# APPLICATION OF $\rho-\omega$ MIXING IN CP VIOLATING $B$-DECAYS* 

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We show how $\rho-\omega$ mixing can be exploited to constrain the strong phase difference between tree and penguin contributions to the decays $B^{ \pm} \rightarrow \pi^{+} \pi^{-} \rho^{ \pm}$and to enhance the CP violation. The resulting CP violation is greater than $20 \%$ and very stable against theoretical uncertainties. Moreover, it should be possible to extract the sign of the CKM angle, $\alpha$, unambiguously.

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## 1. Introduction

Within the Standard Model (SM) CP violation arises from a phase $(\delta)$ in the CKM matrix [1]. Because of its role in generating the apparent baryon number asymmetry of the universe, the origin of this phase and possible alternative explanations of CP violation are the subject of great interest. It is therefore very frustrating that it has so far only been observed in the $K^{0}-\bar{K}^{0}$ system. This also explains the great excitement at the prospects offered by the $B$-factories to give us the first new experimental examples in 30 years.

If the standard model is complete, the CKM matrix must be unitary. This gives a number of constraints which can be tested experimentally. Of those relevant to $b$-quark systems, one has a special status:

$$
\begin{equation*}
V_{u d} V_{u b}^{*}+V_{c d} V_{c b}^{*}+V_{t d} V_{t b}^{*}=0 \tag{1}
\end{equation*}
$$

[^0]Equation (1) can be conveniently represented by a triangle in the complex plane, whose angles are given by

$$
\begin{equation*}
\alpha \equiv \arg \left(-\frac{V_{t d} V_{t b}^{*}}{V_{u d} V_{u b}^{*}}\right), \quad \beta \equiv \arg \left(-\frac{V_{c d} V_{c b}^{*}}{V_{t d} V_{t b}^{*}}\right), \quad \gamma \equiv \arg \left(-\frac{V_{u d} V_{u b}^{*}}{V_{c d} V_{c b}^{*}}\right) . \tag{2}
\end{equation*}
$$

The unitarity condition can then be written in a form independent of the particular convention used for the CKM matrix elements

$$
\begin{equation*}
\alpha+\beta+\gamma=\left.\pi\right|_{\bmod 2 \pi} \tag{3}
\end{equation*}
$$

The equality only holds modulo $2 \pi$ because $\alpha, \beta$ and $\gamma$ can, in principle, be either the internal or external angles of a triangle - depending on the sign of $\sin \delta$ [2]. However, it is generally accepted that $\sin \delta>0$ and hence the angles are internal [3]. Equation(3) therefore provides a significant test of the standard model.

In general CP violation is manifest either indirectly or directly. The neutral kaon system is the classic example of indirect CP violation, in which there is some mixing between CP conjugate states. One can anticipate playing the same game with neutral $B$-mesons. Our concern here is with direct CP violation which has so far never been observed. In order to generate direct CP violation three conditions must be satisfied:

- There must be two different mechanisms for the transition (i.e., at least two different Feynman diagrams).
- These diagrams must have a different strong phase.
- The diagrams must also have a different CP violating phase.

If $\bar{i}$ and $\bar{f}$ are the CP conjugates of states $i$ and $f$, the CP-violating asymmetry in the decay $i \rightarrow f$ is defined through:

$$
\begin{equation*}
A_{С \mathrm{P}}=\frac{\Gamma(i \rightarrow f)-\Gamma(\bar{i} \rightarrow \bar{f})}{\Gamma(i \rightarrow f)+\Gamma(\bar{i} \rightarrow \bar{f})} . \tag{4}
\end{equation*}
$$

The main theoretical obstacle in testing Eq. (3) is the uncertainty introduced by the need to know the strong phase difference between the two diagrams precisely. There has been considerable effort devoted to finding decay channels for which the interpretation of $A_{\varnothing \mathrm{P}}$ is not clouded by uncertainties in the strong phase. Particular attention has focussed on neutral $B$ decay to CP eigenstates and the two initial experiments are likely to be $B_{d} \rightarrow J / \psi K_{S}\left(a_{\mathrm{CP}}=\sin 2 \beta\right)$ and $B_{d} \rightarrow \pi^{+} \pi^{-}\left(a_{\mathrm{CP}} \simeq \sin 2 \alpha\right)$. Although there is still some residual hadronic uncertainty in these extractions, through the sign of the mixing parameter, $B_{B}$, this is believed to be well
understood. On the other hand, measuring $\sin 2 \phi$ only determines $\phi$ up to a four-fold ambiguity $(\phi, \pi / 2-\phi,-\pi / 2-\phi,-\pi+\phi)[2,3]$.

