HOW TO TRACE HEATING AND COOLING OF NUCLEAR MATTER? * **

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Selected studies of different phases of the violent collisions between heavy ions at the intermediate energies are reviewed.

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1. Introduction

The primary aim of studying a hot nuclear matter is to understand the phase diagram (or equation of state) of strongly interacting matter. There are theoretical predictions that nuclear matter undergoes to another phases, namely, nucleon gas phase, hot hadron gas phase and quark–gluon plasma phase.

In this article, I focus on the investigation of nuclear matter through violent collisions between heavy nuclei in the intermediate energy domain. In such collisions, the fragmentation into many pieces of the created nuclear system is observed. This novel process is called the multifragmentation and is considered as a possible manifestation of the liquid-gas phase transition in finite nuclear systems. A collision event can be characterized by following phases:

- The early phase, when the relative kinetic energy of colliding ions is converted into various forms of excitations.
- The freeze-out time instant is defined as such a phase of the reaction when the nuclear interaction between the freeze-out volume constituents terminates.

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• The final phase, the hot reaction products separate by their mutual Coulomb interaction and release their excitation energies by secondary decays.

In following sections, selected studies performed by our group of different phases of the violent collisions between heavy ions at the intermediate energies are reviewed.

2. Early phase

In order to follow the dynamical aspect of the collisions between heavy ions in the early phase, we utilized the molecular dynamics concept. Our model [1], or rather its numerical implementation dubbed CHIMERA (Code for Heavy Ion Medium Energy ReActions,) is a compilation of two recently devised models, namely, the Quantum Molecular Dynamics (QMD) model of Aichelin and Stöcker [2] and the Quasi-Particle Dynamics (QPD) model of Boal and Glosli [3]. The code is described in detail elsewhere [1] and here we shall specify important characteristics only:

- The scattering of the nucleons is treated as if they were free (stochastic scattering with experimental nucleon–nucleon cross section). The collisions are statistically independent and the interference between two different collisions is neglected.
- Nucleons are represented by a constant width Gaussian wave packets which fulfil the requirement of the uncertainty principle.
- The real part of the transition matrix is replaced by an effective potential. The nuclear effective potential was derived from a Skyrme parametrization of the potential energy density and was supplemented with the Coulomb potential and the momentum dependent Pauli potential. The bell shape Pauli potential was implemented to simulate the fermionic nature of the nucleons. It introduces repulsion among the nucleons of the same kind whenever they come too close in the phase space.
- Application of the frictioning cooling method [3] to finding the ground state configuration allowed to reproduce very well the experimental value of the ground state binding energies and rms radii.
- In order to trace the time evolution of the reaction the following procedure was applied. During each event of the simulation, the position and linear momentum of each nucleon were stored at selected time steps. By tracing back through the stored values and averaging over many events, one could determine the time evolution of various quantities.

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Here, we present results of the model calculations for the 35 MeV/nucleon 64 Cu + 232 Th reaction which has been investigated experimentally [4]. The time, t = 0 fm/c, corresponds to the configuration when surfaces of the projectile and target nuclei are separated by 3 fm. The simulations were performed for 4 impact parameter bins of 1 fm width.

Fig. 1(a) depicts the temporal evolution of the total emission rates of nucleons. A large component at early times corresponds to significant preequilibrium emission and a smaller emission at later times represents a statistical evaporation from a thermalized compound system for the violent collisions and from a hot projectile and target like fragments for a larger impact parameters. Fig. 1(b) displays the time evolution of the mean density of the whole system. Here, the most central collisions are characterized by a rather large compression which occurs at the time $t \cong 40 \text{ fm}/c$ followed by an expansion. Fig. 1(c) presents the time evolution of the z-th component of a quadrupole moment tensor of one body density in the momentum space, Q_{zz} . The quadrupole momentum tensor allows to study of the longitudinal momentum component dissipation and reflects the degree of the system equilibration. The amount of the entrance channel kinetic energy converted into the internal excitation energy increases with decreasing impact parameter. For the most central collisions the transversal expansion of the system temporarily dominates the longitudinal expansion, what is indicated by the negative Q_{zz} value. Figs 1(d), (e) and (f) show the time evolution of the number of nucleons bound in the fused system, its normalized longitudinal momentum and velocity, respectively. During the early phase of the collision (t < 120 fm/c), the longitudinal momentum of the fused system quickly decreases reflecting the pre-equilibrium particle emission. A constant velocity of the composite system after t = 120 fm/c indicates the isotropic emission of particles and that the pre-equilibrium emission has been ceased. Since the composite system continues to decay statistically, its longitudinal momentum decreases slowly. We assumed [5], that the longitudinal momentum carried by the fused system, which is an experimentally investigated observable, is directly related to the incompressibility of nuclear matter. This quantity is also called the compression modulus and denoted by K. Since the CHIMERA code explicitly includes incompressibility, we examined different combinations of the projectiles, targets and incident energies with respect to the longitudinal momentum transfer and in consideration of two types of the compression modulus, namely, K = 380 MeV and K = 200 MeV.

