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CHIRAL DISORDER AND DIFFUSION OF LIGHT QUARKS IN THE QCD VACUUM*

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We give a pedagogical introduction to the concept that light quarks diffuse in the QCD vacuum following the spontaneous breaking of chiral symmetry. By analogy with disordered electrons in metals, we show that the diffusion constant for light quarks in QCD is $D = 2F_\pi^2/|\langle\bar{q}q\rangle|$ which is about 0.22 fm. We comment on the correspondence between the diffusive phase and the chiral phase as described by chiral perturbation theory, as well as the cross-over to the ergodic phase as described by random matrix theory. The cross-over is identified with the Thouless energy $E_c = D/\sqrt{V_4}$ which is the inverse diffusion time in an Euclidean four-volume V_4.

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1.

In this talk and borrowing from our recent work [1], we explain using theoretical arguments how the QCD vacuum can be viewed as a chirally disordered medium, with chiral quarks diffusing through Euclidean four-space V_4 with diffusion constant $D = 2F_\pi^2/|\langle\bar{q}q\rangle|$.

In Section 2 we define the basic concepts of diffusion, a phenomenon well known in 1,2,3-dimensional electronic systems, where the coherent description of electron waves fails at transport distances greater than the mean free path. In Section 3 we argue that a similar behavior in 4-dimensional

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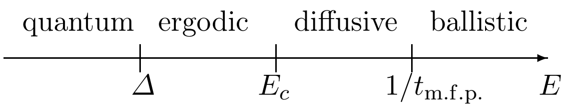
Euclidean space allows us to interpret the result of multiple scatterings of originally coherent quark waves, in the form of a diffusion pole. In Section 4 we show how the diffusive picture merges with the universal predictions of random matrix models, provided that the eigenvalues of the Dirac operator are smaller than the Thouless energy $E = D/\sqrt{V_4}$. In Section 5 we comment on the generic character of this description in comparison with chiral perturbation theory, the instanton model and lattice QCD simulations. In Section 6 we list some plausible extensions of our description to important issues in QCD.

2.

The typical distance of a diffusing particle grows as a function of time as $r^2(t) = Dt$, where D is a diffusion constant. One could therefore identify a time scale in which a particle is diffusing through the sample of a linear size L as $\tau_c = L^2/D$. The corresponding energy scale known as the Thouless energy, is therefore $E_c = 1/\tau_c = D/L^2$. For energies *lower* than the Thouless energy (times greater than τ_c), particle probes the whole volume of the system, defining in this way an ergodic regime. Decreasing the energy yields a minimal scale of the order of the mean quantum spacing Δ, where the quantum description of the system becomes dominant. On the other side, for energies *larger* than the Thouless energy, a particle has time to probe only small domains of the whole volume. This is a diffusive regime. For even larger energies the diffusive regime breaks down, and we enter a ballistic regime — simply times are so short, that the particle cannot even travel the mean free path (m.f.p.) between elastic collisions. This scale hierarchy is shown beneath.



Quantitatively, the diffusion is easily described in terms of the retarded and advanced Green's function $G^{\pm}(r, r', E) = \langle r|(E - H \pm i\varepsilon)^{-1}|r'\rangle$, where r, r' are positions in d-dimensional space, and where the Hamiltonian H contains the details of the disorder (*e.g.* H could be an Anderson Hamiltonian). With the help of the Green's function, one could easily define [2] the return probability $P(t)$ to the origin in fixed time t for the diffusing particle with energy E_F. Specifically,

$$P(t) = \frac{V}{2\pi\rho} \int d\lambda \mathrm{e}^{-i\lambda t} \langle G^+(r, r, E_F + \lambda/2) G^-(r, r, E_F - \lambda/2)\rangle\,, \tag{1}$$

where ρ is an average spectral density, and $\langle ... \rangle$ is the connected average over the underlying disorder. Ergodicity implies that the disorder average and spectral average are the same.