In charged $B$ decay only direct CP violation is possible and the resultant asymmetries are proportional to $\sin \phi$ (where $\phi=\alpha, \beta$ or $\gamma$ ). They are also necessarily dependent on hadronic quantities and hence, due to the strong interaction uncertainty, cannot be used in isolation to determine the unitarity angles. However the extraction of the $\operatorname{sign}$ of $\sin \phi, S(\sin \phi)$, would complement the results from neutral systems and reduce the four-fold uncertainty to two-fold. The measurement of $\cos 2 \phi$, would then completely determine $\phi[3]$.

In Ref. [7] we sketched the argument leading to the conclusion that the particular decay channel $B^{ \pm} \rightarrow \rho^{ \pm} \pi^{+} \pi^{-}$should allow us to determine the sign of $\sin \alpha$ unambiguously. Here we provide a more detailed discussion of the role of $\rho-\omega$ mixing in this particular channel. This work followed the original suggestion by Lipkin [4] that $\rho-\omega$ mixing (for a review see, e.g., Ref. [5]) might provide a phenomenologically well constrained contribution to the strong phase leading to CP asymmetries. Enomoto and Tanabashi [6] examined this suggestion for decays of the form $B^{ \pm} \rightarrow h^{ \pm} \rho^{0}(\omega) \rightarrow h^{ \pm} \pi^{+} \pi^{-}$ and found a considerable CP asymmetry around the $\omega$ peak.

## 2. Direct CP violation in $B^{ \pm} \rightarrow \rho^{ \pm} \pi^{+} \pi^{-}$

The first discussion of direct CP violation in $b$-decays came from Bander, Silverman and Soni (BSS) [8] in 1979. They suggested that CP violation could be observed in charged $B$ systems, in particular those involving the decay $b \rightarrow f+q+\bar{q}$, through interference between the tree and penguin contributions shown in Fig. 1.


Fig. 1. The tree and penguin contributions to $B$ decay.
We have already reviewed the conditions necessary to produce direct CP violation. Let us concentrate first on the need for a well understood strong phase difference. The usual source of this strong phase is the absorptive part of the penguin quark loops corresponding to $j$ and $\bar{j}$ being on massshell [8]. Unfortunately this phase is small and somewhat uncertain. The
process which we consider, namely $B^{ \pm} \rightarrow h^{ \pm} \rho^{0}(\omega) \rightarrow h^{ \pm} \pi^{+} \pi^{-}$, solves this problem beautifully. As long as charge symmetry is respected by the strong interactions, the penguin process produces only a $\rho^{0}$ final state, which decays to $\pi^{+} \pi^{-}$. The tree diagram, on the other hand, produces either a $\rho^{0}$ or an $\omega$. The latter can decay to a $\pi^{+} \pi^{-}$final state through mixing with a $\rho^{0}$. This process has been extensively studied with $e^{+} e^{-}$colliding beams and the mixing amplitude is well known - modulo subtleties that we discuss below, which do not alter this conclusion. The relative phase of these two processes differs by the presence of the $\omega$ propagator and hence the relative strong phase difference passes through $\pi / 2$ at or near the $\omega$ mass. As a result, the relative phase is under control and the signal is maximised because $\sin \pi / 2$ is one.

With respect to the CP violating phase difference, we note that in the penguin graph $j$ runs over $u, c, t$. The top contribution, which is proportional to $V_{t d} V_{t b}^{*}\left(P_{t}-P_{u}\right)$, is generally assumed to dominate because of the GIM cancellation. Although this is exact only for $m_{u}=m_{c}$, corrections to it are expected to be small and we neglect them. In this case the CP violating phase is exactly $\alpha$. We note also that in addition to strong penguins there are electroweak penguins where the gluon in Fig. 1 is replaced by a photon or $Z^{0}$ and we also include those.

