

STATUS OF EFFECTIVE FIELD THEORY  
OF  $NN$  SCATTERING \*

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I review recent progress in developing a systematic power counting scheme for scattering processes involving more than one nucleon.

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**1. Why effective field theory?**

There exist many nucleon–nucleon potentials which reproduce phase shifts and nuclear properties with remarkable accuracy (an extensive reference list can be found in Ref. [1]). Three fundamental features are shared by these potential models:

- (i) pions are important at long distances,
- (ii) there is a source of intermediate-range attraction, and
- (iii) there is a source of short-distance repulsion.

However, in general, distinct physical mechanisms in these models account for the same feature of the nuclear force. Agreement with experiment is maintained in spite of these differences because of the large number of fit parameters.

Systematic approaches to the scattering of strongly interacting particles, such as chiral perturbation theory, are based on the ideas of effective field theory (EFT). The fundamental premise of EFT is that when a system is probed at momentum  $k \ll M$ , details of the dynamics at scale  $M$  are unimportant. What is important at low energies is the physics that can be captured in operators of increasing dimensionality which take the form of a power-series in  $k/M$  [2, 3]. It is entirely possible that EFT fits to phase

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shifts will ultimately not be as good as those produced by conventional  $NN$  potentials with the same number of parameters. So then, what can be gained from such an enterprise?

Consider the following questions: Is it possible to account for short distance physics at low energies systematically, using power counting arguments? What is the minimal set of parameters required to describe data at low energies? Or rather, what is the minimal short distance physics required? Can we fit some processes to experiment and use that information to predict other processes? For instance, one would like to relate  $NN$  scattering systematically to scattering processes with more nucleons, such as  $NNN$  scattering, and to make predictions for processes involving electromagnetic and pionic probes of few-nucleon systems. The underlying theory, QCD, has one scale,  $\Lambda_{\text{QCD}}$ . Why are characteristic nuclear binding energies  $\ll \Lambda_{\text{QCD}}$ ? These are the sort of questions that EFT can help answer. In what follows I will review recent progress in answering some of these questions.

## 2. The Weinberg program

Naive application of EFT ideas to nuclear physics immediately suggests a puzzle. In nuclear physics there are bound states whose energy is unnaturally small on the scale of hadronic physics. In order to generate such bound states within a “natural” theory it is clear that one must sum some operators to all orders. Weinberg proposed [4, 5] implementing the EFT program in nuclear physics by applying the power counting arguments of chiral perturbation theory to an  $n$ -nucleon effective potential rather than directly to the  $S$ -matrix. Only  $n$ -nucleon irreducible graphs should be included in the  $n$ -nucleon effective potential. The potential obtained in this way is then to be inserted into a Schrödinger equation and iterated to all orders. See Fig. 1. There will of course be unknown coefficients in the effective potential, but these can be fit to experimental data as in ordinary chiral perturbation theory [6–10]. Perhaps the most powerful result to emerge from Weinberg’s power counting is the hierarchy of  $n$ -body forces (*e.g.* three-body forces are small) [4, 5, 11, 12].

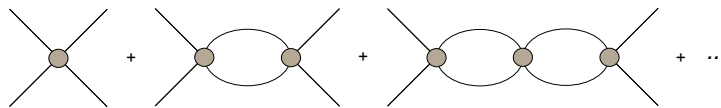


Fig. 1. The diagrammatic solution of the Schrödinger equation with the effective  $NN$  potential represented by the shaded blob.

Weinberg did not specify a regularization and renormalization scheme. Yet, as we will see, an understanding of regularization and renormalization appears to be crucial in the identification of a consistent power counting scheme.

### 3. Dissecting the Weinberg program — the pionless EFT

In Ref. [7] Kaplan, Savage and Wise (KSW) considered  $NN$  scattering in the  $^1S_0$  ( $np$ ) channel at momentum scales  $k \ll m_\pi$ . The EFT at these scales involves only nucleons since the pion is heavy and may therefore be “integrated out”. The effective Lagrangian thus consists of contact operators of increasing dimensionality constrained by spin and isospin. This EFT is useful because scattering amplitudes can be calculated analytically. It therefore allows one to address issues of principle in EFT for  $NN$  scattering.

