagrangian, whose terms contain several free parameters, usually denoted L_i. Within our approach, these coefficients will be fitted to data and denoted by $\hat{L}_i$ since they do not have to coincide with those used in χPT, as we shall see. Actually, the L_i coefficients depend on a regularization scale (μ). In our scheme this scale dependence appears through the cut-off.

The difference between [9] and Eq. (10) is that in [9] tadpoles and loops ind the cross channels are evaluated explicitly at $\mathcal{O}(p^4)$ while here they are absorbed into the $\hat{L}_i$ coefficients, so then the values of L_i in both approaches are somewhat different.

Using Eqs. (11) and former equations, our final formula for the T matrix is given by

$$T = T_2 \cdot [T_2 - T_4^P - T_2 \cdot G \cdot T_2]^{-1} \cdot T_2 \,. \tag{11}$$

3. Results and comparison with experiment

Detailed calculations are presented in [10] for the different channels. So here we just show some selected results in Fig. 1. They are obtained using a cut off for the three momentum integration variable, $q_{\max} = 1.02$ GeV.

As one can see, the results obtained are in good agreement with experiment up to about 1.2 GeV. In addition one also obtains poles in all the meson resonances below that energy, the $\sigma(500), f_0(980), a_0(980), K(800)$ in the scalar sector ($J = 0$) plus the $\rho(770)$ and $K^*(800)$ in $J = 1, I = 1$. A pole in $J = 1, I = 0$ corresponding to an $SU(3)$ octet and close to the ϕ meson is also obtained. Partial decay widths are also calculated and are in fair agreement with experiment [10]. The values of the $\hat{L}_i$ parameters are of the same order as those of χPT for a scale corresponding to our cut off $q_{\max}$, with some discrepancies in L_5 and $2L_6 + L_8$, but as mentioned, tadpoles and crossed loops are incorporated at $\mathcal{O}(p^4)$ by means of changes in these coefficients.