A comparison of the CHIMERA calculations [5] with the results of the fractional linear momentum transfer measurements shows that the model calculations with the compression modulus K = 200 MeV very well reproduce the experimental data. On the contrary, in the case of K = 380 MeV, the model calculations considerably overestimate the linear momentum



Fig. 1. Time evolution of the emission rate (a), mean density (b), the relative quadrupole moment in momentum space (c), the number of nucleons bound in the fused system (d), the linear momentum transfer (e) and the longitudinal velocity of the fused system in the laboratory frame (f). The calculations have been performed for the system produced in the $^{64}Cu+^{232}Th$ reaction at 35 MeV per nucleon. The respective lines depict the evolution for different impact parameter bins.

transfer to the fused system. Concluding, the model calculations undoubtfully favor the soft equation of states ($K \cong 200$ MeV).

3. Freeze-out stage

It was shown in the previous section that composite system created in the violent heavy ion collision reaches a high degree of equilibrium in the early stage of the reaction. This means that the use of the conventional thermodynamic concepts of equilibrium states is justified. In our study [6], we applied the quantum statistical thermodynamics approach to investigate hot finite nuclear systems.

Quantum statistical mechanics allows calculation of the number of available states and their occupation for the particle of mass m_i , spin degeneracy g_i , and chemical potential, μ_i in the volume \mathcal{V} . The number of i^{th} species of particle, N_i , is given for bosons and fermions by the Bose–Einstein [7] and Fermi–Dirac [8] integral functions, respectively.

In our calculations we assumed:

- Isotopes with mass A up to 19 are allowed to appear in thermally and chemically equilibrated system (68 isotopes are included).
- Each isotope in an excited state is treated as a separate species. The excited states of the fragments with $\Gamma \leq 1$ MeV are taken into account $(t_{1/2} > 5 \cdot 10^{-22} \text{ sec}).$
- The Coulomb interaction energy between the fragments is taken into account using the Bondorf *et al.*, prescription [9].

Moreover, we considered the freeze-out stage of the system for which the surfaces of the fragments become well separated from each other, on average, by a distance d, of the order of the range of nuclear forces. The freeze-out volume is dependent on the total fragment multiplicity. An estimation of the freeze-out volume may be obtained using the prescription of Bondorf *et al.*, [9].

We have to emphasis that our model is designed to investigate the disintegration of highly excited nuclei as vaporization is approached. Due to the condition that only isotopes with $A \leq 19$ are allowed to exist in the hot nucleus considered, our model can only be applied to systems at temperatures such that the probability of the fragment population with A > 19 is negligible. The low temperature limit of the model was established by studying the fragment mass population distributions in thermally and chemically equilibrated ¹⁹⁷Au and ¹⁰⁰Mo nuclei over a broad temperature range. The results presented in Fig. 2 show that for freeze–out temperatures, $T_{\rm fo} \geq 5$ MeV, the modification of the primary fragment mass distribution caused by extension of the fragment mass range from 17 (dashed line) to 19 (solid line) is negligible. So, the extension of the model to the heavier species than A = 19 for the system temperatures, $T_{\rm fo} \geq 5$ MeV, would produce a negligible effect on the fragment population probability distribution. Our results are, therefore,



Fig. 2. The fragment mass population distributions in the thermodynamically and chemically equilibrated $^{197}{\rm Au}$ and $^{100}{\rm Mo}$ nuclei at the temperatures indicated.

assumed to be valid for nuclear systems hotter than 5 MeV. This temperature coincides with the temperature at which a wide plateau in the caloric curve was reported experimentally (see *e.g.* [10]). The plateau was ascribed to a liquid–vapor phase and our calculations predict condensation of large nuclear drops at this temperature.