The process of averaging is equivalent to integrating over "fast" variables of the system. As a result (for details see *e.g.* [2]) the return probability is given by a classical formulae (in t space and Fourier space, respectively)

$$P(t) = \sum_Q \mathrm{e}^{-DQ^2 t}\,, \qquad P(\lambda) = \sum_Q \frac{1}{-i\lambda + DQ^2}\,, \tag{2}$$

where the details of integration over the fast variables are hidden in the diffusion constant D, and the diffusion modes Q are "slow" (effective) degrees of freedom. Since in the infinite volume limit the solution (2) reduces to the classical solution of the Helmholtz equation, the effective "slow" variable is sometimes called a "diffuson".

The diffusion pole could easily develop a gap, $-i\lambda+DQ^2 \to -i\lambda+\gamma+DQ^2$ if an exponential damping $\exp(-\gamma t)$ tantamount of loss of coherence, multiplies $P(t)$. The damping is related to a finite coherence length $L_{\rm coh}$ as

$$\gamma = D/L^2_{\rm coh}\,. \tag{3}$$

In the ergodic regime (and ignoring the damping γ), the return probability is always 1. Technically, one could recover this result from (2) by restricting the sum to *zero modes* Q=0. In this way the results are space independent (infinitely long range, or equivalently, no derivatives), and therefore belong to the same universality class as ensembles of random matrices (*i.e.* a field theory in 0 dimensions). What only matters are the underlying symmetries of the Hamiltonian H, leading in this way to the famous Dyson's threefold path: Gaussian Unitary (GUE), Orthogonal (GOE) and Symplectic (GSpE) Ensembles, for hermitian, real and skew-symmetric Hamiltonians, respectively [3].

3.

The idea that light quarks in the QCD vacuum might undergo (one-dimensional) random walk was pointed out by Banks and Casher [4]. It was later on suggested in the context of the instanton liquid model [5, 6] that light quarks may behave like in a 'semiconductor' depending on the instanton density in the vacuum. This point was established using numerical simulations with instantons [6, 7]. In this context it is suggestive to compare the Einstein–Kubo–Greenwood formula for the conductivity σ of a dc-current to the to the Banks-Casher relation for the quark condensate as suggested explicitly in [8]

$$\sigma = D\rho(E_F) \qquad \leftrightarrow \qquad |\langle\bar{q}q\rangle| = \frac{1}{\pi V_4}\rho(0)\,, \tag{4}$$

where $\rho(E_F)$ is the average electronic density at the Fermi level, and $\rho(0)$ is the average spectral quark density of the eigenvalues of the Dirac operator in QCD, averaged over all gluonic configurations.

What is new in our present arguments is that the analogy is indeed quantitatively realized in the QCD vacuum. Despite the fact that they confine, light quarks diffuse in d=4 in the QCD vacuum [1]. The conductivity is played by the pion decay constant F^2, and the Einstein–Kubo–Greenwood formula is precisely realized in QCD in the form of the (GOR) Gell-Mann–Oakes–Renner relation [9] [1]. The idea that light quarks diffuse in d=4 is key to understanding a number of phenomena in QCD in light of results in disordered electronic systems. More importantly, it allows us to organize certain aspects of infrared QCD as corrections to the Ohmic conductivity, thereby providing a new and nonperturbative calculational scheme.

Indeed, the eigenvalue equation of the massless Dirac operator for fundamental quarks in a fixed gluon field A in Euclidean volume V_4

$$i\nabla\!\!\!/[A]\, q_k = \lambda_k[A]\, q_k \tag{5}$$

allows us to extend the theory into 4+1 dimension with proper time t, and define the probability $P(t)$ for a light quark to start at $x(0)$ in V and return back to the same position $x(t)$ after a proper time duration t. Here the four-dimensional "Hamiltonian" $i\nabla\!\!\!/[A]$ acts as a generator for the evolution operator along the additional time t, so the diffusion picture dwells in 4+1 dimensions.

