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RELATIVISTIC THEORY OF DRIP-LINE NUCLEI*

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Relativistic Mean Field (RMF) theory is used for an investigation or nuclei in the vicinity of the drip-lines Pairing correlations are taken into account by Hartree-Bogoliubov theory based on *pp*-interaction of finite range. Phenomena as neutron skin and neutron halo are discussed within a self-consistent framework.

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1. Introduction

In recent years Relativistic Mean-Field (RMF) models have been successfully applied in calculations of nuclear matter and properties of finite nuclei throughout the periodic table [1]. Using only six or seven parameters they allow a fully self-consistent description of many nuclear properties with high accuracy. As compared to other non-relativistic mean field approximations such as density dependent Hartree–Fock calculations these models have the advantage to provide a consistent description of the spin–orbit term and its isospin dependence.

An essential problem in the theoretical description of drip-line nuclei arises from the closeness of the Fermi level to the particle continuum: particle-hole and pair excitations reach the continuum. The coupling between bound states and the particle continuum has to be explicitly taken into account. The Relativistic Hartree Bogoliubov (RHB) theory [2, 3], which is a relativistic extension of the Hartree Fock Bogoliubov theory, provides a unified description of mean-field and pairing correlations. A fully self-consistent RHB theory in coordinate space [4, 5] correctly describes the coupling between bound and continuum states. The theory provides a framework for describing the nuclear many-body problem as a relativistic system of baryons

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and mesons not only in the valley of β-stability but also in regions with large neutron of proton excess even close to the drip-lines.

2. The relativistic Hartree–Bogoliubov model

In comparison with conventional non-relativistic approaches, relativistic models explicitly include mesonic degrees of freedom and describe the nucleons as Dirac particles. Nucleons interact in a relativistic covariant manner through the exchange of virtual mesons: the isoscalar scalar σ-meson, the isoscalar vector ω-meson and the isovector vector ρ-meson. The model is based on the one boson exchange description of the nucleon-nucleon interaction. We start from the effective Lagrangian density

$$\begin{aligned}\mathcal{L} \;=&\; \bar{\psi}\left(i\gamma\cdot\partial - m\right)\psi + \frac{1}{2}(\partial\sigma)^2 - U(\sigma)\\ &- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{4}\vec{\mathrm{R}}_{\mu\nu}\vec{\mathrm{R}}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^{\,2} - \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\\ &- g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma\cdot\omega\psi - g_\rho\bar{\psi}\gamma\cdot\vec{\rho}\vec{\tau}\psi - e\bar{\psi}\gamma\cdot A\frac{(1-\tau_3)}{2}\psi\,. \end{aligned} \tag{1}$$

The lowest order of the quantum field theory is the *mean-field* approximation. The Dirac equation reads

$$\left\{-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta(m+g_\sigma\sigma) + g_\omega\omega^0 + g_\rho\tau_3\rho_3^0 + e\frac{(1-\tau_3)}{2}A^0\right\}\psi_i = \varepsilon_i\psi_i \tag{2}$$

The effective mass $m^*(\mathbf{r})$ is defined as

$$m^*(\mathbf{r}) = m + g_\sigma\,\sigma(\mathbf{r}), \tag{3}$$

and the potential $V(\mathbf{r})$ as

$$V(\mathbf{r}) = g_\omega\,\omega^0(\mathbf{r}) + g_\rho\,\tau_3\,\rho_3^0(\mathbf{r}) + e\frac{(1-\tau_3)}{2}A^0(\mathbf{r}). \tag{4}$$

In order to describe ground-state properties of spherical open-shell nuclei, pairing correlations have to be taken into account. For nuclei close to the β-stability line, pairing has been included in the relativistic mean-field model in the form of a simple BCS approximation [6]. However, for nuclei far from stability the BCS model presents only a poor approximation. In particular, in drip-line nuclei the Fermi level is found close to the particle continuum. The lowest particle-hole or particle-particle modes are often embedded in the continuum, and the coupling between bound and continuum states has to be taken into account explicitly. The BCS model does not provide a correct description of the scattering of nucleonic pairs from bound states to the positive energy continuum. It leads to an unbound system, because levels in the continuum are partially occupied. Including the system in a box of finite size leads to unreliable predictions for nuclear radii depending on the size of this box. In the non-relativistic case, a unified description of mean-field and pairing correlations is obtained in the framework of the Hartree–Fock–Bogoliubov (HFB) theory in coordinate space [7].

$$\begin{pmatrix} \hat{h}-\lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}+\lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}. \tag{5}$$

HFB-theory, being a variational approximation, results in a violation of basic symmetries of the nuclear system, among which the most important is the non conservation of the number of particles. In order that the expectation value of the particle number operator in the ground state equals the number of nucleons, equations (5) contain a chemical potential λ which has to be determined by the particle number subsidiary condition. The column vectors denote the quasi-particle wave functions, and E_k are the quasi-particle energies.

