BARYON–BARYON MIXING IN HYPERNUCLEI*

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Implications of few-body hypernuclei for our understanding of the baryon-baryon interaction are examined. Octet-octet coupling effects not present in conventional, non strange nuclei are the focus. The need to identify strangeness −2 hypernuclei to test model predictions is emphasized.

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1. Introduction

A primary impetus for investigating the structure and reactions of baryon systems is to understand the fundamental baryon-baryon force in the realm of nonperturbative QCD. Few-baryon systems play an important role, because one can calculate complete solutions to test a particular baryon-baryon interaction ansatz. Hypernuclei are crucial to this investigation, because they permit one to probe models based upon our non strange sector experience outside of the conventional world where the models were developed. That is, we can test whether sophisticated models of the nucleon-nucleon (NN) interaction extrapolate successfully beyond the strangeness 0 region in which the parameters were fitted, or whether the models are only interpolative. For these reasons, the highlights of this brief discussion are the strange ($S=-1$ and $S=-2$) few-body hypernuclei.

In an attempt to relate nonperturbative QCD to physical observables, a number of theorists have turned to chiral perturbation theory and effective field theory. However, in the case of the NN interaction we have already available sophisticated meson-exchange and one-boson-exchange potentials which embody the desired characteristics of these approaches: a one-pion-exchange tail and a quantitative fit to the low-energy scattering parameters and deuteron properties. Moreover, substantial partial-wave-analysis of the

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NN scattering data have already specified the number of free parameters required to represent a particular partial wave. Therefore, the potentials developed by Speth and his collaborators such as Holinde, or the Argonne group, or the Paris group, or the Nijmegen group provide superb models which satisfy the constraints of chiral perturbation theory or finite range effective field theory. These potential models form the basis for our successful "traditional approach" to calculating physical observables for few-nucleon systems with strangeness zero ($S = 0$).

2. Our strangeness zero experience

The traditional approach to nuclear physics can best be defined in terms of the model assumptions:

- Nuclei consist of nucleons — other degrees of freedom are suppressed.
- Nucleons move slowly within the nucleus — nonrelativistic dynamics prevails.
- Nucleons interact primarily via pairwise (two-body) forces.

This is an enormous simplification of the physics, but it accounts amazingly well for much experimental data. Nonetheless, our calculational ability has achieved the precision required to see differences between the traditional model predictions and experiment. Much research during the past decade has been focussed upon extensions: meson exchange currents (MEC), three-body forces (3BF), NN-$N\Delta$ coupling, relativistic dynamics, quark-gluon substructure, ...

It has now been reasonably well established [1, 2] that the low energy observables "scale" with the trinucleon binding energy. A summary of results for charge radii, wave function probabilities, magnetic moments, Coulomb energy, asymptotic normalization constants, and Nd scattering lengths can be found in Ref. [3]. Because of this scaling property, any triton discussion can be limited to results for the binding energy. Benchmark results exist for a variety of realistic potentials, where realistic implies

- strong spin-isospin dependence ($\mathrm{V}_{nn} \neq \mathrm{V}_{np}$),
- strong tensor force (OPEP is essential, providing up to 3/4 of the potential energy in ^{3}H and ^{4}He),
- strong short range repulsion (the probability of NN overlap at such separations should be small),

in addition to a reasonable fit to the NN scattering data. Charge dependent potential models (with $V_{nn} \neq V^s_{np} \neq V^t_{np}$) have now been used to estimate the triton binding energy to be B(^{3}H) = 7.6 MeV. That is, a local potential model, which fits the NN observables as well as a proper phase shift analysis implies is possible, leads to underbinding of the triton by about 0.85 MeV, and a corresponding failure to describe the low energy physical observables. This missing energy is less than 2% of the 50 MeV of potential energy in the system, and provides a more quantitative description of the triton than we had any right to expect *a priori*.

Such underbinding of the triton by local potential models led theorists to ask [4] about the role of three-nucleon forces. Adding a two-pion-exchange three-body force (3BF) to the Hamiltonian, adjusted to reproduce the triton binding energy, indeed scales the other physical observables into agreement with experiment. *Apparently, a non relativistic Hamiltonian composed of a local NN potential plus a suitable 3BF can yield approximately the correct value for* B(^{3}H). Moreover, it leads to the correct binding of ^{4}He (see, *e.g.*, the GFMC results of Carlson [5]), enhancing the binding by some 3 MeV, as predicted by the strong correlation among the ^{3}H and ^{4}He binding energies established by Tjon [6]. Similar results were later confirmed by Gloeckle and coworkers. Although the 3BF approach is but one means of achieving the desired increase in binding, it seems not unreasonable when the model 3BF contributes some 18 MeV to the binding of ^{7}Li while the nucleus is underbound by only 2 MeV, of the order that one might expect of higher order forces. Nevertheless, it should be noted that the short range regularization of the potentials ensures that estimates of relativitic effects are small.