## 3. The short distance physics

Using the renormalization group we can evolve the pieces of Fig. 1 involving a weak boson from the $W$ mass scale to the $b$ scale, contracting the $W$ propagators to obtain an effective Hamiltonian for both the tree and penguin diagrams. This effective Hamiltonian involves a set of local four-quark operators and Wilson coefficients, $c_{i}(\mu)$ [11]

$$
\begin{equation*}
\mathcal{H}^{\mathrm{eff}}=\mathcal{H}^{T}+\mathcal{H}^{P}=\frac{4 G_{F}}{\sqrt{2}}\left(V_{u b} V_{u q}^{*} \sum_{i=1}^{2} c_{i} \mathcal{O}^{i}-V_{t b} V_{t q}^{*} \sum_{i=3}^{10} c_{i} \mathcal{O}^{i}\right)+\text { h.c. } \tag{5}
\end{equation*}
$$

where $G_{F}$ is the Fermi couping constant. The 4-quark operators are given by $\left(q_{L, R}=\left(1 \mp \gamma_{5}\right) q / 2\right)$

$$
\begin{array}{ll}
\mathcal{O}_{1}^{u}=\bar{q}_{L \alpha} \gamma^{\mu} u_{L \beta} \bar{u}_{L \beta} \gamma_{\mu} b_{L \alpha} & \mathcal{O}_{2}^{u}=\bar{q}_{L \alpha} \gamma^{\mu} u_{L \alpha} \bar{u}_{L \beta} \gamma_{\mu} b_{L \beta} \\
\mathcal{O}_{3}=\bar{q}_{L \alpha} \gamma^{\mu} b_{L \alpha} \Sigma_{q^{\prime}} \bar{q}_{L \beta}^{\prime} \gamma_{\mu} q_{L \beta}^{\prime} & \mathcal{O}_{4}=\bar{q}_{L \alpha} \gamma^{\mu} b_{L \beta} \Sigma_{q^{\prime}} \bar{q}_{L \beta}^{\prime} \gamma_{\mu} q_{L \alpha}^{\prime} b_{L \beta} \\
\mathcal{O}_{5}=\bar{q}_{L \alpha} \gamma^{\mu} b_{L \alpha} \Sigma_{q^{\prime}} \bar{q}_{R \beta}^{\prime} \gamma_{\mu} q_{R \beta}^{\prime} & \mathcal{O}_{6}=\bar{q}_{L \alpha} \gamma^{\mu} b_{L \beta} \Sigma_{q^{\prime}} \bar{q}_{R \beta}^{\prime} \gamma_{\mu} q_{R \alpha}^{\prime} b_{L \beta} \\
\mathcal{O}_{7}=\frac{3}{2} \bar{q}_{L \alpha} \gamma^{\mu} b_{L \alpha} \Sigma_{q^{\prime}} e_{q^{\prime}} \bar{q}_{L \beta}^{\prime} \gamma_{\mu} q_{L \beta}^{\prime} & \mathcal{O}_{8}=\frac{3}{2} \bar{q}_{L \alpha} \gamma^{\mu} b_{L \beta} \Sigma_{q^{\prime}} e_{q^{\prime}} \bar{q}_{L \beta}^{\prime} \gamma_{\mu} q_{L \alpha}^{\prime} b_{L \beta} \\
\mathcal{O}_{9}=\frac{3}{2} \bar{q}_{L \alpha} \gamma^{\mu} b_{L \alpha} \Sigma_{q^{\prime}} e_{q^{\prime}} \bar{q}_{R \beta}^{\prime} \gamma_{\mu} q_{R \beta}^{\prime} & \mathcal{O}_{10}=\frac{3}{2} \bar{q}_{L \alpha} \gamma^{\mu} b_{L \beta} \Sigma_{q^{\prime}} e_{q^{\prime}} \bar{q}_{R \beta}^{\prime} \gamma_{\mu} q_{R \alpha}^{\prime} b_{L \beta}, \tag{6}
\end{array}
$$

where $\alpha$ and $\beta$ are color indices and $e_{q^{\prime}}=2 / 3(-1 / 3)$ for $q^{\prime}=u, c(d, s)$.
The Wilson coefficients, $c_{i}$, to be used in Eq. (5), are effective coefficients, independent of renormalization scheme, and are given in standard works. We therefore do not repeat them here. As noted above, the phase occuring in the effective Wilson coefficient of the penguin terms arises from the absorptive part of the charm loops in the BSS mechanism [8].