The most general effective Lagrangian consistent with spin and isospin, including only operators relevant to  $^1S_0$  scattering is

$$\begin{aligned} \mathcal{L} = & N^\dagger i\partial_t N - N^\dagger \frac{\nabla^2}{2M} N - \frac{1}{2} C (N^\dagger N)^2 \\ & - \frac{1}{2} C_2 (N^\dagger \nabla^2 N) (N^\dagger N) + \text{h.c.} + \dots \end{aligned} \tag{1}$$

It is important to realize that all of the coefficients in the effective theory,  $C, C_2, \dots$ , are renormalization scheme dependent. This means that power counting will necessarily look different in different schemes. It is clearly fruitful to choose a scheme which maintains the power counting hierarchy of operators; although ultimately the scattering amplitude which is calculated to a given order in the EFT is scheme independent, the power counting is transparent in some schemes while requiring counterintuitive cancellations in others [3]. For instance, in a perturbative EFT expansion — such as Fermi theory — power counting is transparent in dimensional regularization (DR) with minimal subtraction (MS), while somewhat mysterious using a cut-off [3].

Ultimately what one would like to reproduce in the  $NN$  EFT is the effective range expansion, written here as

$$\frac{1}{T^{\text{on}}(k)} = -\frac{M}{4\pi} \left[ -\frac{1}{a} + \frac{1}{2} r_e k^2 + O(k^4) - ik \right], \tag{2}$$

where  $T(k)$  is the scattering amplitude,  $a$  is the scattering length and  $r_e$  is the effective range. Experiment determines (in the  $^1S_0$  ( $np$ ) channel)  $a = -23.714 \pm 0.013$  fm and  $r_e = 2.73 \pm 0.03$  fm. The extremely large value of the scattering length implies that there is a virtual bound state in this

channel very near zero energy. While the value of  $r_e$  is consistent with what one might expect for a natural theory where pions dominate the low-energy physics ( $r_e \sim 1/M_\pi$ ), the value of  $a$  is far from being natural ( $a \gg 1/M_\pi$ ). As seen in Fig. 2 this scattering amplitude (neglecting  $O(k^4)$  terms) compares favorably with data up to center of mass momenta of order  $M_\pi$ . Phase shift data of Fig. 2 are taken from Ref. [13].

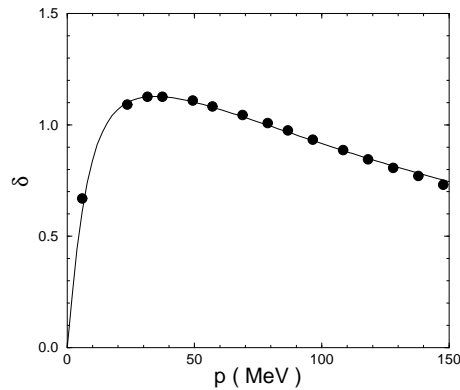


Fig. 2. The effective range expansion with the extracted phase-shift data up to center-of-mass momenta of order  $M_\pi$ .

I will proceed in the spirit of Weinberg power counting. The effective potential in the pionless EFT is simply the sum of all tree graphs extracted from the Lagrangian of Eq. (1). It is straightforward to find the “second-order” potential:

