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MIXING ANGLES AND ELECTROMAGNETIC PROPERTIES OF GROUND STATE PSEUDOSCALAR AND VECTOR MESON NONETS IN THE LIGHT-CONE QUARK MODEL*

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Both the mass spectra and the wave functions of the light pseudoscalar (π, K, η, η') and vector$(\rho, K^*, \omega, \phi)$ mesons are analyzed within the framework of the light-cone constituent quark model. A gaussian radial wave function is used as a trial function of the variational principle for a QCD motivated Hamiltonian which includes not only the Coulomb plus confining potential but also the hyperfine interaction to obtain the correct $\rho - \pi$ splitting. The mixing angles of $\omega - \phi$ and $\eta - \eta'$ are predicted and various physical observables such as decay constants, charge radii, and radiative decay rates *etc.* are calculated. Our numerical results have a good agreement with the available experimental data.

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Using the quark potential model, we attempt to fill the gap between the model wave function and the QCD motivated potential, which includes not only the Coulomb plus confining potential but also the hyperfine interaction to obtain the correct ρ–π splitting. For the confining potential, we take (1) harmonic oscillator potential and (2) linear potential and compare the numerical results for these two cases. We use the variational principle to solve the equation of motion. Accordingly, our analysis covers the mass spectra of light pseudoscalar(π, K, η, η') and vector$(\rho, K^*, \omega, \phi)$ mesons and the mixing angles of $\omega - \phi$ and $\eta - \eta'$ as well as other observables such as charge radii, decay constants, radiative decay widths *etc.*

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TABLE I
Optimized quark masses (m_q, m_s) [GeV] and the gaussian parameters β [GeV] for both harmonic oscillator and linear potentials obtained from the variational principle. q=u and d.

Potential	m_q	m_s	$\beta_{q\bar{q}}$	$\beta_{s\bar{s}}$	$\beta_{q\bar{s}}$
H.O.[Lin.]	.25[.22]	.48[.45]	.3194[.3659]	.3681[.4128]	.3419[.3886]

The QCD motivated effective Hamiltonian for the description of the meson mass spectra is given by[1]

$$H_{q\bar{q}} = H_0 + V_{q\bar{q}} = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{q\bar{q}}\,. \qquad (1)$$

We use the usual confining interaction potential $V_{q\bar{q}} = V_0(r) + V_{\rm hyp}(r)$ given by

$$V_{q\bar{q}} \;=\; a + b\mathcal{V}_{\rm conf.}(r) - \frac{4\kappa}{3r} + \frac{2\vec{S}_q\cdot\vec{S}_{\bar{q}}}{3m_q m_{\bar{q}}}\nabla^2 V_{\rm Coul}\,, \qquad (2)$$

where $\mathcal{V}_{\rm conf.}(r) = r[r^2]$ is linear[harmonic oscillator] type potential. Our variational method first evaluates $\langle\phi|[H_0 + V_0]|\phi\rangle$ with a trial function $\phi(k^2) = Ne^{-k^2/2\beta^2}$ that depends on the parameters (m,β) and varies these parameters until the expectation value of $H_0 + V_0$ is a minumum. Once these model parameters are fixed, then, the mass eigenvalue of each meson is obtained by $M_{q\bar{q}} = \langle\phi|[H_0 + V_0]|\phi\rangle + \langle\phi|H_{\rm hyp}|\phi\rangle$.

Our determination of model parameters is summarized in Table I. The mixing angles from the mass spectra of (ω,ϕ) and (η,η') were also determined by incorporating the quark-annihilation diagrams mediated by gluon exchanges including the SU(3) symmetry breaking effect, *i.e.*, $m_{u(d)} \neq m_s$. Identifying (f_1, f_2)=(ϕ,ω) and (η,η') for vector and pseudoscalar nonets, the physical meson states f_1 and f_2 are given by

$$\begin{aligned} |f_1\rangle &= -\sin\delta|n\bar{n}\rangle - \cos\delta|s\bar{s}\rangle, \\ |f_2\rangle &= \cos\delta|n\bar{n}\rangle - \sin\delta|s\bar{s}\rangle, \end{aligned} \qquad (3)$$