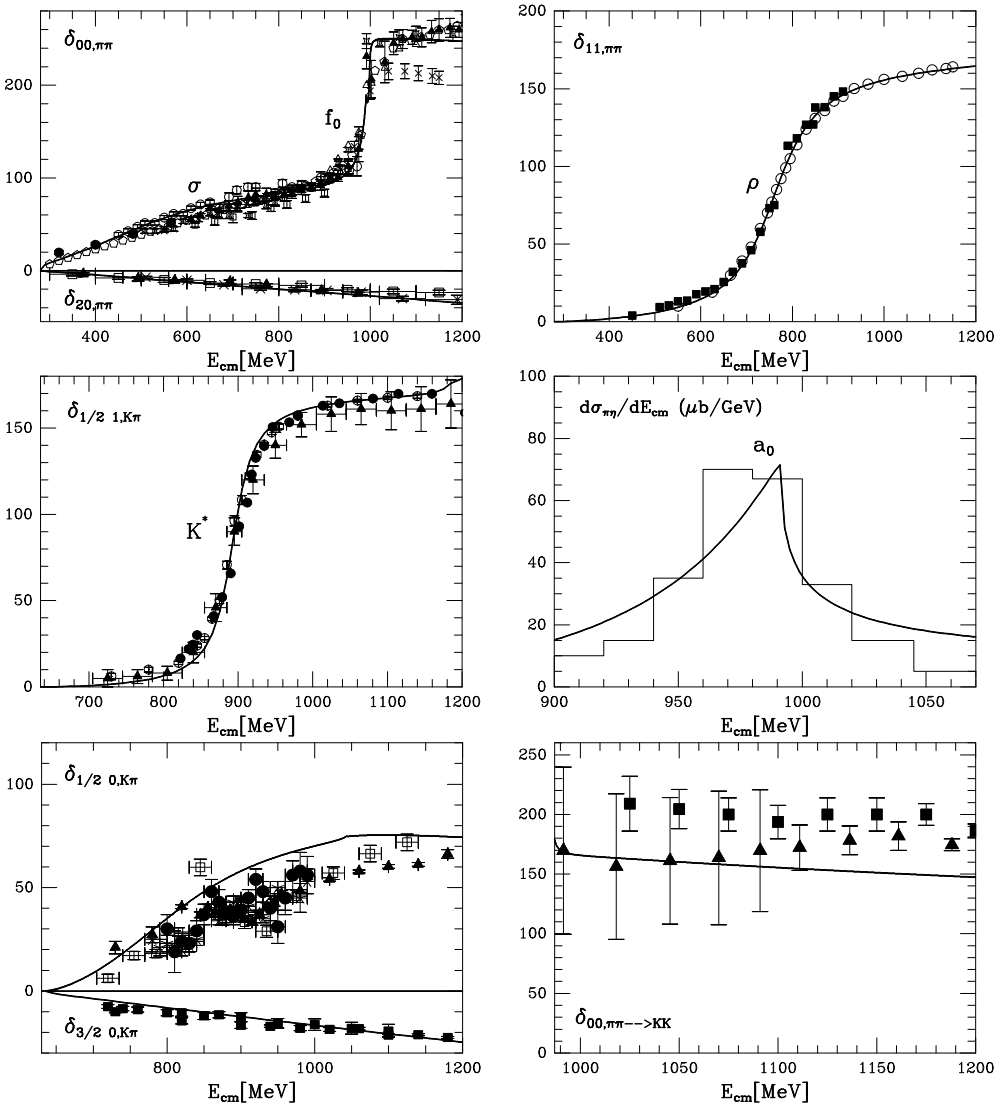


Fig. 1. We display the results of our method for the phase shifts of $\pi\pi$ scattering in the $(I,J)=(0,0),(1,1),(2,0)$ channels, where the σ, f_0 and ρ resonances appear, together with those of $\pi\pi \to K\bar{K}$, as well as the phase shifts of πK scattering in the $(3/2,0),(1/2,0)$ and $(1/2,1)$ channels, where we can see th appearance of the K^* resonance. The results also include the $\pi^-\eta$ mass distribution for the a_0 resonance in the $(I,J)=(1,0)$ channel from $K^-p \to \Sigma(1385)\pi^-\eta$. For reference to the data, see [4] and [7] and references therein.

In the calculation of Ref. [9], where tadpoles and crossed loops are explicitly evaluated, the agreement between the $\hat{L}_i$ and L_i coefficients is better.

4. Coupled channel approach to s-wave $\bar{K}N$ interactions

Here we follow the steps of the former section and include the coupled chanels $K^-p, \bar{K}^0 n, \pi^0\Lambda, \pi^+\Sigma^-, \pi^0\Sigma^0, \pi^-\Sigma^+, \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$ in order to study K^-p elastic and inelastic scattering close to threshold. The success of the approximation of Eq. (8) for the $J = 0$ meson-meson interaction suggests this should be sufficient here as it is indeed the case. Hence one uses the coupled channel Bethe Salpeter eqns. of Eq. (9) and uses the cut off $q_{\max}$ as a parameter. A value of $q_{\max} = 630\, MeV$ together with a value for $f = 1.15\, f_\pi$, between the f_π and f_K, was used in the calculations in [11].

The lowest order χPT amplitudes [1, 2] for these channels are easily evaluated and are given by

$$V_{ij} = -C_{ij}\frac{1}{4f^2}\bar{u}(p')\gamma^\mu u(p)(k_\mu + k'_\mu)\,, \tag{12}$$

where $p, p'(k, k')$ are the initial, final momenta of the baryons (mesons). Also, for low energies one can safely neglect the spatial components in Eq. (12) and only the γ^0 component becomes relevant, hence simplifying Eq. (12) which becomes

$$V_{ij} = -C_{ij}\frac{1}{4f^2}(k^0 + k'^0) \tag{13}$$

with C_{ij} a symmetric matrix which is given in [11].

The scheme followed here is in the spirit of the one of refs. [12, 13]. The novelties here are the consideration of all the meson channels in the coupled channel approach, while in [12,13] only six channels were considered, omitting the η and Ξ channels. In addition, a careful treatment of the renormalization of the lowest order constants when solving the scattering equations is done. While the Ξ channels are of no practical relevance, the η channels are important and change some cross sections by about a factor three. The results presented in [12, 13] are very similar to those obtained in [11] because higher order terms in the chiral Lagrangians are included in [12, 13] by fitting some parameters and the effects of the η channels are thus phenomenologically included.