In the coordinate space representation of the pairing field $\hat{\Delta}$ is

$$\Delta_{ab}(\mathbf{r},\mathbf{r}') = \frac{1}{2}\sum_{c,d} V_{abcd}(\mathbf{r},\mathbf{r}')\kappa_{cd}(\mathbf{r},\mathbf{r}'). \tag{6}$$

where a,b,c,d denote all quantum numbers, apart from the coordinate $\mathbf{r}$, that specify the single-nucleon states. $V_{abcd}(\mathbf{r},\mathbf{r}')$ are matrix elements of a general two-body pairing interaction, and the pairing tensor is defined as

$$\kappa_{cd}(\mathbf{r},\mathbf{r}') = \sum_{E_k>0} U^*_{ck}(\mathbf{r})V_{dk}(\mathbf{r}'). \tag{7}$$

3. Surface diffuseness and spin–orbit reduction at the drip-line

The spin–orbit interaction plays a central role in the physics of nuclear structure. It is rooted in the basis of the nuclear shell model, where its inclusion is essential in order to reproduce the experimentally established magic numbers. In non-relativistic models based on the mean field approximation, the spin–orbit potential is included in a phenomenological way. Of course such an ansatz introduces an additional parameter, the strength of the spin–orbit interaction. The value of this parameter is usually adjusted to the experimental spin–orbit splittings in spherical nuclei, for example ^{16}O. On the other hand, in the relativistic framework the nucleons are described as Dirac spinors. This means that in the relativistic description of the nuclear many-body problem, the spin–orbit interaction arises naturally from the Dirac-Lorenz structure of the effective Lagrangian. No additional strength parameter is necessary, and relativistic models reproduce the empirical spin–orbit splittings.

Many properties of nuclei along the line of beta stability have been successfully described in the framework of models based on the mean-field approximation. Conventional non-relativistic models that include density dependent interactions with finite range (Gogny) or zero-range (Skyrme) forces, have been extensively used to describe the structure of stable nuclei. More recently, it has been shown that models based on the relativistic mean-field theory [9] provide an elegant and economical framework, in which properties of nuclear matter and finite nuclei, as well as the dynamics of heavy-ion collisions, can be calculated (for a recent review see [1]). In comparison with conventional non-relativistic approaches, relativistic models explicitly include mesonic degrees of freedom and describe the nucleons as Dirac particles. Non-relativistic models and the relativistic mean-field theory predict very similar results for many properties of beta stable nuclei. However, cases have been found where the non-relativistic description of nuclear structure fails. An example is the anomalous kink in the isotope shifts of Pb nuclei [10]. This phenomenon could not be explained neither by the Skyrme model, nor by the Gogny approach. Nevertheless, it is reproduced very naturally in relativistic mean-field calculations. A more careful analysis [11] has shown that the origin of this discrepancy is the isospin dependence of the spin–orbit term.

In the following we present results for the chain of Li and Zr isotopes. We find that in the framework of relativistic mean field theory, the magnitude of the spin–orbit potential is considerably reduced in light drip line nuclei. With the increase of the neutron number, the effective one-body spin–orbit interaction becomes weaker. This result in a reduction of the energy splittings between spin–orbit partners. The reduction of the spin–orbit potential is especially pronounced in the surface region, and does not depend on a particular parameter set used for the effective Lagrangian. These results are at variance with those calculated with the non-relativistic Skyrme model. It has been shown that the differences have their origin in the isospin dependence of the spin–orbit terms in the two models. If the spin–orbit term of the Skyrme model is modified in such a way that it does not depend so strongly on the isospin, the reduction of the spin–orbit potential is comparable to that observed in relativistic mean-field calculations.

4. Halo-phenomena at the neutron drip-line

In Fig. 1 we show she calculated density distribution for the neutrons in the isotopes ^{9}Li and ^{11}Li. It is clearly seen that the increase of the matter radius is caused by a large neutron halo in the nucleus ^{11}Li. Its density distribution is in very good agreement with the experimental density of this isotope show with its error bars by the shaded area.

