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ON SELF-CONSISTENCY IN THE THEORY OF FINITE FERMI SYSTEMS*

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It is shown how single-particle and collective excitations can be treated consistently in the framework of an extended Landau–Migdal theory.

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1. Introduction

While most of the traditional nuclear-structure theories start from a bare nucleon-nucleon interaction, the theory of Fermi liquids, originally developed by Landau [1] and adapted to the nucleus by Migdal [2] and his followers, aimed at a description of the low-energy nuclear spectrum in terms of a universal effective two-body interaction amplitude. In infinite systems this was sufficient to describe a whole series of low-energy phenomena, since the few free one-body parameters of an isotropic liquid, *e.g.* the effective mass, could be directly linked to specific parts of the universal amplitude. In nuclei however, surface effects are so much more important that a concept as that of the effective mass is by far to crude to account for the complex interplay between fermionic and bosonic degrees of freedom. Consequently, for a reliable microscopic description a link is needed between the mass operator and the two-body interaction amplitude.

2. The consistent system of equations

In the end-sixties the number-operator method was introduced [3] to discuss the over-estimation of the nuclear ground-state correlations by the

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quasi-boson (RPA) method. Formulated in the language of Green's functions, the method amounts to calculating the nuclear response to the number operator

$$\widehat{N} = \sum_{\mu} a_{\mu}^{\dagger} a_{\mu}$$

which plays the role of an external field. The result is the difference

$$\langle \Phi_s(A+1)|\widehat{N}|\Phi_s(A+1)\rangle - \langle \Phi_0(A)|\widehat{N}|\Phi_0(A)\rangle = 1\,.$$

Using the many-body T matrix, which is the correlated part of the two-body Green function, this leads to an equation for the single-particle strength z_s,

$$1 - z_s \;=\; z_s \sum_{1234} \langle\sigma|1\rangle\langle 2|\sigma\rangle \oint \frac{\mathrm{d}\Omega}{2\pi \mathrm{i}}\, T_{1423}(E_s,\Omega,0) \sum_{5} G_{54}(\Omega) G_{35}(\Omega)\,,$$

which is related to the derivative of the mass operator by

$$z_s = \left|\langle \Phi_s(A+1)|a_{\sigma}^{\dagger}|\Phi_0\rangle\right|^2 = \left(1 - M'_{\sigma\sigma'}(E_s)\right)^{-1}\,.$$

In this equation (where the Ω-integration is meant to be closed in the upper half plane) the T matrix (not the mass operator as one might have expected) contains all the nuclear information.

What was not known at that time, and was found later by the author [4], is that there is another operator that plays a role complementary to the number operator, namely the Hamiltonian. Assuming that it has the form of a sum of a kinetic and a (two-body) potential-energy part one can write it as follows:

$$H = \sum_{12} a_1^{\dagger} a_2 + \tfrac{1}{2} \sum_{1234} V_{1234} a_1^{\dagger} a_2^{\dagger} a_4 a_3 = \tfrac{1}{2} \sum_{12} (t_{12} a_1^{\dagger} a_2 + \mathrm{i} a_1^{\dagger} \dot{a}_1 \delta_{12})\,.$$

At the cost of having introduced a time dependence (or an ω dependence after Fourier transformation) the Hamiltonian has adopted a single-particle-like form here. Using it in the same way as the number operator just before, we can calculate the difference

$$\langle \Phi_s(A+1)|H|\Phi_s(A+1)\rangle - \langle \Phi_0(A)|H|\Phi_0(A)\rangle = E_s\,.$$

After using Ritz's variational principle, in order to determine the optimal single-particle basis, this gives the following eigenvalue equation:

$$\begin{aligned} E\langle 1|\sigma\rangle \;=\; \sum_{2} \Bigg\{ t_{12} + \sum_{3456} \oint \frac{\mathrm{d}\Omega}{2\pi \mathrm{i}} T_{1625}(E,\Omega,0) G_{46}(\Omega) G_{53}(\Omega) \\ \times \Big(t_{34} + (\Omega - 2E)\delta_{34} \Big) \Bigg\} \langle 2|\sigma\rangle\,. \end{aligned}$$

Comparing with Dyson's equation,

$$E_\lambda \langle 1|\lambda\rangle = \sum_2 (t_{12} + M_{12}(E_s))\langle 2|\lambda\rangle \,,$$

we see that we have got an expression for the mass operator on the energy shell in terms of the T matrix, which contains all nuclear-structure information. In fact, it can be shown that Landau's basic amplitude is closely related to its particle-hole irreducible part. Now, however, one has to expand it into all three two-body channels. The resulting equation is

$$\begin{aligned}
&T_{1423}(\Omega,\Omega',\omega) = \; K_{1423}(\Omega,\Omega',\omega)\\
&+\mathrm{i}\sum_{1'2'3'4'}\int\frac{\mathrm{d}\Omega''}{2\pi}\int\frac{\mathrm{d}\Omega'''}{2\pi}F_{121'2'}(\Omega,\Omega'',\omega)R_{1'2'3'4'}(\Omega'',\Omega''',\omega)F_{3'4'34}(\Omega''',\Omega',\omega)\\
&-\mathrm{i}\sum_{1'2'3'4'}\int\frac{\mathrm{d}\varepsilon}{2\pi}\int\frac{\mathrm{d}\varepsilon'}{2\pi}F_{131'3'}\left(\tfrac{1}{2}(\Omega+\Omega'+\omega),\varepsilon,\Omega-\Omega'\right)\\
&\times R_{1'3'2'4'}(\varepsilon,\varepsilon',\Omega-\Omega')F_{2'4'24}\left(\varepsilon',\tfrac{1}{2}(\Omega+\Omega'-\omega),\Omega-\Omega'\right)\\
&+\tfrac{1}{4}\mathrm{i}\sum_{1'2'3'4'}\int\frac{\mathrm{d}\varepsilon}{2\pi}\int\frac{\mathrm{d}\varepsilon'}{2\pi}I_{141'4'}\left(\tfrac{1}{2}(\Omega-\Omega'+\omega),\varepsilon,\Omega+\Omega'\right)\\
&\times G^{II}_{1'4'2'3'}(\varepsilon,\varepsilon',\Omega+\Omega')I_{2'3'23}\left(\varepsilon',\tfrac{1}{2}(\Omega-\Omega'-\omega),\Omega+\Omega'\right)
\end{aligned}$$

with

$$R_{1234}(\Omega,\Omega',\omega) = G^{II}_{1423}(\Omega,\Omega',\omega) - 2\pi\delta(\omega)G_{12}(\Omega)G_{43}(\Omega')$$

being what is called the response function, which is the non-decomposing part of the two-body Green function. The new fundamental interaction amplitude is now the K matrix, which is the totally irreducible part of the T matrix. The remaining three terms on the right-hand side of the equation are each reducible in exactly one of the three expansion channels, the corresponding effective interactions being the particle-hole irreducible amplitude F and the particle-particle irreducible amplitude I.

In the simplest possible approximations the above equation leads to Hartree–Fock and Tamm–Dancoff. The next step gives the shifts of the single-particle energies due to particle-phonon coupling, as well as the single-particle strength factors z_s. Already at this level it can be shown that, unlike in conventional theories, no Pauli-principle violation problems arise.

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