3. The charm of hypernuclei

As stated above the question to be addressed is whether our models developed to describe conventional nuclei and nuclear reactions extrapolate beyond the $S = 0$ realm? Or are they merely exquisite interpolation schemes?

Pure one-boson-exchange (OBE) potential models provide both a quantitative fit to the extensive NN data base and a means to transform from $S = 0$ into $S = -1$, -2. In particular, the Nijmegen models [7] satisfy that criteria. (Other contemporary potential models yield similar few-nucleon system results.) Let me briefly recall triton results which suggest an approach to Λ hypernuclei with a somewhat surprising outcome. The Argonne V_{14} potential [8] has been thoroughly studied and yields a triton binding energy of $\simeq$ 7.7 MeV. The V_{14} model is particularly interesting because there also exists a V_{28} model, one which includes $NN - N\Delta$ mixing and is fitted to the same NN data set. Surprisingly, the triton binding energy is essentially unchanged. Why is the octet-decuplet ($NN - N\Delta$) coupling so well modeled implicitly by V_{14}? Can one extend this approach to the $S = -1$ octet-octet coupling? That is, can one represent the hyperon-nucleon coupled-channel

$N\Lambda$ potential

$$\begin{pmatrix} V_{N\Lambda} & V_{NX} \\ V_{NX} & V_{N\Sigma} \end{pmatrix}$$

by an effective single-channel potential $\overline{V}_{N\Lambda}$?

Even at first glance, one sees quickly that the physics of the s-shell hypernuclei is puzzling. The experimental observations show that the physics is novel. In particular, one observes:

- anomalous binding energies
- important 3BF effects
- striking charge symmetry breaking
- puzzling non mesonic weak decays
- anomalous π^+ decays
- $\Lambda\Lambda$ hypernuclei and not the H dibaryon

The hypernuclear sector of hadronic physics is apparently not just a simple extension of $S = 0$ phenomena.

4. $S = -1$ binding energy systematics

The available data on the few-body Λ hypernuclei come primarily from emulsion experiments [9–11, 13] – binding energies and weak decay properties. We limit discussion to the binding energies, because the $S = 0$ sector has taught us that binding energies determine the low energy observables. In the study of hypernuclei, it is customary to quote the Λ-separation energies

$$\mathrm{B}_\Lambda({}_\Lambda\mathrm{A}) = \mathrm{B}({}_\Lambda\mathrm{A}) - \mathrm{B}(\text{A-1}).$$

In the non strange sector we observe that the ratio of neutron separation energies for neighboring s-shell nuclei is approximately 3: $B_n(^3\mathrm{H})/\mathrm{B_n}(^2\mathrm{H}) \simeq 6/2 = 3$, and $B_n(^4\mathrm{He})/\mathrm{B_n}(^3\mathrm{H}) \simeq 20/6 \simeq 3$. If the physics of few-body systems is similar, then we might anticipate a factor of 3 in the ratio of Λ separation energies for neighboring Λ hypernuclei. Using $B_\Lambda(^4_\Lambda\mathrm{H}) \simeq 2$ MeV as our basis, we would then predict $B_\Lambda(^5_\Lambda\mathrm{He}) \simeq 3\times B_\Lambda(^4_\Lambda\mathrm{H}) \simeq 6$ MeV and $B_\Lambda(^3_\Lambda\mathrm{H}) \simeq \frac{1}{3}\times\ B_\Lambda(^4_\Lambda\mathrm{H}) \simeq \frac{2}{3}$ MeV. Simple, central force calculations using $\overline{V}_{N\Lambda}$ fitted to $B_\Lambda(^4_\Lambda\mathrm{H})$ plus low-energy scattering data confirm [15] this simple analysis.

However, the real world is more complex. The accepted values for the s-shell systems are quoted in Table I along with the measured γ-ray deexcitation energies [13] for the two species with particle-stable excited states. The A=6 entry [14] is $\mathrm{B}_{\Lambda\Lambda} = \mathrm{B}({}_{\Lambda\Lambda}^{\ 6}\mathrm{He}) - \mathrm{B}(^4\mathrm{He})$.