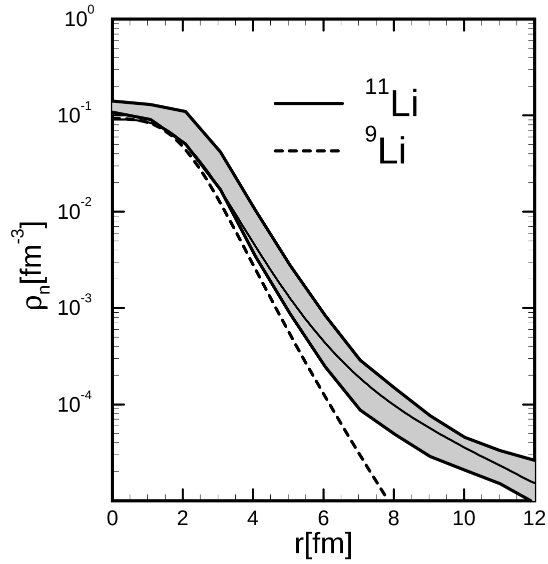

Fig. 1. The density distribution of the halo nucleus ^{11}Li as compared to that of the core ^{9}Li. The shaded area corresponds to the experimental error bars.

The microscopic structure of this halo can be understood by the fact that the Fermi level for the neutrons is very close to the continuum limit in close vicinity to the $\nu 1p_{1/2}$ below the continuum and to the $\nu 2s_{1/2}$ level in the continuum. Pairing correlations cause a partial occupation of both the $\nu 1p_{1/2}$ and the $\nu 2s_{1/2}$ level, *i.e.* a scattering of Cooper pairs into the continuum. This is in contrast to earlier calculations using Skyrme forces and relativistic mean field without pairing, where the last occupied $\nu 1p_{1/2}$ level had to be shifted artificially very close to the continuum limit by a adjustment of the potential. In contrast to these investigations the halo is not formed by two neutrons occupying the $1p_{1/2}$ level very close to the continuum limit, but is is formed by Cooper-pairs scattered mainly in the two levels $1p_{1/2}$ and $2s_{1/2}$. This is made possible by the fact that the $2s_{1/2}$ comes down close to the Fermi level in this nucleus and by the density dependent pairing interaction coupling the levels below the Fermi surface to the continuum. In contrast to the previous explanation which use the accidental coincidence that one single particle level is so close to continuum threshold so that the tail of its wave function forms a halo, this is a much more general mechanism, which could possibly be observed in other halo nuclei also. One needs only several single particle levels with small orbital angular momenta and correspondingly small centrifugal barrier close, but not directly at, the continuum limit.

In fact going along the neutron drip line there are several such regions, in particular the region where the $2p$ and the $3p$ levels come close to the continuum limit. In the first case a multi-particle halo in the region of heavy Ne-isotopes has been predicted [5]. Here we discuss in more detail the region where the $3p$ orbits are in the vicinity of the continuum limit. Here he have predicted a giant halo with up to 6 neutrons in the halo.

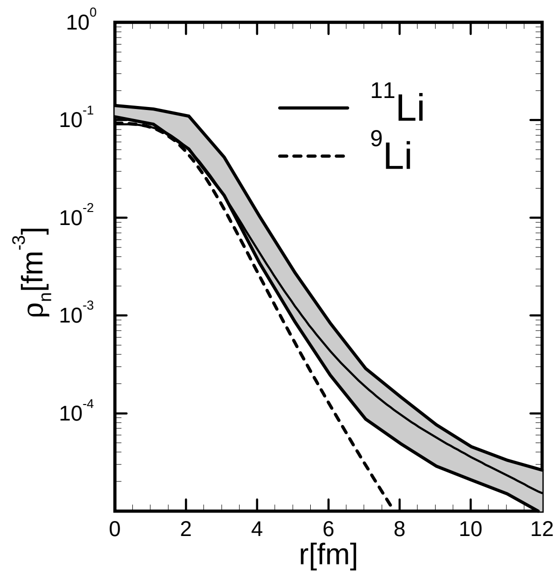

Fig. 2. Upper part: *rms*-radii for neutrons and protons in Zr isotopes close to the neutron drip line as a function of the mass number A. Lower part: single particle energies for neutrons in the canonical basis as a function of the mass number. The dashed line indicates the chemical potential.

In the upper panel of Fig. 2 we show the *rms* radii of the protons and neutrons for the Zirconium isotopes with mass numbers $A = 110$ to $A = 140$, the drip line nucleus. To guide the eye we also give a dashed line with a $N^{1/3}$-dependence. It clearly shows a kink for the neutron *rms*-radius at the magic neutron number $N = 82$.

These results can be understood more clearly by considering the microscopic structure of the underlying wave functions and the single particle energies in the canonical basis [8]. Therefore, in the lower panel of Fig. 1 we show the single particle levels in the canonical basis for the isotopes with an even neutron number as a function of the mass number. Going from $N = 70$ to $N = 100$ we observe a big gap above the $1h_{11/2}$ orbit. For neutron numbers larger than the magic number $N = 82$, the neutrons are filled to the levels in the continuum or weakly bound states in the order of $3p_{3/2}$, $2f_{7/2}$, $3p_{1/2}$, $2f_{5/2}$ and $1h_{9/2}$.