We restrict our description to zero virtuality, *i.e.* we focus on the diffusion of quarks corresponding to $E_F \sim 0$, by analogy with (4). Therefore $P(t)$ reads [1], in analogy to (1)

$$P(t) = \frac{V_4}{2\pi\rho}\lim_{y\to x}\int d\lambda \mathrm{e}^{-i\lambda t}\Big\langle \mathrm{Tr}\left(S(x,y;z)S^{\dagger}(x,y;\bar{z})\right)\Big\rangle_A \tag{6}$$

with *complex* $z = m - i\lambda/2$, and Green's function (quark propagator)

$$S(x,y;z) = \langle x|\frac{1}{i\nabla\!\!\!/[A] + iz}|y\rangle\,. \tag{7}$$

[1] Recently, Stern [10] has presented arguments in which F^2 is also interpreted as a conductivity but challenged the conventional form of the GOR relation and the Einstein–Kubo–Greenwood formula for light quarks.

Here $\langle ... \rangle$ denotes *connected* average over all gluonic configurations in the QCD vacuum, making at first sight the r.h.s. intractable. Nevertheless, it is possible to relate the classical return probability to the properties of the pion, since $S^\dagger(x,y;\bar{z}) = -\gamma_5 S(y,x;z)\gamma_5$. Therefore the return probability reads

$$P(t) \sim \lim_{y\to x} \int d\lambda\, \mathrm{e}^{-i\lambda t} \mathbf{C}_\pi(x,y;z)\,, \tag{8}$$

where

$$\mathbf{1}^{ab}\, \mathbf{C}_\pi(x,y;z) = \left\langle \mathrm{Tr}\left(S(x,y;z) i\gamma_5 \tau^a S(y,x;z) i\gamma_5 \tau^b \right) \right\rangle_A . \tag{9}$$

We recognize that $\mathbf{C}_\pi$ is the analytically continued (with $z = m - i\lambda/2$) pion correlation function[2], given (for $z = m$), due to pion-pole dominance, by

$$\mathbf{C}_\pi(x,y;m) \approx \frac{1}{V_4} \sum_Q \mathrm{e}^{iQ\cdot(x-y)} \frac{\Sigma^2}{F_\pi^2} \frac{1}{Q^2 + m_\pi^2} \tag{10}$$

with $Q_\mu = n_\mu 2\pi/L$ in $V_4 = L^4$ and $\Sigma = |\langle \overline{q}q \rangle|$.

Using the GOR [9] relation $F_\pi^2 m_\pi^2 = m\Sigma$, and the analytical continuation $m \to z = m - i\lambda/2$, we find [1]

$$\mathbf{C}_\pi(x,y;z) \approx \frac{1}{V_4} \sum_Q \mathrm{e}^{iQ\cdot(x-y)} \frac{2\Sigma}{-i\lambda + 2m + DQ^2} \tag{11}$$

with the diffusion constant $D = 2F_\pi^2/\Sigma$, in full analogy to (2). Inserting (11) into (8), and using the Banks–Casher relation we conclude, after a contour integration that indeed

$$P(t) = \mathrm{e}^{-2mt} \sum_Q \mathrm{e}^{-DQ^2 t}\,. \tag{12}$$

For very large current quark masses m, the diffusion is suppressed. For small m, we see that twice the current quark mass plays the role of the damping γ, introducing a cutoff on infinitely long diffusion paths. For $m \neq 0$, the coherence length corresponding to the collective excitations of the QCD vacuum is finite, $L_{\mathrm{coh}} = 1/m_\pi$, and we see that this is again consistent with the GOR formula (*cf.* (3) with $D = 2F_\pi^2/\Sigma$). The analytically continued pion propagator plays the role of the "diffuson" degrees of freedom, after integrating the QCD fast variables. The effective Lagrangian for pions is organized

[2] The insertion of the isospin generators τ^a is a formal 'trick' to project onto the connected part of the correlation function.

on the basis of chiral counting [11], and similarly effective Lagrangian for diffusions are basically sigma models [2].