TABLE I

Hypernuclear Λ-separation energies and excitation energies in MeV

Hypernucleus	B_Λ	E_γ
${}^{3}_{\Lambda}$H	0.13±.05	
${}^{4}_{\Lambda}$H	2.04±.04	1.04±.04
${}^{4}_{\Lambda}$He	2.39±.03	1.15±.04
${}^{5}_{\Lambda}$He	3.10±.02	
${}^{6}_{\Lambda\Lambda}$He	10.9	

Experimentally we know that $B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) \simeq 3.1$ MeV and $B_\Lambda({}^{3}_{\Lambda}\mathrm{H}) \simeq 0.13$ MeV. Our $S = 0$ model experience does not extrapolate to $S = -1$. Explicit $N\Lambda - N\Sigma$ (octet-octet) mixing appears to play a large role whereas $NN - N\Delta$ (octet-decuplet) mixing does not. Moreover, there exist π^+ decay data that suggest the importance of explicit $N\Lambda - N\Sigma$ mixing in Λ hypernuclei. The open channels for Λ mesonic decay are $\Lambda \to \pi^- p$ and $\Lambda \to \pi^0 n$. However, experimentally [16] there is observed a 5% branching ratio for ${}^{4}_{\Lambda}\mathrm{He} \to \pi^+ + X$. Second order pion processes such as charge exchange ($\pi^- pp \to \pi^+ nn$) are too small [17] to explain more than 1%. The virtual $p\Lambda \to n\Sigma^+$ transition followed by $\Sigma^+ N \to \pi^+ nN$ decay appears [18] to play a significant role.

Explicit $N\Lambda - N\Sigma$ mixing was demonstrated in Ref. [19] to play a crucial role in driving the hypertriton Λ separation energy from 2/3 MeV toward 0.1 MeV. Furthermore, these separable potential model Faddeev-type calculations demonstrated that the binding of the halo-like hypertriton system was due to the existence of an attractive $NN\Lambda$ $3BF$ when the $N\Sigma$ channel was formally eliminated. Gloeckle and coworkers [20] have since established that the $S = -1$ Nijmegen soft-core potential yields a value for $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$ which agrees with experiment. The Juelich potential models [21] of Speth and co-workers have also been at the forefront of the hypertriton analysis. In addition, a correct ordering of the $A = 4$ isodoublet 0^+ and 1^+ states appears to require explicit $N\Lambda - N\Sigma$ mixing [22]. Simple single-channel four-body calculations would produce a ground state with 1^+ quantum numbers. Finally, Monte Carlo calculations [23] have indicated that suppression of $\Lambda \otimes {}^4\mathrm{He} \leftrightarrow \Sigma \otimes {}^4\mathrm{He}^*$ mixing, because of the large excitation energy of the $T = 1$ even parity ${}^4\mathrm{He}^*$ states that result when the $T = 0$ Λ converts to a $T = 1$ Σ, can account for the anomalously low value of $B_\Lambda({}^{5}_{\Lambda}\mathrm{He}) = 3.1$ MeV. Finally, based upon an analysis of charge-symmetry breaking in the

Nijmegen model D, which is twice as large in a true four-body calculation as it is in a mean-field approximation, it would appear that few-body dynamics can be crucial in such analyses.

One should not infer from this discussion that all questions have been resolved. For example, in the ${}^{10}\mathrm{B}(\mathrm{K}^-,\pi^-){}^{10}_{\Lambda}\mathrm{B}$ reaction, the 3^+ ground state of the target ensures that one reaches the 2^- state of the hypernucleus. Mean field, shell model calculations, based upon the 0^+, 1^+ splitting in the $A = 4$ isodoublet, predict that the ground state of ${}^{10}_{\Lambda}\mathrm{B}$ should be 1^-. A search for the $2^- \rightarrow 1^-$ γ transition [24] proved negative. Does $N\Lambda - N\Sigma$ mixing hold the key?

5. $S = -2$ puzzles

Given that explicit octet-octet mixing plays a key role in $S = -1$ physics, let us turn to the interesting puzzle that the single reported ${}^{\;6}_{\Lambda\Lambda}\mathrm{He}$ event [14] presents. Assuming that the $\Lambda\Lambda$ separation energy $B_{\Lambda\Lambda}({}^{\;6}_{\Lambda\Lambda}\mathrm{He}) = B({}^{\;6}_{\Lambda\Lambda}\mathrm{He})$ - $B({}^4\mathrm{He}) \simeq 10.9$ MeV is accurate (this interpretation is consistent with the two other accepted $\Lambda\Lambda$ events), we see that the matrix element $\langle V_{\Lambda\Lambda}\rangle_{A=6}$ is weak: $-\langle V_{\Lambda\Lambda}\rangle = B_{\Lambda\Lambda}({}^{\;6}_{\Lambda\Lambda}\mathrm{He})\ -2 \times B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He}) \simeq 10.9 - 2(3.1) = 4.7$ MeV. This value is comparable with that of the $N\Lambda$ interaction:
$\langle V_{\Lambda\Lambda}\rangle_{A=6} \simeq \langle V_{N\Lambda}\rangle_{A=4}$. Both the $\Lambda\Lambda$ and $N\Lambda$ matrix elements are relatively small [25] compared with that of the nn interaction: $\langle V_{nn}\rangle \simeq -7$ MeV. However, $\Lambda\Lambda$ and nn are analogs, belonging to the same ${}^1S_0$ multiplet. Why is $\langle V_{\Lambda\Lambda}\rangle_{A=6}$ much smaller?