$$V^{(2)}(p', p) = C + C_2(p^2 + p'^2). \quad (3)$$

The Schrödinger equation iterates this potential to all orders (see Fig. 1). The divergences get worse as one goes to higher order in the potential. All divergences are of power-law type. Therefore *DR* with *MS* has the effect of unitarizing the scattering amplitude with the potential from Eq. (3) [1, 9]. The problem is that the resulting scattering amplitude only matches to the effective range expansion for momenta  $k \ll 1/\sqrt{ar_e}$ . Given our working assumption that all renormalization schemes give equivalent results but generally have different power counting, this means that *DR* with *MS* is not particularly well suited to the problem since higher order operators must be highly correlated in this scheme in order to ensure that the EFT matches to the effective range expansion [7]. In Ref. [14] a novel way of reproducing the effective range expansion within the *DR* with *MS* scheme was proposed. The main idea is that the effective range expansion can be viewed as arising from the exchange of a di-baryon field (transvestite in the vernacular) which is included in the EFT as a fundamental field and “dressed” via its interactions with the nucleons.

Following the work of Ref. [7] many authors argued that the pathological features of DR with MS are a good reason to work with a cut-off EFT [9, 15–17]. The problem is that, unless one is willing to carry out all analysis numerically, not much insight is gained into power counting; the unpleasant features that one has in cut-off Fermi theory are present in  $NN$  scattering with a vengeance. There are other pathologies as well which force the cut-off to be very low [1, 8, 15–17], unless the bare coefficients in the Lagrangian are chosen to be imaginary. We will return to the issue of cut-off EFT below.

#### 4. Resolution

The physical scattering amplitude that is generated when an effective potential is iterated in the Schrödinger equation is exactly unitary (like Eq. (2)) and therefore necessarily contains arbitrarily high powers in momentum. This occurs regardless of the order to which one is working in the momentum expansion of the potential  $V$ . Therefore, the scattering amplitude thus obtained samples arbitrarily short-distance scales. Such a scattering amplitude is not necessarily in contradiction with the EFT approach since short-distance physics included in the amplitude might be small in a power counting sense. But if it is small it is not clear why it should be included in the scattering amplitude.

An important observation in this spirit was made by van Kolck [18] and KSW [19]. Given the experimentally established hierarchy of scales  $a \gg r_e \sim 1/M_\pi$ , what the effective theory should be reproducing is

$$T(k) = -\frac{4\pi}{M} \frac{1}{(1/a + ik)} \left[ 1 + \frac{r_e/2}{(1/a + ik)} k^2 + O(k^4) \right], \quad (4)$$

and not necessarily the full effective range expansion of Eq. (2). This form of the scattering amplitude can be reproduced in a scheme independent way by summing the  $C$  operator of Eq. (1) to all orders and treating all higher order derivative operators as perturbations. Say  $\aleph$  represents the long-distance nonperturbative scale [18]. If  $\Lambda$  represents the scale of short distance physics, then the effective expansion parameter is  $\aleph/\Lambda \sim 1/aM_\pi$ . Summing to all orders in  $1/aM_\pi$  gives the effective range expansion, or equivalently, the transvestite.

A scheme in which this power counting is manifest was found by KSW in Ref. [19], which gives an elegant renormalization group analysis of the coefficients in the EFT. The regularization and renormalization scheme in which the power counting is manifest is  $DR$  with power divergence subtraction (PDS). As opposed to MS, in which counterterms are added which subtract the poles in three space dimensions, in PDS the poles in two space dimensions are also subtracted by counterterms. This scale-dependent scheme is

similar to performing a momentum subtraction at  $p^2 = -\mu^2$  [20]. In this scheme, the fine-tuning in the underlying theory which gives rise to a large scattering length is identified with a single operator in the Lagrangian, the  $C$  operator of Eq. (1).

### 5. The three-body force

One of the most important results that has emerged from EFT in nuclear physics is due to Bedaque, van Kolck and Hammer [21, 22]. These authors consider  $N$ -deuteron scattering. There are two channels, a quartet of total spin  $J = 3/2$  and a doublet of  $J = 1/2$ . The leading interactions involve two-body interactions whose low-energy parameters have been fit to  $NN$  scattering. Recall that this is EFT at its best; parameters fit to one process predict an independent process. The two-body interactions are accounted for using transvestite fields and iterated using a Fadeev equation. This is not strictly systematic in the sense of  $\mathbb{N}/\Lambda$  power counting; however, including some of the higher order terms in  $\mathbb{N}/\Lambda$  via the transvestite does not make the results any less accurate. Specifically, the transvestite should be considered accurate to second order in the  $\mathbb{N}/\Lambda$  power counting scheme. Only the transvestite with spin one, isospin zero contributes to the quartet scattering length, giving a theoretical prediction of  ${}^4a = 6.33$  fm as compared to the experimental value of  ${}^4a = 6.35 \pm 0.02$  fm.