Using this short distance Hamiltonian, the two body hadronic decay amplitude for $B$ mesons is now given by

$$
\begin{equation*}
\mathcal{A}_{h_{1} h_{2}}=\left\langle h_{1} h_{2}\right| \mathcal{H}^{\mathrm{eff}}|B\rangle . \tag{7}
\end{equation*}
$$

This is calculated using the factorization approximation which assumes that the decay $B \rightarrow h_{1} h_{2}$ can be considered as a product of two individual processes by taking pairwise combinations of the quarks and antiquarks in the effective Hamiltonian operators

$$
\begin{align*}
& \left\langle h_{1} h_{2}\right| \mathcal{H}|B\rangle=\sum_{i} v_{i} c_{i}\left\langle h_{1} h_{2}\right| \mathcal{O}^{i}|B\rangle  \tag{8}\\
& \equiv \sum_{i} v_{i} c_{i}\left[\left\langle h_{1}\right| J_{i}(1)_{\mu}|0\rangle\left\langle h_{2}\right| J_{i}(2)^{\mu}|B\rangle+\left\langle h_{2}\right| j_{i}(2)_{\mu}|0\rangle\left\langle h_{1}\right| j_{i}(1)^{\mu}|B\rangle\right]
\end{align*}
$$

where $j_{\mu}$ and $J_{\mu}$ are currents constructed from the constituents of the four quark operators $\mathcal{O}^{i}$ of Eq. (5). Note that currents of the type $\bar{q}_{\alpha} \gamma_{\mu} q^{\beta}$ are suppressed by a factor $1 / N_{c}$ with respect to those like $\bar{q}_{\alpha} \gamma_{\mu} q^{\alpha}$, after Fierz rearrangement.

As factorization is only an approximation [12], $N_{c}$ is treated as a phenomenological parameter. Rather than the exact value, $N_{c}=3$, of QCD, phenomenological fits to $B$ decay prefer $N_{c}$ between 2 and 3 [13]. It is useful to introduce the parameters:

$$
\begin{align*}
a_{i} & =c_{i}+c_{i+1} / N_{c} & & i \text { odd }  \tag{9}\\
a_{i} & =c_{i}+c_{i-1} / N_{c} & & i \text { even. } \tag{10}
\end{align*}
$$

In factorizing the hadronic matrix elements all information on the gluon momentum, $q^{2}$, is lost. It is customary to estimate the uncertainty by letting it vary over the range $\frac{m_{b}^{2}}{4}<q^{2}<\frac{m_{b}^{2}}{2},\left(2<N_{c}<3\right)$.

In order to deal with the strong phase difference we need to isolate the tree terms (those involving $a_{1}$ and $a_{2}$ ) from the penguins (those with $a_{3-10}$ ). We, therefore, rewrite Eq. (9) as

$$
\begin{equation*}
\left\langle f_{1} f_{2}\right| \mathcal{H}|B\rangle=\left(a_{T} V_{T}+a_{P} V_{P}\right) \mathcal{A}+\left(b_{T} V_{T}+b_{P} V_{P}\right) \mathcal{B}, \tag{11}
\end{equation*}
$$

where $\mathcal{A}$ and $\mathcal{B}$ are the amplitudes for $\left(B \rightarrow h_{1}\right)\left(0 \rightarrow h_{2}\right)$ and $\left(B \rightarrow h_{2}\right)(0 \rightarrow$ $h_{1}$ ), respectively. The coefficients $a_{T}$ and $b_{T}$ are the tree terms composed
of $a_{1}$ and $a_{2}$, while $a_{P}$ and $b_{P}$ are the analogous penguin terms. For the $B$ decay amplitude and vacuum to meson matrix element that constitute $\mathcal{A}$ and $\mathcal{B}$ we use the model of Bauer, Stech and Wirbel (BSW) [15].

## 4. The role of $\rho-\omega$ mixing

We now consider the strong phase phase difference between the tree and penguin diagrams. We have already seen there is a short distance contribution from absorptive quark loops. However, there is an additional mechanism for the case of interest to us, where we have resonant final states. In particular, the propagator of a resonance such as the $\rho$ or $\omega$ is complex, with a phase that passes through $\pi / 2$. This can provide a vital constraint on the CP invariant strong phase which is usually so difficult to determine accurately [16].