It is now easy to identify all the relevant length scales defined in Section 2 and separating the different regimes of the QCD vacuum viewed as a disordered medium. The scale which separates the quantum regime from the ergodic one is the average level spacing Δ from the spectral function of the Dirac operator, $\Delta=\pi/\Sigma V_4$. The Thouless energy equals $D/\sqrt{V_4}$ with $D=2F_\pi^2/\Sigma$. In this way we define the ergodic regime. The diffusive regime starts to be relevant for eigenvalues of the Dirac operator greater than the Thouless energy. The diffusive regime merges with the ballistic regime when the eigenvalues approach twice the value of the *constituent* quark mass. Indeed, if the propagation in time is not enough to cover a mean free path, the concept of "dressing" the quark through multiple collisions becomes obsolete. We note that the segregation of scales take place naturally in a *finite* Euclidean volume V_4, hence the usefulness of the 'box' [1].

4.

It is inspiring to see how the universal (ergodic) regime appears as a limit of the diffusive regime. This regime gained recently a lot of attention in 'random' QCD, due to several remarkable agreements between predictions based on chiral random matrix models and lattice QCD simulations. Like in disordered metallic systems, where the ergodic regime appears as a consequence of restricting to zero modes of diffusion, a similar simplification operates in QCD. For energy scales *smaller* than the Thouless energy, the quark return probability is equal to 1, which again corresponds to the zero-mode approximation. Since restricting to $Q=0$ is equivalent to keeping only constant modes (no derivatives), much of the QCD dynamics becomes irrelevant. The partition function for *full* QCD in this regime is simply [11]

$$Z(m)=\int dU \mathrm{e}^{\Sigma V_4 \mathrm{Tr}\,(m(U+U^\dagger))}\,. \tag{13}$$

One sees that the r.h.s. depends *solely* on the way chiral symmetry is spontaneously broken (the integration dU over the coset space of Goldstone modes) and on the explicit pattern of breaking chiral symmetry (exponent), here in the $(N_f,\bar{N}_f)+(\bar{N}_f,N_f)$ representation. Like in the nonchiral case, here also only three generic scenarios are possible, depending on the symmetries of the Dirac operator [12]. For QCD with $N_c\geq 3$ where quarks are in fundamental representation, theory belongs to the universality class of Chiral GUE. For QCD with $N_c=2$ the theory belongs to the universality class of Chiral GOE. For QCD with quarks in the adjoint representation the theory belongs to the universality class of Chiral GSpE.

The "chirality" (off-block diagonal structure) of the random ensembles is the remnant of the chiral property $[i\not{D}, \gamma_5]_+ = 0$. All three possibilities have a simple interpretation from the point of view of approaching the ergodic regime from the diffusive regime [1]. For QCD with three or more colors, each quark orbit is traversed once. For QCD with two colors, each quark orbit is traversed twice, since $SU(2)$ does not 'distinguish' between quarks and antiquarks. Finally, the adjoint representation of quarks means that we have supersymmetric QCD (same group representation for quarks and gluons). Intuitively, in this case the quark traverses only "half" of the orbit, due to the fact that supersymmetry is "taking a square root" from the Dirac equation. More formally, in the diffusive regime of QCD one could postulate following semiclassical arguments from classical chaos [13]

$$K(t) \approx 2t\Delta^2/(4\pi^2\beta)P(t)\,, \tag{14}$$

where the spectral form factor $K(t)$ is directly related to two-level quantum correlation function, and $\beta = 1, 2, 4$ for Chiral GUE,GOE,GSpE, respectively.