Can we measure $\Lambda\Lambda$ scattering? “Yes, indirectly.” Two example Ξ^- capture reactions that could measure $a_{\Lambda\Lambda}$ are $\Xi^- d \rightarrow \Lambda\Lambda n$ and $\Xi^-{}^7\mathrm{Li} \rightarrow \Lambda\Lambda{}^6\mathrm{He}$, where the spectator particle is detected in analogy with the a_{nn} measurements from $nd \rightarrow nnp$ and ${}^3\mathrm{H}\ {}^3\mathrm{H} \rightarrow nn{}^4\mathrm{He}$. Alternatively, one could try to analyze the final state in the reaction ${}^2\mathrm{H}(\mathrm{K}^-,\mathrm{K}^0)n\Lambda$, provided one can extract information about the strong $\mathrm{K}^0\Lambda$ from another source.

Short of such data we ask about the constraints that an $\alpha\Lambda\Lambda$ model of ${}^{\;6}_{\Lambda\Lambda}\mathrm{He}$ provides. In the analysis by Carr *et al.* [26], octet-octet mixing is essential. For an effective $\Lambda\Lambda$ potential whose strength is comparable to that of the NN force ($\overline{V}_{\Lambda\Lambda} \simeq V_{nn}$), overbinding of ${}^{\;6}_{\Lambda\Lambda}\mathrm{He}$ is obtained. In contrast, a coupled-channel ($\Lambda\Lambda - N\Xi$) potential of similar overall strength yields binding comparable to experiment, because of Pauli blocking. The α core saturates the $(1\mathrm{s})^4$ shell, forcing a 5th nucleon into a higher shell and significantly weakening the $N\Xi$ part of the force. In other words, by including $\Lambda\Lambda - N\Xi$ coupling explicitly, one can accommodate a relatively weak $\langle V_{\Lambda\Lambda}\rangle_{A=6}$ even though the free space $\Lambda\Lambda - N\Xi$ potential is comparable in strength to the nn.

In contrast, the $S = -2$ ${}_{\Lambda\Lambda}^{5}\mathrm{H}$ hypernucleus should show evidence of enhanced binding, and ${}_{\Lambda\Lambda}^{4}\mathrm{H}$ may give the best estimate of the free interaction. Whereas the ${}^{4}\mathrm{He}$ core in ${}_{\Lambda\Lambda}^{6}\mathrm{He}$ must be excited by 40 MeV to permit the $\Lambda\Lambda \to N\Xi$ transition, the ${}^{2}\mathrm{H}$ (${}^{3}\mathrm{H}$) core in ${}_{\Lambda\Lambda}^{4}\mathrm{He}$ (${}_{\Lambda\Lambda}^{5}\mathrm{H}$) is bound by an additional 6 (20) MeV following $\Lambda\Lambda \to N\Xi$ ($\Lambda\Lambda \to p\Xi^{-}$) conversion, to form a ${}^{3}\mathrm{H}$ or ${}^{3}\mathrm{He}$ (${}^{4}\mathrm{He}$) core.

6. Conclusions

In concluding, let me return to the question of why the role of octet-decuplet mixing in the S=0 sector ($NN - N\Delta$) appears relatively unimportant, whereas that of octet-octet mixing in the S≠0 sector is essential. There certainly exist alternate possibilities: (*i*) the large Δ width, (*ii*) the large $N - \Delta$ mass difference, and (*iii*) the duality of particle physics. The first two are obvious; the third may not be. In the Maldelstam representation one can write the scattering amplitude M equivalently as $M(t,u)$ or $M(s,u)$. That is, we can use either t-channel/u-channel variables or s-channel/u-channel variables. Does the t-channel meson-exchange picture of the OBE potential essentially subsume the s-channel resonance picture of the $NN - N\Delta$ conversion process, so that explicit octet-decuplet coupling is not required? The question begs to be answered.

In summary, hypernuclear physics continues to be novel and exciting. New questions continue to arise. As a testing ground for $S = 0$ based concepts, hypernuclei are unsurpassed.

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