Figure 3 depicts the relationships between the freeze-out temperature and the double isotope ratios (plotted as $1/\mathcal{R}$) where \mathcal{R} represents the yield



Fig. 3. The system temperature at the freeze-out as a function of the double yield ratios of two final isotope pairs.

ratios of cold final products, i.e.

$$\mathcal{R}_{\mathrm{H/He}} = \frac{Y(^{2}\mathrm{H}) \cdot Y(^{4}\mathrm{He})}{Y(^{3}\mathrm{H}) \cdot Y(^{3}\mathrm{He})}$$
(1)

and

$$\mathcal{R}_{\rm He/Li} = \frac{Y({}^{4}\rm He) \cdot Y({}^{6}\rm Li)}{Y({}^{3}\rm He) \cdot Y({}^{7}\rm Li)}$$
(2)

The cold fragment yields were obtained from the freeze–out fragment population using the code GEMINI [11]. One can see that both combinations of the double yield ratios of two pairs of isotopes, $\mathcal{R}_{\rm H/He}$ (see Fig. 3 upper panel) and $\mathcal{R}_{\rm He/Li}$ (see Fig. 3 lower panel) show a smooth and monotonic dependence on the temperature of the disintegrating system. Fig. 3 also shows that the calculated double yield ratios of final cold isotope pairs depends very weakly on the size of the disintegrating system (compare the circles and squares plotted for ¹⁹⁷Au and ¹⁰⁰Mo nuclei, respectively). Here, we conclude that the isotopic yield ratios of the final cold products can be used as the sensitive gauge of the system temperature at the freeze-out time. This result encourages us to derive a phenomenological formula which will allow calculation of the temperature of the decaying nuclear system at the freeze-out time from the experimentally observed double yield ratios of two

isotope pairs. A simple formula given below can be applied for both pairs of the isotope combinations and for light and heavy nuclei:

$$T_{\rm fo} = p_0 + p_1 \cdot \mathcal{R}^{-1} + p_2 \cdot \mathcal{R}^{-2} + p_3 \cdot \mathcal{R}^{-3}, \qquad (3)$$

where \mathcal{R} is defined in Eq. (1) or (2) and parameters p_i are given in Table I. TABLE I

A	\mathcal{R}	p_0	p_1	p_2	p_3
197	${\mathcal R}_{ m H/He} \ {\mathcal R}_{ m He/Li}$	$3.13 \\ 3.07$	$70.77 \\ 34.51$	$146.62 \\ -24.32$	-325.88 21.90
100	${{{\cal R}_{ m H/He}} \over {{{\cal R}_{ m He/Li}}}}$	$3.30 \\ 2.98$	$56.25 \\ 37.84$	$305.32 \\ -30.23$	$-683.42 \\ 27.06$

Excellent fits of the polynomial parametrization (Eq. (3)) to the calculated dependence of the freeze–out temperature on the observed double yield ratios are shown by the dotted lines in Fig. 3. In conclusion, a phenomenological formula was found, which allows derivation of the temperature of the decaying nuclear system at the freeze-out time from the measured double yield ratios of two isotope pairs.

4. De-excitation phase

In this section, a reconstruction of the primary fragment excitation energy is presented. Our procedure [12] is applied to the $^{84}Kr + ^{93}Nb$ reaction at 45A MeV [13]. We focus on the violent collisions which has been selected using the total collected charge condition.