To summarise: we would like to stress once more, that several nontrivial results for QCD below the Thouless energy, like the existence of the Leutwyler-Smilga sum rules [14], the existence of universal microscopic correlators suggested by the Stony Brook group [15] and proven to be universal by the Copenhagen group [16], the observation of universal oscillations in lattice spectra [17] and other results, are all various facets of the same fact that "$P(t) = 1$ in the ergodic regime."

5.

The diffusive picture of the QCD vacuum, sketched here on the basis of analogies to condensed matter physics, has several relations to existing descriptions of the spontaneous breakdown of chiral symmetry. In the language of chiral power counting, the ergodic regime corresponds to the limit $mV_4 \sim 1$, whereas the diffusive regime to the limit $m^2V_4 \sim 1$. The first counting dwarfs the contribution of the non-zero modes to the pion propagator $\mathbf{C}_\pi$, leaving only the zero mode. The second counting enhances the contribution of the non-zero modes over the zero mode, leading to standard chiral perturbation theory [11].

It is interesting to compare the diffusive picture to the model of instantons, since the analytical scenario put forward by Diakonov and Petrov [5] and numerical one suggested by Shuryak [6] are (in our knowledge) the first detailed realization of some of the ideas on disorder in a QCD model. In the instanton model, the average $\langle ... \rangle$ over all gluonic configurations is performed explicitly, assuming an ansatz for a random model of instantons and antiinstantons. Disorder comes from random positions and colors of instantons, and chirality is built in through the chiral properties of (right/left) fermionic zero modes for each instanton (antiinstanton). Since in the instanton model one could express parametrically each dimensionfull quantity in terms of the average instanton radius $\bar{r}$ and average inter-instanton distance $R \sim 1$ fm, the smallness of the diffusion constant $D = 0.22$ fm reflects simply the diluteness of the instanton vacuum, where $\bar{r}/R = 0.2 - 0.3$. The fact that the instanton vacuum has a regime consistent with (13) is well known. A recent numerical study by Osborn and Verbaarschot [18] has also confirmed the existence of the Thouless energy in this model, as we originally predicted [1].

Perhaps the most interesting comparison of our predictions is a direct lattice simulation. From (14), a direct comparison to the lattice is possible since the l.h.s. is explicitly measured on the lattice, and the r.h.s. is explicitly calculable for all regimes of disorder. The first interesting investigation confirming some of the results presented here was recently carried out by Berbenni–Bitsch *et al.* [19] In particular, they have considered the dimensionless ratio $\lambda_{\rm Max}/\Delta$, where $\lambda_{\rm Max}$ is the *maximal* eigenvalue of the Dirac operator for which the universal predictions based on random matrix theory still hold. Since $\lambda_{\rm Max}$ is nothing but the Thouless energy, this dimensionless ratio reads

$$\frac{E_c}{\Delta} = \frac{2}{\pi} F_\pi^2 L^2 \,, \tag{15}$$

where we used the Banks–Casher relation and the preceding definitions for the Thouless energy E_c and the diffusion constant D, respectively. By studying various lattice sizes, the authors [19] have observed that the scaling is consistent with (15) and identified the Thouless energy for the lattice.

6.

The present ideas on chiral disorder translate concepts of disorder from condensed matter physics to low-energy chiral QCD. Most of the results are directly amenable to lattice studies.[3] A number of non-perturbative investigations can be carried out in the present context. In particular,

— the effects of finite temperature and chemical potential on the diffusive properties of the QCD vacuum;

— the role of the number of flavors and finite θ (strong CP violating) angle on the diffusion scenario;

[3] We also hope that the ballistic regime is amenable to a lattice investigation despite the proximity of this regime to the UV-spectrum.

— the change of the spectral properties of the 'diffusion' at the critical temperature and the dependence of the return probability on the critical exponents in QCD phase transitions;

— the modifications of the diffusion due to the addition of electromagnetic and chromomagnetic external fields (negative magnetoresistance of the QCD vacuum, Bohm-Aharonov like effects due to the presence of fluxons or maximally abelian-projected monopoles).

Some of these issues will be brought up next.

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