One might wonder about the doublet channel in  $N$ -deuteron scattering. Unlike the quartet channel, the scattering length in this channel is not well described in the EFT because the absence of Pauli blocking (which is present in the quartet channel) renders physics at short distances potentially relevant to long distance observables. Bedaque and van Kolck have pointed out that the problem might be remedied by inclusion of a 3-body contact interaction in the EFT which “summarizes” the effects of this short-distance physics.

### 6. The role of the pion — a challenge for nuclear theorists

It is desirable to push the short distance cut-off of the  $NN$  EFT to as high a momentum scale as possible. In a realistic EFT of  $NN$  scattering it is important to include the pion. The lightness of the pion in itself guarantees that it should play a fundamental role in nuclear physics. However, it is the fact that chiral symmetry is spontaneously broken — implying a light pion interacting weakly at low energies — that allows pion effects to be included in an EFT description.

KSW have pointed out that Weinberg’s power counting arguments are problematic when computations are performed using dimensional regularization. The fundamental problem is that pion exchange effects in the  ${}^3S_1 - {}^3D_1$

channel that are leading order in Weinberg's power counting require counterterms at all orders in the momentum expansion, suggesting that Weinberg's power counting scheme is not consistent.

Given the pathologies of the nonperturbative pion, KSW have proposed a radical power counting scheme which fuses the  $\mathbb{N}$ /PDS power counting of the pionless effective theory with a perturbative pion [19]. To date, phase shifts in the  $^1S_0$  and  $^3S_1 - ^3D_1$  channels have been computed in this scheme at next-to-leading order. The  $^3S_1 - ^3D_1$  mixing parameter  $\epsilon_1$  is a prediction at this order. Agreement with experiment is reasonable. Moreover, KSW have calculated the electromagnetic form factors of the deuteron by using the parameters fit to scattering data and have found good agreement with experiment [23]. This is a true test of the EFT.

The idea of a nuclear force with a perturbative pion is anathema to most nuclear physicists. However, given that a consistent power counting scheme has been proposed and nontrivial calculations have been performed with good experimental agreement, it would seem incumbent on traditionalists to propose low-energy observables whose description requires a nonperturbative pion.

## 7. Conclusion

There has been remarkable progress made in the last few years in developing systematic power counting technology for scattering processes involving more than a single nucleon. A new power counting scheme, which is consistent in the sense of renormalization, has emerged to challenge the original Weinberg power counting proposal.

Is Weinberg power counting wrong? Is a nonperturbative pion truly incompatible with EFT ideas? The work of Refs [6, 8–10, 24] using cut-off EFT suggests otherwise. These numerical analyses include nonperturbative pions and yet exhibit universal low-energy behavior: low-energy physics is insensitive to the specific choice of regulator. One way of gaining insight into this issue might be to unitarily transform the effective potential (which is unobservable) to a new effective potential which by construction involves only momenta less than a fixed value [25]. In my view, understanding why EFT with a nonperturbative pion works in spite of the failure implied by dimensional regularization is an important issue, and not purely academic. In losing Weinberg power counting we lose his beautiful explanation of the hierarchy of n-body forces, which evidently has no explanation in the new power counting scheme.

Be that as it may, it is clear that Kaplan, Savage and Wise have introduced a consistent power counting scheme which is economical in the sense that it appears to include only minimal short distance physics and not the

barrage of short distance physics which is inherent to any solution of the Schrödinger equation.

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