We see that while the $\omega$ penguin term comes largely from the strong interaction, $p_{\rho}$ survives only through the electroweak interaction. The suppression of $p_{\rho}$ can be understood in general terms. The $B$ is a $0^{-}$state and therefore decays to a $J=0$ final state, which is symmetric for integer spin particles. Therefore the spin degrees of freedom must conspire to produce an overall symmetric state. Applying this to $B^{+} \rightarrow \rho^{+} \rho^{0}$ we note that the final state has isospin $M_{I}=1$ and is therefore either $I=1$, which is antisymmetric and therefore forbidden, or $I=2$, which cannot be accessed in penguin decay as $\Delta I=3 / 2$ is greatly suppressed compared with $\Delta I=1 / 2$ [17].

We now come to the essential feature of this channel. What is actually seen in this particular $B^{ \pm}$decay channel is, for our purposes, $B^{ \pm} \rightarrow$ $\pi^{+} \pi^{-} \rho^{-}$. The production of $\pi^{+} \pi^{-}$in the $\rho$ resonance region is very well known from $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$experiments [18] and can be fixed by fitting to this data [19]. A notable feature of this reaction is the $G$-parity violating appearance of the isosinglet $\omega$, which is clearly visible in the data. The $\omega$ formed in the penguin process can couple to the $\pi^{+} \pi^{-}$final state through either the direct decay $\omega \rightarrow \pi^{+} \pi^{-}$(which has often been neglected) or by mixing, through the function, $\Pi_{\rho \omega}\left(q^{2}\right)$, to a $\rho^{0}\left(\omega \rightarrow \rho^{0} \rightarrow \pi^{+} \pi^{-}\right)$. The role of $\rho-\omega$ mixing in the decay we are considering is illustrated in Fig. 2. The mixing function, $\Pi_{\rho \omega}\left(q^{2}\right)$, is necessarily momentum dependent as a result of current conservation [20]. Therefore, we may write the pion form-factor as [21]
$F_{\pi}(s)=\frac{f_{\rho \gamma} g_{\rho \pi \pi}}{s-m_{\rho}^{2}+i m_{\rho} \Gamma_{\rho}}\left(1+\frac{f_{\omega \gamma}}{f_{\rho \gamma}} \frac{\Pi_{\rho \omega}(s)}{s-m_{\omega}^{2}+i m_{\omega} \Gamma_{\omega}}\right)+\frac{f_{\omega \gamma} g_{\omega \pi \pi}}{s-m_{\omega}^{2}+i m_{\omega} \Gamma_{\omega}}$,
where the photon couples to a vector meson, $V$, with strength $e f_{V \gamma}$.

It is a vital feature of Eq. (12) that the unknown $\omega \rightarrow \pi^{+} \pi^{-}$coupling enters in exactly the same way here as in the pion form factor. We can therefore simplify Eq. (12) by introducing an effective mixing matrix element, $\widetilde{\Pi}_{\rho \omega}$, which incorporates both direct ( $\omega \rightarrow \pi^{+} \pi^{-}$) and mixing induced ( $\omega \rightarrow \rho^{0} \rightarrow \pi^{+} \pi^{-}$) $G$-parity violation [19]. We can then write the pion electromagnetic form-factor as:

$$
\begin{equation*}
F_{\pi}(s)=\frac{f_{\rho \gamma} g_{\rho \pi \pi}}{s-m_{\rho}^{2}+i m_{\rho} \Gamma_{\rho}}\left(1+\frac{f_{\omega \gamma}}{f_{\rho \gamma}} \frac{\widetilde{\Pi}_{\rho \omega}(s)}{s-m_{\omega}^{2}+i m_{\omega} \Gamma_{\omega}}\right) . \tag{13}
\end{equation*}
$$



Fig. 2. The contributions to $B^{-} \rightarrow \rho^{-} \pi^{+} \pi^{-}$in the $\rho-\omega$ interference region.
In principle, $\widetilde{\Pi}_{\rho \omega}$ can be both momentum dependent and complex, as we might expect from a comparison of Eqs. (12) and (13). This has been tested in the most recent analysis of pion form factor data [19], with the result that there are very good limits on the extent of its possible momentum dependence (or its imaginary piece). Note that, whereas isospin violation is typically very small [22], the $\omega$ contribution to $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$accounts for roughly $20 \%$ of $\left|F_{\pi}(s)\right|^{2}$ at the $\omega$ mass pole, in part due to the narrowness of the $\omega$. We note, following an old argument of Renard, that even if $g_{\omega \pi \pi}$ is non-zero, it would only effect the real part of $\widetilde{\Pi}_{\rho \omega}[21]$. The most recent value of $\widetilde{\Pi}_{\rho \omega}$, which we use here, is [19]

$$
\begin{equation*}
\widetilde{\Pi}_{\rho \omega}=-3920 \pm 330 \mathrm{MeV}^{2} . \tag{14}
\end{equation*}
$$