The results obtained in [11], which are in good agreement with data, are elastic K^-p cross section, $K^-p \to \bar{K}^0\pi, \pi^0\Lambda, \pi^+\Sigma^-, \pi^0\Sigma^0, \pi^-\Sigma^+$ cross sections, K^-p and K^-n scattering lengths, the $\Lambda(1405)$ resonance, which is generated dynamically, plus the threshold ratios $\gamma = \Gamma(K^-p \to \pi^+\Sigma^-)/\Gamma(K^-p \to \pi^-\Sigma^+)$, $R_c = \Gamma(K^-p \to \text{charged particles})/\Gamma(K^-p \to \text{all})$, $R_n = \Gamma(K^-p \to \pi^0\Lambda)/\Gamma(K^-p \to \text{all neutral states})$.

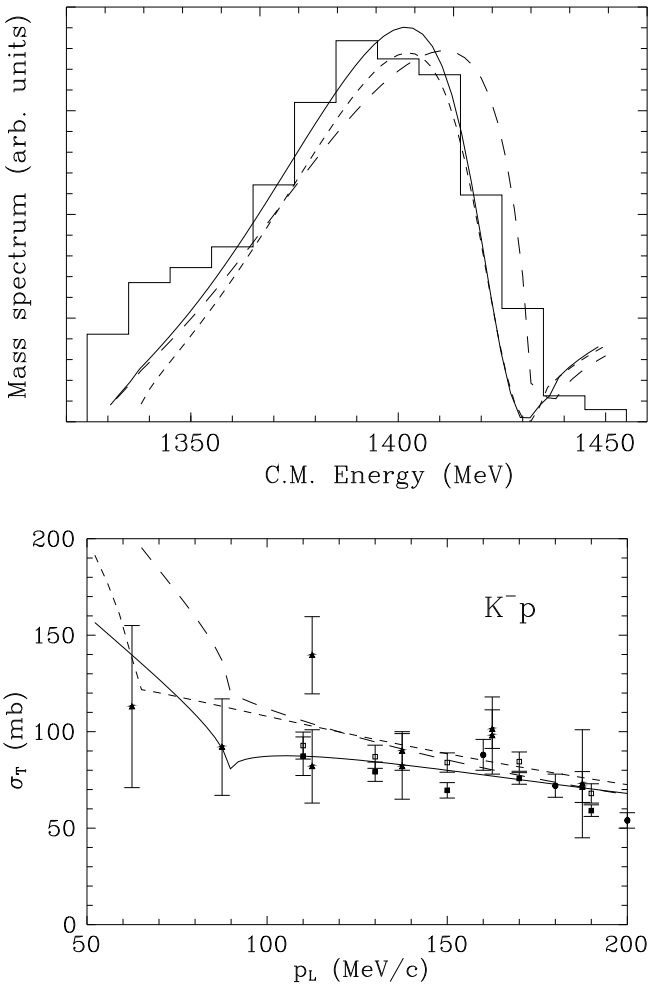


Fig. 2. From up to down. Mass spectra of $\pi\Sigma$ production corresponding to the $\Lambda(1405)$ resonance. Elastic cross section for K^-p collisions at low energies. The solid lines are the final results.

In Fig. 2 we show the results obtained for the $\pi\Sigma$ mass distribution around the $\Lambda(1405)$ resonance plus the elastic K^-p cross section. The quality of the agreement with data in the other channels is similar.

5. Conclusions

We have shown how the inverse amplitude method in coupled channels, respecting unitarity, allows one to go to higher energies, extracting more information contained in the chiral Lagrangians than is possible using χPT.

The description of the meson meson data upto 1.2 GeV requires the use of the $O(p^2)$ and $O(p^4)$ chiral Lagrangians. However, it is remartable to see that both for the meson-meson interaction and for the $\bar{K}N$ interaction in $J = 0$, the use of the lowest order Lagrangian and a suitable cut off is enough to reproduce the experimental results with high accuray.

The results obtained here obviously allow one to tackle typical problems of χPT at higher energies. Examples of that are the $\gamma\gamma \to MM$ reaction which has been worked out in [14], the $\phi \to \gamma K^0 \bar{K}$ decay worked out in [15] and the $K^- p \to \gamma\Lambda, \gamma\Sigma^0$ worked out in [16]. The good results obtained for these reactions suggest that the nonperturbative chiral scheme developed is an ideal tool to extend the ideas of χPT to much higher energies than are possible with the perturbative scheme.

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