The neutron chemical potential is given in this Figure by a dashed line. It approaches rapidly the continuum already shortly after the magic neutron number $N = 82$ and it crosses the continuum at $N = 100$ for the nucleus to ^{140}Zr. In this region the chemical potential is very small and almost parallel to the continuum limit. This means that the additional neutrons are added with a very small, nearly vanishing binding energy at the edge of the continuum. The total binding energies E for the isotopes above ^{122}Zr are therefore almost identical. This has been recognized already in Ref. [12] in RMF calculations using the BCS approximation.

The kink in the neutron *rms*-radii shown in Fig. 2 can be understood by the negative parity levels close to the continuum limit. They are responsible for the rapid increase of the neutron radius. Neutrons above the closed neutron core $N = 82$ are filled into these orbits. As more and more neutron are added, $3p_{3/2}$ and $2_{f7/2}$ (after $N > 88$), $3p_{1/2}$ (after $N > 92$) respectively become weakly bound, then the contribution of further continuum $2f_{5/2}$ and $1h_{9/2}$ become more and more important. Going from $A = 122$ to $A = 140$ we observe an almost constant contribution of all the channels to the total *rms* matter radius except a sudden increase in the contribution of the $3p_{3/2}$, $2f_{7/2}$, $3p_{1/2}$ and $2_{f5/2}$ channels. This means that the giant halo in $^{124-140}$Zr are formed by the occupation of all these levels in the respective nucleus.

5. Conclusions

Summarizing we can conclude that we have to go beyond the simple relativistic mean field model in order to describe halo nuclei properly. We have to take into account pairing correlations and the coupling to the continuum in the framework of relativistic Hartree Bogoliubov theory. A density dependent force of zero range has been used in the pairing channel, whose strength is adjusted for the isotope ^{7}Li to a similar calculation with Gogny's force D1S. Good agreement with experimental values is found for the total binding energies and the radii of the isotope chain from ^{6}Li to ^{11}Li. In excellent agreement with the experiment we obtain a neutron halo for ^{11}Li without any artificial adjustment of the potential, as it was necessary in earlier calculations.

In contrast to these investigations the halo is not formed by two neutrons occupying the $1p_{1/2}$ level very close to the continuum limit, but is is formed by Cooper-pairs scattered mainly in the two two levels $1p_{1/2}$ and $2s_{1/2}$. This is made possible by the fact that the $2s_{1/2}$ comes close down close to the Fermi level in this nucleus and by the density dependent pairing interaction coupling the levels below the Fermi surface to the continuum. In contrast to the very accidental fact that one single particle level is such close to continuum threshold, that the tail of its wave function forms a halo, this is a much more general mechanism, which could possibly be observed also in other halo nuclei. One only needs several single particle levels with small orbital angular momenta and correspondingly small centrifugal barrier close, but not directly at, the to the continuum limit.

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REFERENCES

[1] P. Ring, *Progr. Part. Nucl. Phys.* **37**, 193 (1996).

[2] H. Kucharek, P. Ring, *Z. Phys.* **A339**, 23 (1991).

[3] T. Gonzalez-Llarena, J.L. Egido, G.A. Lalazissis, P. Ring, *Phys. Lett.* **B379**, 13 (1996).

[4] J. Meng, P. Ring, *Phys. Rev. Lett.* **77**, 3963 (1996).

[5] W. Pöschl, D. Vretenar, G.A. Lalazissis, P. Ring, *Phys. Rev. Lett.* **79**, 3841 (1997).

[6] Y.K. Gambhir, P. Ring, A. Thimet, *Ann. Phys. (N.Y.)* **198**, 132 (1990).

[7] J. Dobaczewski, H. Flocard, J. Treiner, *Nucl. Phys.* **A422**, 103 (1984).

[8] P. Ring, P. Schuck, *The Nuclear Many-body Problem*, Springer Verlag, Heidelberg 1980.

[9] B.D. Serot, J.D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).

[10] M.M. Sharma, G.A. Lalazissis, P. Ring, *Phys. Lett.* **B317**, 9 (1993); *Phys. Rev. Lett.* **74**, 3744 (1995).

[11] M.M. Sharma, G.A. Lalazissis, J. König, P. Ring, *Phys. Rev. Lett.* **74**, 3744 (1995).

[12] M.M. Sharma, G.A. Lalazissis, W. Hillebrandt, P. Ring, *Phys. Rev. Lett.* **72**, 1431 (1994).