In order to reconstruct the primary fragment excitation energies the multiplicities of the evaporated LCPs from the primary excited fragments have to be determined. Due to overlapping of the LCPs which originate from different sources it is impossible to associate the emitted LCPs to the chosen IMF on an event by event basis. However, if the detected IMF and the detected LCP originate from the same parent then both detected particles are correlated. If we calculate the relative velocities between the selected IMF and all LCPs on an event by event basis then the correlation of the evaporated LCPs from the primary excited fragments is seen as a small excess of counts around the IMF position in v_{\parallel} versus v_{\perp} frame, where v_{\parallel} and v_{\perp} are the projections of the relative velocity into the axis representing the IMF

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direction in the CM system and into a plane perpendicular to that axis, respectively. In order to eliminate uncorrelated emission the background plot has been constructed by replacing the relative velocity of LCP's which coincide with the given IMF by the relative velocity of the uncorrelated LCPs produced in a different event. The result of the subtraction of the background from the total LCP emission shows that the counts clearly surround the IMF's position.

In order to extract the quantitative information on the LCP emission associated with the selected IMFs, a Monte Carlo simulation has been introduced [12]. This simulation begins at the freeze-out time instant assuming that the total excitation energy of the system splits into three modes: (i) a chaotic thermal motion of the point-like objects, *(ii)* a collective expanding mode (flow), and *(iii)* an internal excitation of the fragments. The subsequent evolution of the system involves the Coulomb dynamics and the statistical decays of the excited IMFs. The calculations are performed until the excited fragments become cold and the Coulomb acceleration becomes negligible. Finally, the generated events are filtered through the detection system to match experimental conditions. The results of the background subtraction from the total LCP emission have been used to obtain the experimental correlation function presented in Fig. 4 as the solid line. This correlation function displays the number of events as a function of the relative energy between the LCP and the daughter IMF. In order to reproduce simultaneously the experimental correlation functions for all types of LCPs which are correlated with the selected bin of the detected IMFs, the correlated LCP multiplicities, $M_{exp}(k, j)$ have been searched. The agreement between the experimental correlation functions and those generated using the best set of the multiplicities, $M_{\exp}(k,j)$ is excellent for the protons and the alpha particles and quite satisfactory for the other LCP isotopes with much lower statistics (see dashed lines in Fig. 4).

To convert the LCP multiplicities into the fragment excitation energies, the statistical code GEMINI [11] has been applied. A large set of fragments at selected temperatures has been cooled down. The charge distribution of the excited fragments at given temperature was the same as for the freezeout partition. Neutrons have not been detected in the considered experiment and two assumptions on the N/Z ratio of the primary fragments have been investigated:

- the fragments N/Z ratio is the same as for the beta-stable nuclei
- the fragments N/Z ratio is the same as in the combined target-projectile system (N/Z = 1.3).

The GEMINI calculations supply the average LCP multiplicities for each k bin of the IMF remnants, $M_c(k, j)$ at a given temperature (T) where j



Fig. 4. The results of the substractions of the background from the total LCP emission as a function of the IMS-LCP relative energy. Solid and dotted lines are related to the experimental data and the model calculation, respectively.

TABLE II

valley of st	ability	N/Z = 1.3		
E^*/A [MeV]	$T \; [\text{MeV}]$	$E^*/A \; [\text{MeV}]$	T [MeV]	
$1.95\substack{+0.4 \\ -0.35}$	$4.4_{-0.4}^{+0.45}$	$2.5_{-0.4}^{+0.4}$	$5.0^{+0.4}_{-0.4}$	

represents the type of the LCP. In order to obtain a quantitative comparison between the calculated multiplicities and those derived from the experimental data, the χ^2 criteria has been used. The values of the fragment excitation energies and temperatures at the minima of the χ^2 are listed in Table II. The errors included in the Table II have been obtained assuming that the LCP multiplicities have been deduced from the experimental data with uncertainties of $\pm 25\%$. This analysis shows that assuming the same N/Z ratio in the fragments as in the combined target–projectile system leads to higher fragment excitation energies as compared to the assumption that the fragments

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N/Z ratio is the same as for the beta-stable nuclei. A better agreement between theory and experimental results according to the earlier assumption, indicates that the system preserves the entrance channel N/Z ratio. Further aspects of the primary fragment excitation energies are in progress.

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