We now have all the necessary elements for the decay $B^{-} \rightarrow \rho^{-} \rho(\omega)$ and can derive an expression for the CP violating asymmetry. It is convenient to introduce the ratios of Enomoto and Tanabashi [6]

$$
\begin{equation*}
\arg \left(V_{P} / V_{T}\right) \equiv \phi, \quad \frac{p_{\omega}}{t_{\rho}} \equiv r^{\prime} \mathrm{e}^{i\left(\delta_{q}+\phi\right)}, \quad \frac{t_{\omega}}{t_{\rho}} \equiv \alpha \mathrm{e}^{i \delta_{\alpha}}, \quad \frac{p_{\rho}}{p_{\omega}} \equiv \beta \mathrm{e}^{i \delta_{\beta}}, \tag{15}
\end{equation*}
$$

where $\phi$ is the CP-violating phase. Then, defining $s_{V}=s-m_{V}^{2}+i m_{V} \Gamma_{V}$, the amplitude for $B^{-} \rightarrow \rho^{-} \pi^{+} \pi^{-}$is given by (see Fig. 2)

$$
\begin{align*}
\mathcal{A}_{2 \pi}^{-} & =\frac{t_{\rho}}{s_{\rho}}+\frac{t_{\omega}}{s_{\omega}} \frac{\widetilde{\Pi}_{\rho \omega}}{s_{\rho}}+\frac{p_{\rho}}{s_{\rho}}+\frac{p_{\omega}}{s_{\rho}} \frac{\widetilde{\Pi}_{\rho \omega}}{s_{\rho}} \\
& =\frac{t_{\rho}}{s_{\rho}}\left[1+\alpha \mathrm{e}^{i \delta_{\alpha}} \frac{\widetilde{\Pi}_{\rho \omega}}{s_{\omega}}+r^{\prime} \mathrm{e}^{i\left(\delta_{q}+\phi\right)}\left(\frac{\widetilde{\Pi}_{\rho \omega}}{s_{\omega}}+\beta \mathrm{e}^{i \delta_{\beta}}\right)\right] \tag{16}
\end{align*}
$$

We see that, assuming isospin invariance for the decay amplitude, the only surviving contribution to $\beta$ is from EW penguins. Within factorization $\beta$ is small compared with $\left|\widetilde{\Pi}_{\rho \omega} / m_{\omega} \Gamma_{\omega}\right| \approx 0.6$, and can be ignored. Similarly $\alpha \mathrm{e}^{i \delta_{\alpha}} \sim 1$ and hence the decay amplitude can be approximated by

$$
\begin{equation*}
\mathcal{A}_{2 \pi}^{-}=\frac{t_{\rho}}{s_{\rho}}\left[1+\frac{\widetilde{\Pi}_{\rho \omega}}{\left|s_{\omega}\right|^{2}}\left(s-m_{\omega}^{2}-i m_{\omega} \Gamma_{\omega}\right)\left(1+r^{\prime} \mathrm{e}^{i\left(\delta_{q}+\phi\right)}\right)\right] \tag{17}
\end{equation*}
$$

Therefore, the CP violating asymmetry is given by

$$
\begin{equation*}
A_{\varnothing \mathrm{P}}^{2 \pi}=\frac{4 \widetilde{\Pi}_{\rho \omega} r^{\prime}}{\left|s_{\omega}\right|^{2}} \sin \phi \frac{\left[m_{\omega} \Gamma_{\omega} \cos \delta_{q}+\left(m_{\omega}^{2}-s\right) \sin \delta_{q}\right]}{|D|^{2}} \tag{18}
\end{equation*}
$$

where

$$
\begin{equation*}
|D|^{2}=2+\frac{4 \widetilde{\Pi}_{\rho \omega}}{\left|s_{\omega}\right|^{2}}\left(r^{\prime} \cos \phi\left(\left(s-m_{\omega}^{2}\right) \cos \delta_{q}+m_{\omega} \Gamma_{\omega} \sin \delta_{q}\right)+s-m_{\omega}^{2}\right) \tag{19}
\end{equation*}
$$