where $|n\bar{n}\rangle \equiv 1/\sqrt{2}|u\bar{u} + d\bar{d}\rangle$ and $\delta = \theta_{\rm SU(3)} - 35.26^\circ$ is the mixing angle. These combinations satisfy the (mass)^2 eigenvalue equation given by $\mathcal{M}^2|f_i\rangle = M^2_{f_i}|f_i\rangle (i = 1, 2)$. In order to take into account SU(3) symmetry breaking, we use the following parametrization for $\mathcal{M}^2$ suggested by Scadron[2]

$$\mathcal{M}^2 \;=\; \begin{pmatrix} M^2_{n\bar{n}} + 2\lambda & \sqrt{2}\lambda X \\ \sqrt{2}\lambda X & M^2_{s\bar{s}} + \lambda X^2 \end{pmatrix}. \qquad (4)$$

where the parameter X pertains to SU(3) symmetry breaking such that the quark-annihilation graph factors into its flavor parts, with λ , λX and λX^2 for the $u\bar{u} \to u\bar{u}(d\bar{d})$, $u\bar{u} \to s\bar{s}$(or $s\bar{s} \to u\bar{u}$), and $s\bar{s} \to s\bar{s}$ processes, respectively. The $\omega - \phi$ and $\eta - \eta'$ mixing angles are then predicted from the physical masses of $M_{f_1} = (m_\phi, m_\eta)$ and $M_{f_2} = (m_\omega, m_{\eta'})$ as well as the masses of $M^V_{s\bar{s}}$= 996[952] MeV and $M^P_{s\bar{s}}$= 732[734] MeV determined for harmonic oscillator[linear] potential model. Our predictions for $\omega - \phi$ and $\eta - \eta'$ mixing angles for harmonic oscillator[linear] potential are $\delta_V \approx 4.2^\circ[7.8^\circ]$ and $\theta_{\mathrm{SU(3)}} \approx -19.3^\circ[-19.6.^\circ]$, respectively.

In the light-cone quark model approach, we have investigated the mass spectra, mixing angles, and other physical observables of light pseudoscalar and vector mesons using QCD motivated potentials given by Eq.(2). The variational principle for the effective Hamiltonian was crucial to find the optimum values of our model parameters. As an application[3-5], we computed the observables such as charge radii, decay constants, and radiative decays of $P(V) \to V(P)\gamma^*$ and $P \to \gamma\gamma^*$. In Table II, our predictions of radiative decay widths for $V(P) \to P(V)\gamma$ transitions were summarized and compared with the available experimental data. Our numerical results for these observables in the two cases(harmonic oscillator and linear) are overall not much different from each other and are in a rather good agreement with the available experimental data.

TABLE II

Radiative decay widths for the $V(P) \to P(V)\gamma$ transitions. The mixing angles, $\theta_{\mathrm{SU(3)}} = -19^\circ$ for $\eta - \eta'$ and $\delta_V = +3.3^\circ \pm 1^\circ$ for $\omega - \phi$, are used for both potential models, respectively. The experimental data are taken from Ref.[6].

Widths	H.O.	Linear	Experiment[keV]
$\Gamma(\rho^\pm \to \pi^\pm\gamma)$	76	69	68 ± 8
$\Gamma(\omega \to \pi\gamma)$	730 ∓ 1.3	667 ∓ 1.3	717 ± 51
$\Gamma(\phi \to \pi\gamma)$	$5.6^{+3.9}_{-2.9}$	$5.1^{+3.6}_{-2.6}$	5.8 ± 0.6
$\Gamma(\rho \to \eta\gamma)$	59	54	58 ± 10
$\Gamma(\omega \to \eta\gamma)$	6.9 ∓ 0.3	6.3 ∓ 0.3	7.0 ± 1.8
$\Gamma(\phi \to \eta\gamma)$	49.2 ± 1.6	47.6 ± 1.5	55.8 ± 3.3
$\Gamma(\eta' \to \rho\gamma)$	68	62	61 ± 8
$\Gamma(\eta' \to \omega\gamma)$	7.6 ± 0.4	7.0 ± 0.4	6.1 ± 1.1
$\Gamma(\phi \to \eta'\gamma)$	0.36 ∓ 0.01	0.34 ∓ 0.01	< 1.8
$\Gamma(K^{*0} \to K^0\gamma)$	124.5	116.6	117 ± 10
$\Gamma(K^{*+} \to K^+\gamma)$	79.5	71.4	50 ± 5

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