At the $\omega$ mass, the CP violating asymmetry is maximal and given by (defining $X_{\rho \omega}=\widetilde{\Pi}_{\rho \omega} / m_{\omega} \Gamma_{\omega}$,

$$
\begin{equation*}
A_{\varnothing \mathrm{P}}^{2 \pi}=\frac{4 X_{\rho \omega} r^{\prime} \sin \phi \cos \delta_{q}}{2\left(1+2 r^{\prime} \cos \phi \sin \delta_{q} X_{\rho \omega}+\left[1+2 r^{\prime} \cos \delta_{q} \cos \phi+r^{\prime 2}\right] X_{\rho \omega}^{2}\right)} \tag{20}
\end{equation*}
$$

The $s$-dependent term in Eq. (18), proportional to $\sin \delta_{q}$, induces a "skew" in the shape of the asymmetry.

A similar analysis for the decay $B^{-} \rightarrow \omega \rho^{-}$, where the $\omega$ then decays through its $3 \pi$ decay mode, receives no enhancement from meson mixing. Any strong phase there arises solely from the BSS mechanism, but we can still make use of isospin symmetry for the $B$ decays to write

$$
\begin{equation*}
A_{\varnothing \mathrm{P}}^{3 \pi}=\frac{-4 r^{\prime} \sin \phi \sin \delta_{q}}{2\left[1+2 r^{\prime} \cos \delta_{q} \cos \phi+r^{\prime 2}\right]} \tag{21}
\end{equation*}
$$

We, therefore, have a check on the factorization approximation, because the skew of $A_{\triangle \mathrm{P}}^{2 \pi}$ is related to the sign of $A_{\varnothing \mathrm{P}}^{3 \pi}$. Unfortunately, it does not allow
one to fix the sign of $\sin \phi$ from experimental measurement alone, and we must appeal to factorization for this.

Working within the factorization approximation, we have calculated the expected values for the short distance strong phase, $\delta_{q}$. The results are very stable, $\delta_{q} \sim-170^{\circ}$, because the strong penguin contributions to $p_{\rho}$ are suppressed by isospin and the imaginary parts of the effective Wilson coefficients arising from charm loop corrections, are electromagnetic and hence very small. Using this phase we find that $p_{\omega} / t_{\rho}$ is always negative as is $\cos \delta_{q}$, and hence the sign of $A_{ष \mathrm{P}}^{2 \pi}$ fixes the sign of $\sin \phi$. In addition, the sign of $A_{\varnothing \mathrm{P}}^{3 \pi}$ or the skew of $A_{\subset \mathrm{P}}^{2 \pi}$, acts as a check of factorization. The results of a typical calculation within the factorization approximation are shown in Fig. 3. The comparison between the two-pion and three-pion decay modes is dramatic. In particular, because of the role of $\rho-\omega$ mixing the asymmetry in the two-pion channel is both large and extremely stable in the $\omega$-mass region.


Fig. 3. Results demonstrating the stability of $A_{\varnothing \mathrm{P}}^{2 \pi}$ as compared with $A_{\varnothing \mathrm{P}}^{3 \pi}$ (which is momentum independent) within the factorization approximation. The lines correspond to $\left[N_{c}, q^{2} / m_{b}^{2}\right]=[2,0.5]$ (solid), [2,0.3] (dashed), [3,0.5] (dot-dashed), [3,0.3] (dotted). (We have used the Wolfenstein parameters $A=0.81, \lambda=0.2205$, $\rho=0.12$ and $\eta=0.34$, following Ref.[24].)

## 5. Conclusion

We have shown that $\rho-\omega$ mixing can be used to enhance the CP violation in the decays $B^{ \pm} \rightarrow \pi^{+} \pi^{-} \rho^{ \pm}$in the region where the $\pi^{+} \pi^{-}$pair has invariant mass near $m_{\omega}$. There are sound reasons to expect factorization to fix $\cos \delta_{q} \approx$ -1 in Eq. (18) and under this assumption a measurement of CP violation
in this channel will determine the sign of $\sin \phi$ unambiguously. Finally, the additional measurement of either the skew of $A_{C \mathrm{P}}^{2 \pi}$ or $A_{C \mathrm{P}}^{3 \pi}$ would provide a consistency check of factorization.

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